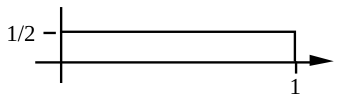
Average Score = 80/100 , Standard Deviation = 16.3, High Score = 101.

Average Scores on each problem:

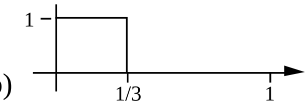
1	2	3	4	5	6	7	8	9	10	11	12	EC
6.0/7	5.8/7	3.4/5	4.2/5	7.6/8	4.5/7	7.3/8	6.1/7	4.7/6	5.2/6	13.8/16	10.7/16	.6/1

Grade Ranges: A = 100-90 (72 A's), B = 90-75 (60 B's), C = 75-55 (60 C's), D = 55- (14 D's)

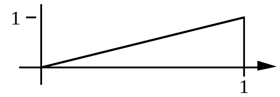
1. (7 points) Which signal has average value $= \frac{1}{2}$ AND average power $= \frac{1}{3}$? The time interval of interest is [0,1] for all signals.

(a) 

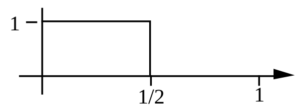
M = 1/2, P = 1/4,

(b) 

M = 1/3, P = 1/3

(c) 

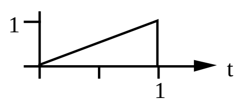
M = 1/2, P = 1/3 **correct answer**

(d) 

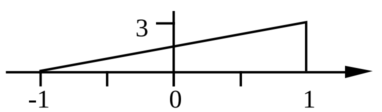
M = 1/2, P = 1/2

(e) None of these signals.

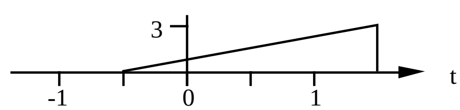
2. *(7 points)*

Let x(t) = 

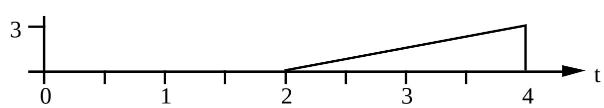
Which of the following shows $y(t) = 3\,x\left(\frac{t}{2} - 1\right)$?

(a) 

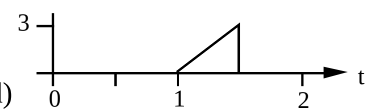
$= 3x\left(\frac{t-1}{2}\right)$,

(b) 

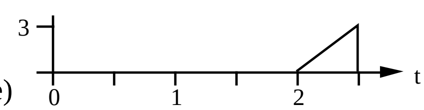
$= 3x\left(\frac{t-1/2}{2}\right)$

(c) 

$= 3\,x\left(\frac{t}{2} - 1\right)$ **correct answer**

(d) 

= 3 x(2(t-1)),

(e) 

= 3 x(2(t-2))

3. (7 points) Continuous-time signals x(t) and y(t) each have support interval [0,3] and average value 2. Their (unnormalized) correlation is C(x,y) = 1. If z = 3y+2, then C(x,z) equals

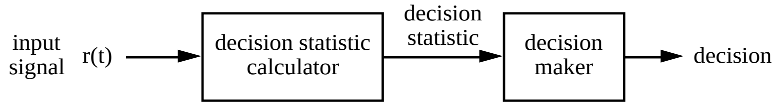
(a) 7, (b) 9, (c) 12, **(d) 15 correct answer,** (e) Not enough information given.

$$C(x,z) = \int_0^3 x(t)\,z(t)\,dt = \int_0^3 x(t)\,(3y(t)+2)\,dt = 3\int_0^3 x(t)\,y(t)\,dt + 2\int_0^3 x(t)\,dt$$

$$= 3\,C(x,y) + 6\,\frac{1}{3}\int_0^3 x(t)\,dt = 3\,C(x,y) + 6\,M(x) = 3\times 1 + 6\times 2 = 15$$

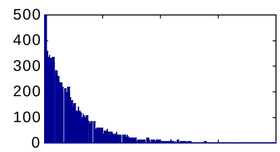
4. (5 points)

Consider the signal/no signal detection system shown below:

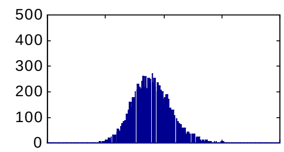


Two histograms for the decision statistic are shown below. Each histogram is based on 10,000 decision statistic measurements.

