EECS 206 – Winter 2002

Homework #6 – Assigned Feb. 15 – Due Friday Feb. 22

Relevant Lectures: 2/11, 2/13, 2/15
Relevant Reading: 4.1 (up to the end of 4.1.1), 9.1.1, 9.3.1, 9.3.2, Section 3 in G. Wakefield's "Primer on DFT", Section 4.2.3 in Lab. 4 handout.
Relevant Items in the DSP First CD: None this week.
Homework Submission Policies: Same as before (see course Web page).

1. Calculate the N-point DFTs of the following signals:

   (a) $s[n] = 1$ for $n = 0$, $s[n] = 0$ for $1 \leq n \leq N-1$

   (b) $s[n] = 1$ for $0 \leq n \leq N-1$

   (c) $s[n] = \sin(2\pi mn/N)$ for $0 \leq n \leq N-1$.
   Hint: See your lecture notes for 2/15!

2. Suppose that the DFT of a periodic signal $x[n]$ is $X[0] = 1$ and $X[k] = 0$ for all other values of k. What is $x[n]$?

3. Suppose that we have a real signal $x[n]$, with fundamental period N_0, whose DFT coefficients are all equal to zero except $X[k]$ and $X[N_0 - k]$. Let $X[k] = Ae^{j\phi}$.

   (a) What is the average value of $x[n]$?

   (b) Find $x[n]$.
   Hint: Find $X[N_0 - k]$ first.

   (c) What is the average power of $x[n]$?

4. Consider the synthesis equation of the DFT:

$$x[n] = \sum_{k=0}^{N_0-1} X[k] e^{j\frac{2\pi}{N_0}kn} .$$

   Use that equation to show that $x[n - N_0] = x[n]$.

5. Prove the time-shift DFT property:

$$y[n] = x[n - n_1] \Longrightarrow Y[k] = e^{-j\frac{2\pi}{N_0}kn_1} X[k]$$

6. Calculate the 4-point DFTs of the following 4-point signals:

   (a) $x[n] = \cos(\pi n/2)$, $n = 0, 1, 2, 3$

   (b) $x[n] = 2^n$, $n = 0, 1, 2, 3$

7. Consider the discrete-time square wave of fundamental period N_0, where it is assumed that N_0 is even:

$$x[n] = \begin{cases} 1 & 0 \leq n \leq \frac{N_0}{2} - 1 \\ 0 & \frac{N_0}{2} \leq n \leq N_0 - 1 \end{cases}$$

   (a) Calculate $X[0]$

   (b) Verify that for $k \neq 0$,

$$X[k] = (\frac{1}{N_0}) e^{-j\frac{2\pi}{N_0}k\frac{1}{2}(\frac{N_0}{2}-1)} \frac{\sin(\frac{2\pi}{N_0}k\frac{N_0}{4})}{\sin(\frac{\pi k}{N_0})}$$

   Hint: After writing the DFT-analysis equation, use the fact below about geometric series and then factor the first term in the given expression for $X[k]$*; the* sin *terms should "appear"...*

$$\sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha} \quad \text{whenever } \alpha \neq 1$$