EECS 210
Introduction to Electrical Engineering I
Fall 2000

Midterm #2
Professors Hero and Stark
November 21, 2000

Rules:

1. **DO NOT OPEN OR START THIS EXAM UNTIL TOLD TO DO SO**
2. The only material that you are allowed to use during this exam are pencils, pens, and calculators. **NO** additional paper will be permitted. You do **NOT** have to write in ink.
3. All your work must be done on the exam and all the pages must be handed in.
4. Place your **ANSWERS** where indicated on the exam and do your calculations on the blank page following the problem. Two extra blank pages are at the end of the exam if you need extra paper for your work. These pages are **NOT** to be removed.
5. You will get **NO** extra credit if you do not show your work.

Good Luck!

Print your name: ______________________________

Print your student ID#: ______________________________

Print your lab section #: ______________________________

Sign the Honor Pledge at the conclusion of the exam:

I have neither given nor received aid on this exam, nor have I concealed any violations of the honor code.

Sign your name here: ______________________________

Exam Number: ____________

Problem: 1 (20 pts)

1.a. (5 pts) Write the current-voltage relationship for a capacitor with capacitance C.

$$i(t) = C \frac{dv(t)}{dt}$$

$i \cdot v$ relation = $i(t) = C \frac{dv(t)}{dt}$

1.b. (5 pts) Write the current-voltage relationship for an inductor with inductance L.

$i \cdot v$ relation = $v(t) = L \frac{di(t)}{dt}$

1.c. (5 pts) Write an equation for the voltage across a capacitance at frequensy $\omega = \infty$ rad/sec.

$v(t) =$ 0

1.d. (5 pts) Write an equation for the voltage across an inductor at frequency $\omega = 0$ rad/sec.

$v(t) =$ 0

1.e. (5 pts) The phasor for the voltage $v(t)$ in a circuit is $V = 3 - j4$. Find the voltage $v(t)$ if $f = 5\text{Hz}$.

$v(t) = 5 \cos(2\pi 5t - .9273)$

$5 = r = \sqrt{3^2 + 4^2}$

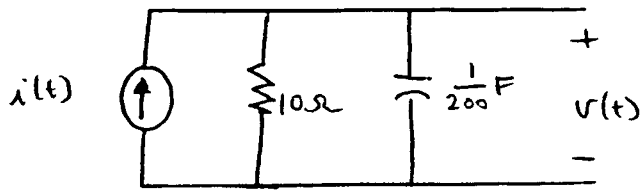
$\theta = \tan^{-1}(-4/3) = -.9273 \text{ rad}$

$= -53.13°$

$v(t) = 5 \cos(2\pi 5t - .9273)$

Problem: 2 (20 pts)

2.a (10 pts) For the circuit shown below determine the voltage $v(t)$ if $i(t) = 10\cos(20t + \pi/4)$.



Use Phasors

$I = 10e^{j\pi/4}$

$Z_R = 10\,\Omega$, $Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$

$= -j\,10\,\Omega$

$Z_R \parallel Z_C = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_C}} = 5 - j5 = 5\sqrt{2}e^{-j\pi/4}$

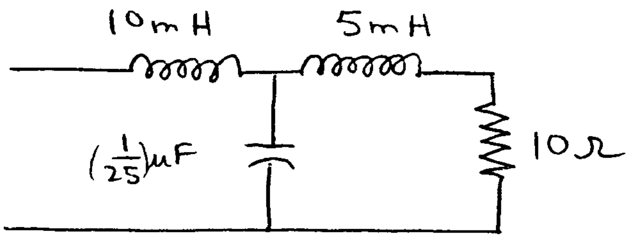
$V = Z_{eq} \cdot I = 5\sqrt{2}e^{-j\pi/4}\,10e^{j\pi/4}$

$= 50\sqrt{2}$

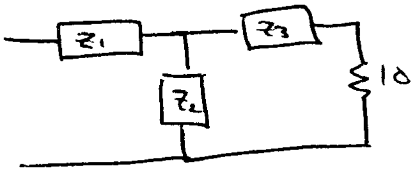
$v(t) = 50\sqrt{2}\cos(20t)$

$v(t) =$ $50\sqrt{2}\cos(20t)$

2.b (10 pts) For the circuit shown below determine the equivalent impedance at $\omega = 4000$ radians/sec.



⇒



$Z_1 = j\omega L_1 = j\,40\ \Omega$

$Z_2 = (j\omega C)^{-1} = -j\,6250\,\Omega$

$Z_3 = j\omega L_2 = j\,20\ \Omega$

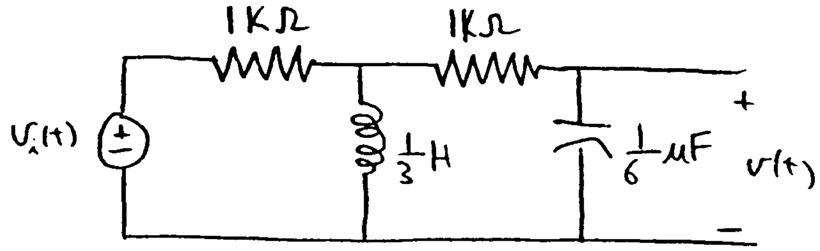
$Z_{eq} = Z_1 + \frac{1}{\frac{1}{Z_2} + \frac{1}{Z_3 + 10}}$

$= 10.06 + j\,60.05\ \Omega$

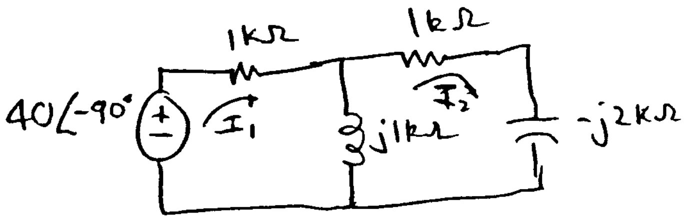
$Z_{eq} =$ $10.06 + j\,60.05\ \Omega$

Problem: 3 (20 pts)

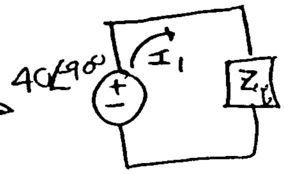
For the circuit shown below determine the voltage $v(t)$ across the capacitor when the source voltage is $v_i(t) = 40\sin(3000t)$. Determine the average power delivered by the source.