"no signal present" histogram



"signal present" histogram



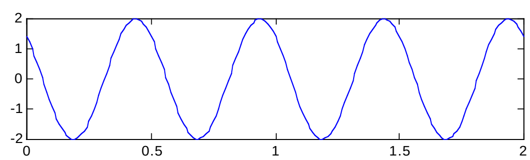
Assuming we want to minimize the number of decision errors, when the decision statistic is 5, the decision maker should output

(a) "signal is present" **(b) "no signal is present"** (c) Not enough information to make this decision.

Notice that for the value 5, the "no signal present" histogram is much larger than the "signal present" histogram. This indicates that the decision statistic value of 5 is much more likely to occur when no signal is present than when signal is present.

Alternatively, the mean of the decision statistic when no signal is present somewhere around 2. The mean of the decision statistic when signal is present is approximately 9. If we set a threshold halfway between these means, the threshold will be approximately 5.5. Since 5 < 5.5, we should decide "no signal present".

5. *(8 points)* Which signal formula describes the signal shown below



(a) $4\cos(4\pi t + \pi/2)$, (b) $2\cos(4\pi t - \pi/4)$, **(c) $2\cos(4\pi t + \pi/4)$ correct answer**

(d) $2\cos(4\pi t - \pi/2)$, (e) $2\cos(2\pi t + \pi/2)$

6. (7 points) The signal

$$x(t) = \text{Imag}\left\{-3\,e^{j(50t+\pi)} + j\,3\,e^{j(50t-\pi/2)}\right\}$$

equals which of the following signals.

(a) $6\sin(50t)$correct answer, (b) $3\sin(50t)$, (c) 0, (d) $3\sin(50t - \pi/2)$

(e) None of the above.

$$x(t) = \text{Imag}\{-3\,e^{j(50t+\pi)} + j\,3\,e^{j(50t-\pi/2)}\}$$

$$= \text{Imag}\{e^{-j\pi}\,3\,e^{j(50t+\pi)} + e^{j\pi/2}\,3\,e^{j(50t-\pi/2)}\} = \text{Imag}\{3\,e^{j\,50t} + 3\,e^{j50t}\}$$

$$= \text{Imag}\{6\,e^{j\,50t}\} = 6\sin(50t)$$

7. (8 points) What is the period of the function f(t) = cos(25) + cos(4t) + cos(6t).
Answer = (d)

$$f(t) = \cos(2t) + \cos(4t) + \cos(6t) = \cos(\frac{2}{2\pi} \times 2\pi t) + \cos(\frac{4}{2\pi} \times 2\pi t) + \cos(\frac{6}{2\pi} \times 2\pi t)$$

$$Thus, T_1 = \pi, T_2 = \pi/2, T_3 = \pi/3.$$

$$l.c.m.of\,(T_1, T_2, T_3) = \pi \Rightarrow \text{T of f(t) is } \pi. \Rightarrow Ans:(d)$$

8. (7 points) Consider the magnitude spectrum depicted below. The phase spectrum is entirely zero. What signal expression matches this spectrum.
Answer = (a)

$$[A + B\cos(w_1 t)]\cos(w_2 t) = [A + \frac{B}{2}e^{-jw_1 t} + \frac{B}{2}e^{jw_1 t}][\frac{1}{2}e^{-jw_2 t} + \frac{1}{2}e^{jw_2 t}]$$

$$= \frac{A}{2}e^{-jw_2 t} + \frac{A}{2}e^{jw_2 t} \quad \text{---------> phase } w_2 = 25,\ \frac{A}{2} = 3 \Rightarrow A = 6$$

$$+ \frac{B}{4}e^{-j(w_1+w_2)t} + \frac{B}{4}e^{j(w_1+w_2)t} \quad \text{----> phase } w_1 + w_2 = 30 \Rightarrow w_1 = 5,\ \frac{B}{4} = 1 \Rightarrow B = 4$$

$$+ \frac{B}{4}e^{-j(w_2-w_1)t} + \frac{B}{4}e^{j(w_2-w_1)t} \quad \text{-----> phase } w_2 - w_1 = 20 \Rightarrow \text{ok},\ \frac{B}{4} = 1 \Rightarrow \text{ok}.$$

Therefore, the signal is $[6 + 4\cos(5t)]\cos(25t). \Rightarrow$ Ans : (a)

9. (6 points) The Fourier series representation of a real periodic signal x(t) contains the following Fourier coefficients:

$$C_0 = 2, C_1 = 10, C_2 = 0, C_3 = ...$$

What is the avreage value of the signal x(t)?

