Midterm 1 Solutions **EECS 206** **March 27, 2000**

Average Score = 67.4 , Standard Deviation = 18.9, High Score = 101.

Average Scores for each problem:

1	2	3	4	5	6	7	8	9	10	11	12	EC
4.7/5	3.9/5	4.9/7	3.8/7	3.3/8	3.7/7	4.5/7	2.5/7	6.3/7	6.4/8	10.4/14	12.3/18	0.5/1

Grade Ranges: A = 100-84 (43 A's), B = 83-66 (73 B's), C = 65-46 (58 C's), D = 45- (28 D's)

$\boxed{\#1}$

$$x[n] = x(nT_s) = \cos\left(2\pi \cdot 300 \cdot n \cdot \frac{1}{800}\right)$$

$$= \cos\left(\frac{3\pi}{4} n\right) \qquad \therefore \boxed{\hat{\omega} = \frac{3\pi}{4} \quad (c)}$$

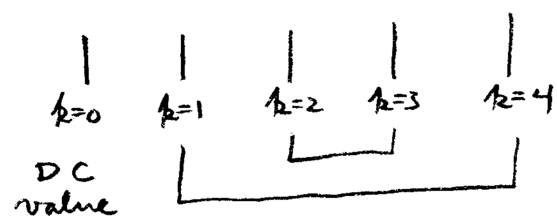
$\boxed{\#2}$ From #1, $x[n] = \cos\left(\frac{3\pi}{4} n\right)$, $\hat{\omega} = \frac{3\pi}{4}$

DT periodicity: $\hat{\omega} \cdot N_0 = 2\pi m$ for smallest integer m

$$\frac{3\pi}{4} \cdot N_0 = 2\pi m \Rightarrow 3\pi N_0 = 8\pi m$$

Pick $m = 3$, $N_0 = 8$ $\qquad \therefore \boxed{N_0 = 8 \quad (b)}$

$\boxed{\#3}$ 5-point DFT:



$$X_4 = X_1^*$$

$$X_3 = X_2^* \qquad \therefore \boxed{X_3 = 4e^{j\pi/4}, \; X_4 = 0.5e^{-j\pi/3} \quad (d)}$$

$\boxed{\#4}$ $\hat{x}[n] = 8 + 2\cos\left(\frac{\pi}{2} n - \pi/2\right)$

$$= 8 + e^{-j\pi/2} e^{j\frac{\pi}{2} n} + e^{+j\pi/2} e^{-j\frac{\pi}{2} n}$$

from neg. to pos. freq.

$$\frac{\pi}{2} n = \frac{2\pi}{8} \cdot k \cdot n \Rightarrow k = 2$$

$$x[n] = \underbrace{8}_{k=0} + \underbrace{e^{-j\pi/2} \cdot e^{j\left(\frac{2\pi}{8}\right) \cdot 2 \cdot n}}_{k=2} + \underbrace{e^{+j\pi/2} e^{j\left(\frac{2\pi}{8}\right)(8-2)n}}_{k=6}$$

$\boxed{\text{Answer: (c)}}$

$\boxed{\#5}$

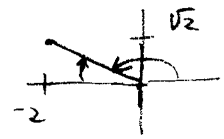
$k=6$, $k = 22-6=16$

$X[6] = (-2+j\sqrt{2})$ $\qquad$ $X[16] = X[6]^* = -2-j\sqrt{2}$

These 2 DFT coefficients come from :

$$2\left|(-2+j\sqrt{2})\right| \cos\left(\frac{2\pi}{22}\cdot 6\cdot n + \angle(-2+j\sqrt{2})\right)$$

$$= 2\sqrt{6}\cdot \cos\left(\frac{6\pi}{11} n + \pi - \underbrace{\tan^{-1}\left(\frac{\sqrt{2}}{2}\right)}_{\neq \frac{\pi}{4}}\right)$$



$k=9$, $k = 22-9 = 13$

$X[9] = 8$ $\qquad$ $X[13] = X[9]^* = 8$

These 2 DFT coefficients come from:

$$2\cdot|8|\cdot\cos\left(\frac{2\pi}{22}\cdot 9\cdot n + \angle(8+0j)\right)$$

$$= 16\cos\left(\frac{9\pi}{11} n\right)$$

None of the answers in (a), (b), (c) match.

$\boxed{\text{Answer: (e)}}$

$\boxed{\#6}$ Since the two phases are non-zero and identical, only case (1) of aliasing seen in class can lead to identical sampled sequences.

(Recall that folding requires $\phi_1 = -\phi_2$.)

$$\therefore \quad f_1 = f_2 + l \cdot f_s \qquad l: \text{integer}$$

$$5500 = 500 + l \cdot f_s$$

The only answer that works is $f_s = 2500$, requiring $l = 2$

$$\boxed{f_s = 2500 \qquad (a)}$$

$\boxed{\#7}$ What is the largest frequency?

$x(t)$ is of the form studied in Amplitude Modulation

$$x(t) = \left[100 + 50 \sin\left(2\pi \cdot (1000)t\right)\right] \cdot \cos\left(2\pi \cdot (5000)t + \frac{\pi}{4}\right)$$

The largest frequency in the Spectrum of $x(t)$ is

$$1000 + 5000 = 6000 \text{ Hz}$$

$$\Rightarrow f_s > 12000 \text{ Samples per Sec.}$$

$\boxed{\text{Answer: (c)}}$

8. *(7 points)* Which of the following describes a system that is linear time-invariant and causal?

