Consider the circuit shown below which contains a voltage controlled current source.

\[ \frac{\sqrt{2}}{\cos(10t + 45^\circ)} \text{ volts} \]

\[ V = \sqrt{2} \angle 45^\circ \]

Find the voltage at node A.

\[ V_A(t) = \cos(10t) \text{ volts} \]

\[ \frac{V - V_A}{1} + (0 - V_A)^3 + 2(V - V_A) = 0 \]

\[ V_A = \frac{V}{1 + j} = \frac{V}{\sqrt{2}} \angle 45^\circ \]

\[ = 1 \angle 0^\circ \]
(20 points)

Problem #2

When an arbitrary voltage $V$ volts (at $\omega=100$ rad/s) is applied to the two terminal circuit shown below a current $I$ amperes flows.

![Two terminal circuit diagram]

It is experimentally found that the relationship between $V$ and $I$ is given by

$$V = (5 + j5) + I(10 + j5)$$

Draw an equivalent two terminal circuit which contains only one independent voltage source ($V_s$), one resistor ($R$) and one inductor ($L$). Specify the values of $V_s$, $R$ and $L$.

$$V_{oc} = (5+j5)$$

$$I_{sc} = \frac{-5+j5}{10+j5}$$

$$Z_T = \frac{V_{oc}}{I_{sc}} = 10+j5$$

![Equivalent circuit diagram]
The frequency transfer function, $H(\omega)$, of a filter is given by the Bode plots shown on the next page.

(a) What kind of filter is this?

(b) If the signal
$$x(t) = 0.1\cos(100t+5^\circ) + 0.1\cos(3100t+53^\circ) + 0.1585\cos(8\times10^5t+47^\circ)$$
is input to this filter, then estimate the filter output $y(t)$ using the Bode plot.

$$y(t) = \cos(100t) + \cos(3100t) - \sin(8\times10^5t)$$

- $|H(100)| = 20 \text{ dB} \ (10)$, $\angle H(100) = -5^\circ \ (5.08^\circ)$
- $|H(3100)| = 10 \text{ dB} \ (3.16)$, $\angle H(3100) = -53^\circ \ (53.29^\circ)$
- $|H(8\times10^5)| = 16 \text{ dB} \ (6.31)$, $\angle H(8\times10^5) = 43^\circ \ (43.57^\circ)$

(c) Find the system transfer function, $H(s)$, from the asymptotic Bode plot.

$$H(s) = \frac{10 \left(1 + \frac{s}{10^4}\right) \left(1 + \frac{s}{10^5}\right)}{\left(1 + \frac{s}{10^3}\right) \left(1 + \frac{s}{10^6}\right)}$$
Bode Plots for Problem # 3

\[ |H(\omega)| \text{(dB)} \]

\[ \omega \text{ (rad/s)} \]

\[ \theta \text{ (degrees)} \]

\[ \omega \text{ (rad/s)} \]

7
(20 points)
Problem #4

Consider the Op-Amp circuit shown below.

(a) Write a node equation at node A.

\[ \frac{V_A - V_i}{R} + \left( \frac{V_A - V_o}{R} \right) CS + \left( \frac{V_A - V_o}{R} \right) = 0 \]  

or \( (2 + RC) V_A = V_i + (1 + RC) V_o \)  

(b) Express the node voltage at node A in terms of \( V_o \) by first computing \( I_1 \).

\[ I_1 = V_o SC \]  

\[ V_A - V_o = R I_1 = V_o RC S \]  

\[ \therefore V_A = (1 + RC) V_o \]  

(c) Combine your results of parts (a) and (b) above to show that the system transfer function is given by

\[ H(s) = \frac{1}{as^2 + bs + 1} = \frac{1}{(1 + RC)^2} \]

Find the values of \( a \) and \( b \) in terms of \( R \) and \( C \).

\[ a = (RC)^2 \]

\[ b = 2RC \]
(d) For $R=1\,\Omega$ and $C=1\,\text{F}$, $H(s)$ is given by

$$H(s) = \frac{1}{(1+s)^2}$$

Sketch the asymptotic Bode plot (magnitude only) of this $H(s)$ on page 12. What type of filter is this?

Low Pass

1 pt.

(e) When $R=1\,\Omega$ and $C=1\,\text{F}$, the 3 dB cut-off frequency $\omega_c$ (i.e., $|H(\omega_c)| = 0.707|H(0)|$) is 0.644 rad/s. Frequency and impedance scaling are used to transform this circuit into a new low-pass filter with $C=10\,\text{nF}$ and 3 dB cut-off frequency 6440 rad/s. Find the new value for $R$.

$$R = 10\,\text{k}\Omega$$

Work Area For Problem # 4

(continued on page 13)
Asymptotic Bode Plot (Magnitude) for Problem #4
Consider the circuit shown below.

\[ V = 1.5 \sqrt{2} \angle 45^\circ \text{ V} \]
\[ \omega = 2 \text{ rad/s} \]
\[ 1.5 \text{ } \Omega \]
\[ 1 \text{ H} \]
\[ 1 \text{ F} \]

8 pts
(a). Compute the power factor seen by the source. Specify whether it is leading or lagging.

\[ Z = 1.5 + j \cdot 2 - 0.5 j = 1.5 (1 + j) \]
\[ \angle pf = \tan^{-1} \left( \frac{-1.5}{1.5} \right) = -45^\circ \]
\[ \therefore \text{ pf} = 0.707 \text{ lagging} \]

4 pts
(b). Find the (i) apparent power, (ii) complex power, and (iii) real power delivered by the source. Don’t forget to include the units.

\[ I = \frac{V}{Z} = \frac{1.5 \sqrt{2} \angle 45^\circ}{1.5 \sqrt{2} \angle 45^\circ} = 1 \angle 0^\circ \text{ A} \]
\[ \text{apparent power} = \frac{1}{2} |I||V| = \frac{1}{2} (1)(1.5 \sqrt{2}) = 1.5 \sqrt{2} \text{ VA} \]
\[ \text{complex power} = S = \frac{1}{2} V I^{*} = \frac{1}{2} (1.5 \sqrt{2} \angle 45^\circ) 1 = 0.75 \sqrt{2} \angle 45^\circ \]
\[ \text{real power} = 0.75 \text{ W} \]

8 pts
(c). A second load is connected in parallel with the series RLC combination. This load dissipates 1.25 W and has a power factor of 0.9806 lagging. Compute the power factor seen by the source with these two loads connected in parallel. Specify whether it is leading or lagging.

\[ S_{2} = 1.25 + j \cdot Q_{2} \]
\[ \angle pf_{2} = \cos^{-1} [0.9806] = -11.30^\circ \]
\[ = \tan^{-1} [-Q_{2}/1.25] \]
\[ Q_{2} = 0.25 \text{ VAR} \]
\[ S_{\text{tot}} = S_{1} + S_{2} = P_{\text{tot}} + j \cdot Q_{\text{tot}} \]
\[ = (0.75 + j \cdot 0.75) + (1.25 + j \cdot 0.25) \]
\[ = 2 + j \cdot 1 \text{ VA} \Rightarrow \angle pf_{\text{tot}} = \tan^{-1} (1/2) = -26.57^\circ \]
\[ p_{f_{\text{tot}}} = 0.894 \text{ lagging} \]