Answer = (c)

Average (x(t)) = C_0 by definition of C_0.

$$C_0 = \frac{1}{T_0}\int_{<T_0>} x(t)dt \quad \Rightarrow \text{Average of } x(t) = 2 \Rightarrow \text{Ans : (c)}$$

10. (6 points) The Fourier series representation of a real periodic signal x(t) contains the following Fourier coefficient for the third harmonic:

$$C_{-3} = 0.5e^{-j\pi/3}$$

What is the value of Fourier coefficient C_{-3}?

Answer = (a)

$C_{-k} = C_k^*$ as proved in class.

Thus, $C_{-3} = 0.5e^{j\frac{\pi}{3}} \Rightarrow$ Ans : (c)

11. *(16 points)*

Express the following signal in standard sinusoidal form

$$\frac{2}{3}\sqrt{3}\ \cos\left(100t + \frac{\pi}{6}\right) - 2\cos\left(100\,t - \frac{\pi}{3}\right)$$

You must show your work for this problem. You will be graded not only on the correctness of your answer, but the correctness of your approach. You will obtain partial credit for partially correct answers and/or approaches.

Answer: $\mathbf{\frac{4}{3}\sqrt{3}\ \cos(100t + \frac{\pi}{2})}$

Approach 1: $\frac{2}{3}\sqrt{3}\ \cos\left(100t + \frac{\pi}{6}\right) - 2\cos\left(100\,t - \frac{\pi}{3}\right)$

corresponding phasors:

$$\frac{2}{3}\sqrt{3}\ e^{j\pi/6} - 2\,e^{-j\pi/3} = \frac{2}{3}\sqrt{3}\,(\cos \pi/6 + j \sin \pi/6) - 2\,(\cos -\pi/3 + j \sin -\pi/3)$$

$$= \frac{2}{3}\sqrt{3}\left(\frac{\sqrt{3}}{2} + j\,\frac{1}{2}\right) - 2\left(\frac{1}{2} - j\,\frac{\sqrt{3}}{2}\right)$$

$$= (1 - 1) + j\left(\frac{\sqrt{3}}{3} + \sqrt{3}\right) = j\,\frac{4}{3}\sqrt{3} = \frac{4}{3}\sqrt{3}\ e^{j\pi/2}$$

finally: $\frac{2}{3}\sqrt{3}\ \cos\left(100t + \frac{\pi}{6}\right) - 2\cos\left(100\,t - \frac{\pi}{3}\right) = \mathrm{Re}\{\frac{4}{3}\sqrt{3}\ e^{j\pi/2}\ e^{j100t}\}$

$$= \frac{4}{3}\sqrt{3}\ \cos\left(100t + \frac{\pi}{2}\right) \quad \left(= 2.31\cos(100t + 1.57) = -\frac{4}{3}\sqrt{3}\ \sin(100t)\right)$$

(the last answer is not in standard sinusoidal form)

Approach 2: $\frac{2}{3}\sqrt{3}\ \cos\left(100t + \frac{\pi}{6}\right) - 2\cos\left(100\,t - \frac{\pi}{3}\right)$

$$= \frac{2}{3}\sqrt{3}\ \cos\left(100t + \frac{\pi}{6}\right) + 2\cos\left(100\,t - \frac{\pi}{3} + \pi\right)$$

$$= \frac{2}{3}\sqrt{3}\ \cos\left(100t + \frac{\pi}{6}\right) + 2\cos\left(100\,t + \frac{2\pi}{3}\right)$$

corresponding phasors:

$$\frac{2}{3}\sqrt{3}\ e^{j\pi/6} + 2\,e^{j2\pi/3} = \frac{2}{3}\sqrt{3}\,(\cos \pi/6 + j \sin \pi/6) + 2\,(\cos 2\pi/3 + j \sin 2\pi/3)$$

$$= \frac{2}{3}\sqrt{3}\left(\frac{\sqrt{3}}{2} + j\,\frac{1}{2}\right) + 2\left(-\frac{1}{2} + j\,\frac{\sqrt{3}}{2}\right) = (1+j.577) - (1-j1.73)$$

$$= (1+j\sqrt{3}/3) - (1-j\sqrt{3}) = (1 - 1) + j\left(\frac{\sqrt{3}}{3} + \sqrt{3}\right) = j\,\frac{4}{3}\sqrt{3}$$

from here the solution is the same as in Approach 1:

Some people attempted and some succeeded at longer approaches, for example expressing each sinusoid using Euler's formulas or using trigonometric identities. These approaches are harder, longer and more prone to errors.

