1.b. (5 pts) Determine the power absorbed by the 5Ω resistor in the following circuit.

\[ P = I^2 R = (10)^2 \cdot 5 = 500 \text{ W} \]

\[ P_2 = 500 \text{ W} \] (Power absorbed by 5Ω resistor)
Problem: 1 (20 pts)

1.a. (5 pts) Determine the equivalent resistance between terminals $a$ and $b$ for the following circuit.

\[ R_{eq} = \frac{4}{3} \ \Omega \]
1.c. (5 pts) State whether the following circuits represent valid or invalid interconnections of sources.

\[ \begin{align*}
\text{Circuit A} & \\
5V & \quad \text{Source} \\
\text{2A} & \\
2V & \quad \text{Source}
\end{align*} \]

\[ \begin{align*}
\text{Circuit B} & \\
4V & \quad \text{Dependent current source} \\
2\Omega & \\
1A & \quad \text{Source}
\end{align*} \]

\textbf{Solu:} currents and voltages throughout circuit are consistent. Hence valid.

\textbf{Solu:} Dependent current source is in series with independent current source so the currents need be the same but of opposite sign. However, using 1A source as reference, \( V = -2V \) so that \( 4V = -8A \) using 1A source as reference.

Circuit A is \textbf{Valid} (Valid/Invalid), Circuit B is \textbf{Invalid} (Valid/Invalid)
1.d. (5 pts) For the circuit below, find the voltage $v_{10\Omega}$ across the 10Ω resistor.

\[ \text{Soln: } 1) \text{ As 2Ω resistor is in parallel with 10V source, } \]
\[ i = \frac{10}{2} = 5 \text{A.} \]
\[ 2) \text{ Thus dependent voltage source generates } 3i = 15V \]
\[ \text{and applying KVL around outer loop: } -10 - 15 + v_{10\Omega} = 0 \text{ or } v_{10\Omega} = 25V \]

\[ v_{10\Omega} = 25V \]
Problem: 2 (20 pts)  
Consider the following circuit:

2.a (5 pts): Label the essential nodes on the circuit diagram and choose a reference node.  
(A, B, C as indicated plus reference node at bottom)  

2.b (10pts) Write down the node equations and simplify.

Solu: Apply KCL to nodes A, B, C:

- **A:** \(15 - \frac{\mathcal{V}_A}{5} - \frac{\mathcal{V}_A - \mathcal{V}_C}{10} = 0\) \[\Rightarrow 3\mathcal{V}_A - \mathcal{V}_C = 150\]
- **B:** \(-15 - 15 - \frac{\mathcal{V}_B}{50} = 0\) \[\Rightarrow \mathcal{V}_B = -50 - 30 = -1500\]  
- **C:** \(15 - \frac{\mathcal{V}_C}{10} - \frac{\mathcal{V}_C - \mathcal{V}_A}{10} = 0\) \[\Rightarrow \mathcal{V}_A - 2\mathcal{V}_C = -150\]

2.d (5 pts) Find the current \(i\) going through the 10\(\Omega\) resistor on top of the circuit diagram.

Solu: Multiply equ. A by 2 and subtract the equ. C:

\[
6\mathcal{V}_A - 2\mathcal{V}_C = 300 \\
- \frac{\mathcal{V}_A - 2\mathcal{V}_C}{5} = -150 \\
5\mathcal{V}_A = 450 \Rightarrow \mathcal{V}_A = 90\text{V}, \mathcal{V}_C = 120\text{V}
\]

Thus: \(i = \frac{\mathcal{V}_C - \mathcal{V}_A}{10} = 3\text{A}\)

\[i = 3\text{A}\]
Problem: 3 (20 pts) Mesh analysis problem

Consider the following circuit:

3.a (5 pts) Label the mesh currents. Are there any supermeshes? Yes

3.b (10 pts) Write down mesh equations and simplify.

\[ KVL \text{ around outer loop (supermesh)} \]

\[-2v_x + 2i_a + 2i_b + 16 + 2i_b + 2i_a = 0 \]

\[ v_x = 2i_b \]

\[ i_b - i_a = 0 \]

3.c (5 pts) Solve mesh equations and find voltage \( v \)

\[-4i_b + 4i_a + 4i_b + 16 = 0 \]

\[ 4i_a + 16 = 0 \Rightarrow i_a = -4 \text{ A} \]

\[ i_b = i_a + 1 = -3 \text{ A} \]

\[ v_x = 2i_b = -6 \text{ V} \]

\[ v = 2i_b + 16 + 2i_b \]

\[ = 4i_b + 16 \]

\[ = 4(-3) + 16 = 4 \text{ V} \]

14
Problem: 4 (20 pts)
Consider the following circuit:

4.a (10 pts) Find the Thevenin equivalent voltage $V_{th}$ and resistance $R_{th}$.

Solu: Find $V_{opencircuit} = V_{th}$: As zero current flows through 5Ω, $V_0$ is the voltage drop across 5A source. Apply KVL around the loop containing 5A source, 10V source, and 3Ω resistor to obtain (note 5A flows through 3Ω resistor):

$$-10 - 3(5) + V_0 = 0 \Rightarrow V_0 = V_{th} = 25V$$

To find $R_{th}$, turn off the sources to obtain equivalent circuit:

$$V_{th} = \frac{25V}{8Ω}, \quad R_{th} = 8Ω$$

4.b (10 pts) Draw the Thevenin equivalent circuit.
Problem: 5 (20 pts) Fourier Series problem

Consider the following periodic waveform $x(t)$.

5.a Determine the period $T$, fundamental frequency $f_0$ and the second harmonic frequency $f_2$ of $x(t)$ (6 pts)

$$T = \frac{5}{5} s, \quad f_0 = \frac{1}{5} \text{ Hz}, \quad f_2 = \frac{2}{5} \text{ Hz}$$

5.b The Fourier series representation of $x(t)$ is $x(t) = A_0 + \sum_{n=1}^{\infty} a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t)$.

Determine whether each statement is true or false (5 pts):

<table>
<thead>
<tr>
<th>Statement</th>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_n = 0$ for all $n$.</td>
<td>✗</td>
<td></td>
</tr>
<tr>
<td>$b_n = 0$ only for odd $n$.</td>
<td></td>
<td>✗</td>
</tr>
<tr>
<td>$a_n = 0$ for all $n$</td>
<td></td>
<td>✗</td>
</tr>
<tr>
<td>$A_0 &lt; 0$</td>
<td>✗</td>
<td></td>
</tr>
<tr>
<td>The average value of $x(t)$ is $2A/5$.</td>
<td>✗</td>
<td></td>
</tr>
</tbody>
</table>

5.c The signal $x(t) = 10 + 12 \cos(2\pi 100t) + 3 \sin(2\pi 200t)$ is the input to a linear time invariant system with transfer function (magnitude and phase) shown below. Determine the output $y(t)$ of the system (9 pts.).

$$y(t) = 5 + 12 \cos \left( 2\pi 100 t + \pi \right)$$