EECS 206 – Winter 2002

Homework #6 – Assigned Feb. 15 – Due Friday Feb. 22

Relevant Lectures: 2/11, 2/13, 2/15
Relevant Reading: 4.1 (up to the end of 4.1.1), 9.1.1, 9.3.1, 9.3.2, Section 3 in G. Wakefield’s “Primer on DFT”, Section 4.2.3 in Lab. 4 handout.
Relevant Items in the DSP First CD: None this week.
Homework Submission Policies: Same as before (see course Web page).

1. Calculate the N-point DFTs of the following signals:
   
   (a) \( s[n] = 1 \) for \( n = 0 \), \( s[n] = 0 \) for \( 1 \leq n \leq N - 1 \)
   
   (b) \( s[n] = 1 \) for \( 0 \leq n \leq N - 1 \)
   
   (c) \( s[n] = \sin(2\pi mn/N) \) for \( 0 \leq n \leq N - 1 \).

   *Hint: See your lecture notes for 2/15!*

2. Suppose that the DFT of a periodic signal \( x[n] \) is \( X[0] = 1 \) and \( X[k] = 0 \) for all other values of \( k \). What is \( x[n] \)?

3. Suppose that we have a real signal \( x[n] \), with fundamental period \( N_0 \), whose DFT coefficients are all equal to zero except \( X[k] \) and \( X[N_0 - k] \). Let \( X[k] = Ae^{j\phi} \).
   
   (a) What is the average value of \( x[n] \)?
   
   (b) Find \( x[n] \).

   *Hint: Find \( X[N_0 - k] \) first.*
   
   (c) What is the average power of \( x[n] \)?

4. Consider the synthesis equation of the DFT:
   
   \[ x[n] = \sum_{k=0}^{N_0-1} X[k] e^{j2\pi k n / N_0}. \]

   Use that equation to show that \( x[n - N_0] = x[n] \).

5. Prove the time-shift DFT property:
   
   \[ y[n] = x[n - n_1] \implies Y[k] = e^{-j\frac{2\pi kn}{N_0}} X[k] \]

6. Calculate the 4-point DFTs of the following 4-point signals:
   
   (a) \( x[n] = \cos(\pi n/2) \), \( n = 0, 1, 2, 3 \)
   
   (b) \( x[n] = 2^n \), \( n = 0, 1, 2, 3 \)
7. Consider the discrete-time square wave of fundamental period $N_0$, where it is assumed that $N_0$ is even:

$$x[n] = \begin{cases} 
1 & 0 \leq n \leq \frac{N_0}{2} - 1 \\
0 & \frac{N_0}{2} \leq n \leq N_0 - 1
\end{cases}$$

(a) Calculate $X[0]$

(b) Verify that for $k \neq 0$,

$$X[k] = \left( \frac{1}{N_0} \right) e^{-j\frac{2\pi k}{N_0} \left( \frac{N_0}{2} - 1 \right)} \frac{\sin \left( \frac{2\pi k}{N_0} \frac{N_0}{2} \right)}{\sin \left( \frac{2\pi k}{N_0} \right)}$$

Hint: After writing the DFT-analysis equation, use the fact below about geometric series and then factor the first term in the given expression for $X[k]$; the sin terms should “appear”...

$$\sum_{n=0}^{N-1} \alpha^n = \frac{1 - \alpha^N}{1 - \alpha} \quad \text{whenever } \alpha \neq 1$$