12. (16 points) Consider the following signal defined over $-\infty < t < \infty$, where it is assumed that $\omega_1 < \omega_2$:

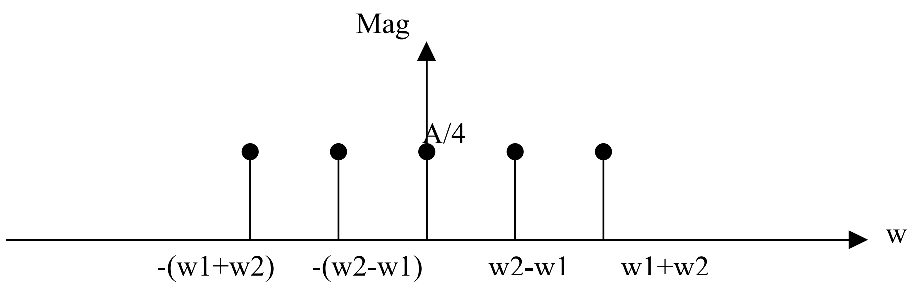
$$x(t) = A\cos(\omega_1 t)\sin(\omega_2 t)$$

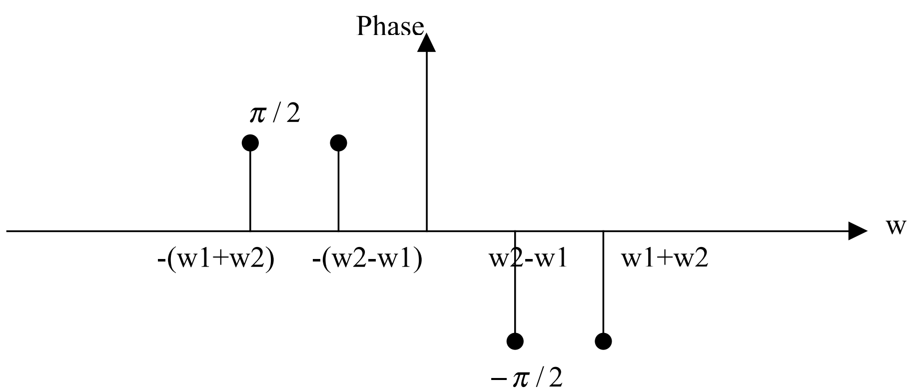
Draw its magnitude and phase spectra (2 plots), for $-\infty < \omega < \infty$. You must show your work for this problem. You will be graded not only on the correctness of your answer, but the correctness of your approach. You will obtain partial credit for partially correct answers and/or approaches.

$$x(t) = A\cos(w_1 t)\cos(w_2 t - \frac{\pi}{2}) = \frac{A}{4}(e^{-jw_1 t} + e^{jw_1 t})(e^{-jw_2 t}e^{j\frac{\pi}{2}} + e^{jw_2 t}e^{-j\frac{\pi}{2}})$$

$$= \frac{A}{4}[e^{-j(w_1+w_2)t}e^{j\frac{\pi}{2}} + e^{-j(w_2-w_1)t}e^{j\frac{\pi}{2}} + e^{j(w_2+w_1)t}e^{-j\frac{\pi}{2}} + e^{j(w_2-w_1)t}e^{-j\frac{\pi}{2}}]$$

$$= \frac{A}{4}e^{j\frac{\pi}{2}}e^{-j(w_1+w_2)t} + \frac{A}{4}e^{j\frac{\pi}{2}}e^{-j(w_2-w)t} + \frac{A}{4}e^{-j\frac{\pi}{2}}e^{j(w_1+w_2)t} + \frac{A}{4}e^{-j\frac{\pi}{2}}e^{j(w_2-w)t}$$





Extra Credit (1 point) How many bits are used per sample when music is encoded on a compact disc?
(a) 8 **(b) 16 correct answer** (c) 32

Each sample, whether it comes from the left or the right channel, is encoded with 16 bits.