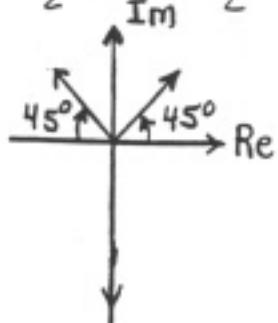


- 10 (a) $A \cos(10t + \theta) = 5 \cos(10t + 45^\circ) + 5 \cos(10t + 135^\circ) + 10(2)^{1/2} \sin(10t)$
Find A.

$$A =$$

$$5\sqrt{2} = 7.07$$

$$\begin{aligned} A \angle \theta &= 5 \angle 45^\circ + 5 \angle 135^\circ + 10\sqrt{2} \angle -90^\circ \\ &= \left(5 \frac{\sqrt{2}}{2} + j \frac{5\sqrt{2}}{2}\right) + \left(-5\frac{\sqrt{2}}{2} + j \frac{5\sqrt{2}}{2}\right) - 10\sqrt{2}j \\ &= -5\sqrt{2}j \end{aligned}$$



Convert to phasor form
& then add phasors.

Find magnitude of resultant phasor.

10 (b)

$$x + jy = \frac{(4.33 + j 2.5)(1.414 + j 1.414)(-2.121 + j 2.121)}{(2.5 - j 4.33)(3 - j 5.196)}$$

Find x and y.

$$\begin{aligned} &= \frac{(5 \angle 30^\circ)(2 \angle 45^\circ)(3 \angle 135^\circ)}{(5 \angle 300^\circ)(6 \angle 300^\circ)} = 1 \angle -30^\circ \\ &= .866 - j .5 \end{aligned}$$

$$x =$$

$$.866$$

$$y =$$

$$-.5$$

Do computation
using polar
form.

5 (c) $x + jy = (1 + j.176325)^{30}$ Find x and y.

$$z = (1 + j.176325)$$

$$= 1.01543 \angle 10^\circ$$

$$x =$$

$$0.7915$$

$$y =$$

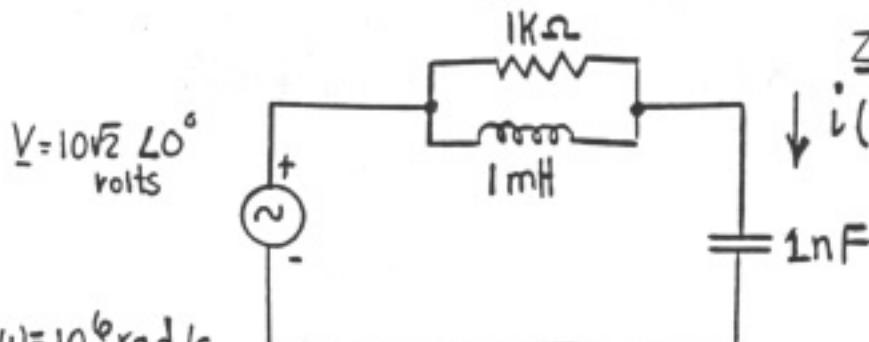
$$-1.371$$

$$\begin{aligned} z^{30} &= (1.01543)^{30} \angle 300^\circ \\ &= 1.583 \angle 300^\circ \\ &= 0.7915 - j 1.371 \end{aligned}$$

(25 points)

Problem # 2

Consider the following circuit.



$$\omega = 10^6 \text{ rad/s}$$

(a) Find $i(t)$.

$$i(t) = \boxed{20 \cos(10^6 t + 45^\circ) \text{ mA}}$$

$$1k\Omega \parallel 1mH = 500 \angle j 500^\circ$$

$\underline{Z} = (1nF)$ series with

$$1k\Omega \parallel 1mH = -j1000 + 500 + j500 \\ 500 - j500 \Omega$$

$$\underline{I} = \underline{V} / \underline{Z} = 20 \angle 45^\circ \text{ mA}$$

↑
impedance seen
by source

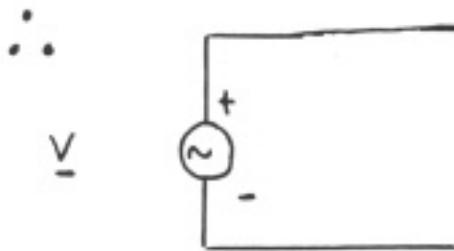
(b) Find the average power supplied by the voltage source.

$$P_{av} = \boxed{10 \text{ mW}}$$

$$P_{av} = \frac{1}{2} \operatorname{Re} [\underline{V} \underline{I}^*] \\ = \frac{1}{2} (10\sqrt{2}V) (20 \text{ mA}) \cos(-45^\circ) \\ = 100 \text{ mW}$$

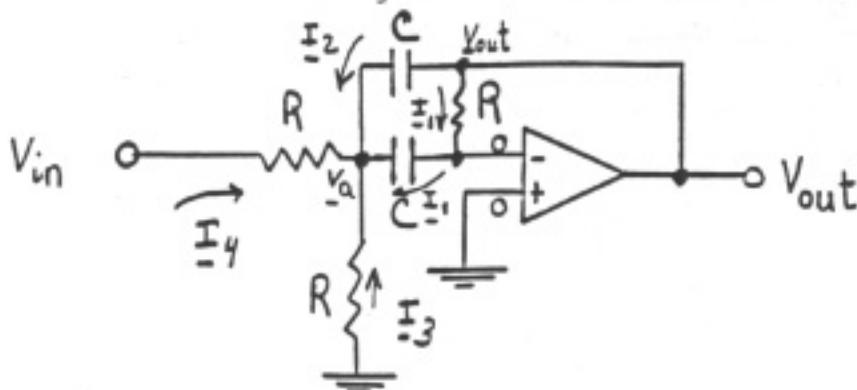
(c) Sketch an equivalent circuit when $\omega \rightarrow 0$.