(a) $y[n] = \frac{1}{3}x[n-1] + \frac{1}{3}x[n] + \frac{1}{3}x[n+1]$ *not causal*

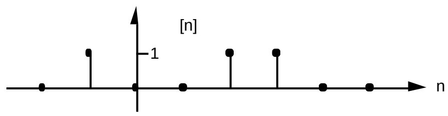
(b) $y[n] = \dfrac{3}{x[n-1]+2x[n]}$ *not linear*

(c) $y[n] = 3\,x[2n]$ *not time-invariant, not causal*

(d) $y[n] = 4$ *not linear*

(e) none of the above ***correct answer***

9. *(7 points)* If the impulse response h[n] of a filter is shown below



then the difference equation is

(a) $y[n] = x[n-1] + x[n+2] + x[n+3]$

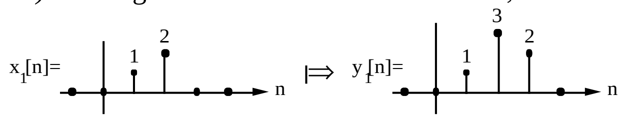
(b) $y[n] = x[n+1] + x[n-2] + x[n-3]$ ***correct***

(c) $y[n] = x[n-3] + x[n-2] + x[n-1]$

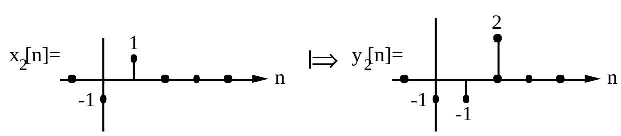
(d) $y[n] = x[n+1] + x[n+2] + x[n+3]$

(e) None of the above

10. *(7 points)* For a given linear time-invariant filter, we are told that



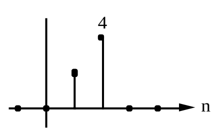
and



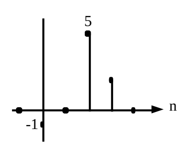
The response to $x_1[n+1] + x_2[n]$ is

(a)

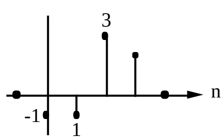
correct



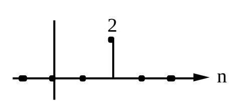
(b)



(c)



(d)



(e) None of the above

$\boxed{\# 11}$

$x(t)$: continuous-time signal.

$T_o = 3 \quad \Rightarrow \quad \omega_o = \frac{2\pi}{T_o} = 2\pi/3$

(1) $C_o = \frac{1}{T_o} \int_{<T_o>} x(t)\,dt = \text{Average of } x(t)$

$= \frac{1}{3} \left[\int_0^1 2\,dt + \int_1^2 1 \cdot dt + \int_2^3 0 \cdot dt \right] = \frac{1}{3} \cdot 3 = 1$

$\boxed{C_o = 1}$

(2) $C_k = \frac{1}{T_o} \int_{<T_o>} x(t)\, e^{-jk\omega_o t}\, dt$

$C_3 = \frac{1}{3} \int_{<T_o>} x(t)\, e^{-j\, 3 \cdot \frac{2\pi}{3} \cdot t}\, dt$

$= \frac{1}{3} \left[\int_0^1 2 e^{-j2\pi t}\, dt + \int_1^2 1 \cdot e^{-j2\pi t}\, dt \right]$

$= \frac{1}{3} \left[\left(\frac{-2}{j2\pi}\right) e^{-j2\pi t} \Big|_0^1 + \left(\frac{-1}{j2\pi}\right) e^{-j2\pi t} \Big|_1^2 \right]$

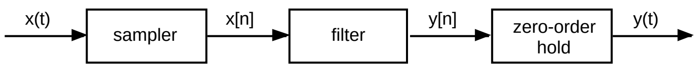
$= 0$ since $e^{-j2\pi} = e^{-j4\pi} = 1$.

$\boxed{C_3 = 0}$

12. *(18 points)* The continuous-time signal

$$x(t) = \begin{cases} 3t, & 0 \le t \le 5 \\ 0, & \text{else} \end{cases}$$

is the input to the system shown below, which consists of a sampler that samples at $f_s = 1/2$ sample/sec, a discrete-time filter with coefficients $b_0 = 2$, $b_1 = 1$, and a zero-order hold interpolator.



Find and accurately plot the signal $y(t)$. Make sure that the key points of your plot are labelled.

You must show your work. You will be graded not only on the correctness of your answer, but also on the correctness of your approach. You will obtain partial credit for partially correct answers and/or approaches.

Solution:

sampling rate $f_s = 1/2$, which implies sampling interval $T_s = 2$ sec

sampled signal: $x[n] = x(nT_s) = x(2n) = 6n, \; 0 \le 2n \le 5$, i.e. $0 \le n \le 2$.

Therefore,

$\ldots, \; x[0] = 0, \; x[1] = 6, \; x[2] = 12, x[3] = 0, \ldots$

the filtered output

$y[n] = 2x[n] + x[n-1]$

$\ldots, \; y[0] = 0, \; y[1] = 12, \; y[2] = 30, y[3] = 12, \; y[4] = 0, \; \ldots$

the zero-order hold produces

$$y(t) = \sum_n y[n] \; p(t-nT_s) \; , \quad \text{where} \quad p(t) = \begin{cases} 1, & -T_s/2 \le t \le T_s/2 \\ 0, & \text{else} \end{cases}$$