When $\omega \rightarrow 0$, capacitor is open & inductor is short



(25 points)
Problem # 3

Consider the Op-Amp circuit shown below.



$$V_{in}(t) = \sqrt{5} \cos(10^6 t + \pi/3) \text{ Volts}$$

$$R = 1 \Omega$$

$$C = 1 \mu\text{F}$$

Find $v_{out}(t)$.

$$v_{out}(t) = \boxed{\cos(10^6 t + 266.565^\circ)}$$

Volts

$$\underline{I}_1 = \underline{V}_{out}/R = \underline{V}_{out} \quad (1)$$

$$\underline{V}_a = - \underline{I}_1 \left(\frac{1}{j\omega C} \right) = - \underline{I}_1 \left(\frac{1}{j} \right) = j \underline{I}_1 \quad (2)$$

$$\therefore \boxed{\underline{V}_a = j \underline{V}_{out}} \quad (3)$$

Combine Eqs. (5) & (3)

$$\underline{KCL \text{ @ node } a} \quad \underline{I}_4 + \underline{I}_2 + \underline{I}_1 + \underline{I}_3 = 0 \quad (4)$$

$$\therefore (\underline{V}_{in} - \underline{V}_a)/R + (\underline{V}_{out} - \underline{V}_a)j\omega C + \underline{V}_{out}/R - \underline{V}_a/R = 0 \quad (5)$$

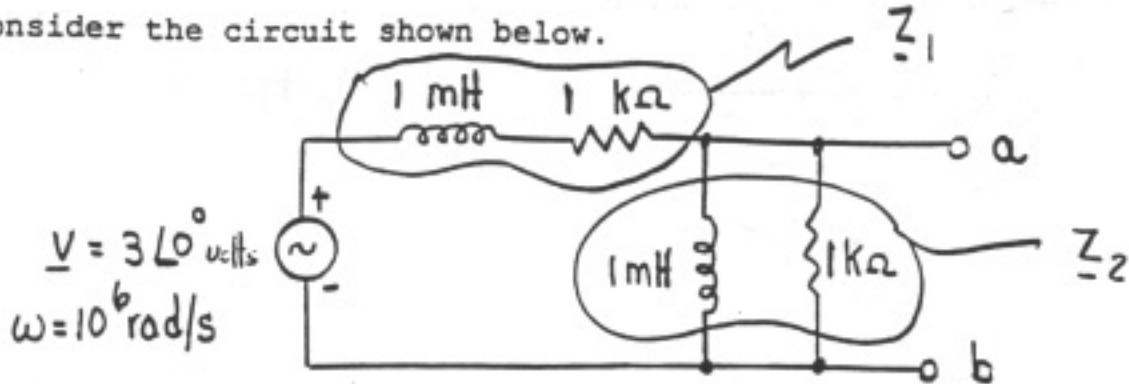
$$\rightarrow (\underline{V}_{in} - j\underline{V}_{out}) + j(\underline{V}_{out} - j\underline{V}_{out}) + \underline{V}_{out} - j\underline{V}_{out} = 0 \quad (6)$$

$$\Rightarrow \underline{V}_{out} = \frac{1}{-2+j} \underline{V}_{in} = \left(\frac{1}{\sqrt{5}} \angle 206.565^\circ \right) \underline{V}_{in}$$

$$\therefore v_{out}(t) = \cos(10^6 t + 266.565^\circ)$$

(25 points)
Problem # 4

Consider the circuit shown below.



10 (a) Find the Thevenin equivalent voltage v_T .

$$v_T = \boxed{1\angle 0^\circ \text{ Volts}}$$

$$\underline{Z}_2 = 1\text{mH} \parallel 1\text{k}\Omega = 500 + j500 \text{ }\Omega$$

$$\begin{aligned}\underline{Z}_1 &= 1\text{mH} \text{ series } 1\text{k}\Omega \\ &= 1000 + j1000\end{aligned}$$

$$\text{Voltage divider} \Rightarrow v_{oc} = \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} \underline{V} = \frac{1}{3} \underline{V} = 1\angle 0^\circ$$

10 (b) Find the Thevenin equivalent impedance \underline{z}_T .

$$\underline{z}_T = \boxed{1000/3 + j1000/3 \Omega}$$

$$= 477.3 \angle 45^\circ$$

$$\underline{z}_T = \underline{z}_2 \parallel \underline{z}_1 = \frac{\underline{z}_1 \underline{z}_2}{\underline{z}_1 + \underline{z}_2} = \frac{2}{3} \underline{z}_2$$

(turn-off the independent voltage source)

↙ i.e., short circuit it

5(c) A load impedance $\underline{z}_L = 500 + j1500 \Omega$ is connected across terminals a and b. Find the power, P_L , delivered to the load.

$$P_L = \boxed{61.64 \mu W}$$

$$\underline{I}_L = \frac{\underline{V}_T}{\underline{z}_T + \underline{z}_L} = \frac{1}{833.3 + j1833.3}$$

$$P_L = \frac{1}{2} |\underline{I}_L|^2 R_L = \frac{1}{2} (2.466 \times 10^{-7}) 500$$