Use Phasors



⟺



$$Z_{eq} = 1k + \frac{j1k(1k - j2k)}{j1k + 1k + (-j2k)}$$

$$= 1500 + j1500\ \Omega = 1500\sqrt{2}\,e^{j\pi/4}$$

$$I_1 = \frac{V}{Z_{eq}} = \frac{40e^{-j\pi/2}}{1500\sqrt{2}\,e^{j\pi/4}} = \frac{40}{1500\sqrt{2}}e^{-j3\pi/4}$$

$$P_{av} = \frac{1}{2}\text{Re}\left[V \cdot I^*\right] = \frac{1}{2}\text{Re}\left[\frac{40e^{-j\pi/2} \cdot 40e^{j3\pi/4}}{1500\sqrt{2}}\right]$$

$$= \frac{4}{15}\text{W}$$

$$I_2 = \frac{j1}{j1 + 1 - j2}I_1$$

$$= -\frac{1+j}{2}I_1 =$$

$$V_c = I_2(Z_c) = I_2(-j2k)$$

$$= \left(-\frac{1+j}{2}\right)I_1(-j2k)$$

$$= (1+j)I_1 k = \sqrt{2}e^{j\pi/4}I_1 k$$

$$= \frac{\sqrt{2}\,40}{1500\sqrt{2}}e^{j\pi/4}e^{-j3\pi/4} \times 1000$$

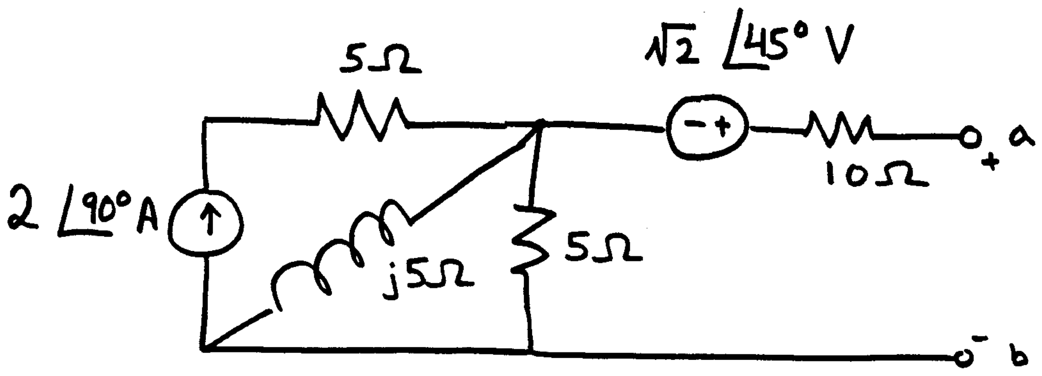
$$= \frac{40}{1500}e^{-j\pi/2} \times 1000$$

$v(t) =$ $\frac{80}{3}$ sin(3000t) Volts

$P_{av} =$ 4/15 Watts

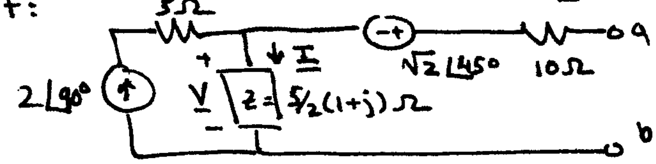
Problem: 4 (20 pts)

Consider the following circuit:



4.a (10 pts) Find the Thevenin equivalent voltage V_{th} and impedance Z_{th}.

Note that j5Ω and 5Ω (vertical) resistor are in parallel with equivalent $Z = (j5)5/(5+j5) = 5/2\, j(1-j) = 5/2(1+j)$. Thus redraw circuit:



$V_{ab}^{open} = V_{th} = V + \sqrt{2}\angle 45°$ $(I_{10\Omega} = 0)$

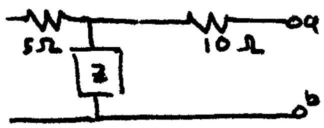
$V = 2\angle 90° \cdot 5/2(1+j) = 5(-1+j)$ V

(2∠90° = 2j; √2∠45° = 1+j)

$\therefore V_{th} = 5(-1+j) + (1+j) = 2(-2+3j)$

$= 2\sqrt{13}\angle 123.7°$ V

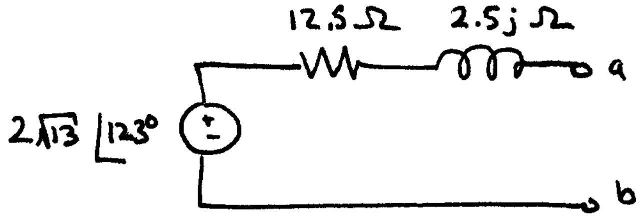
For Z_{th} redraw circuit w/ sources off



$Z_{th} = Z + 10 = \frac{25}{2} + \frac{5}{2}j$

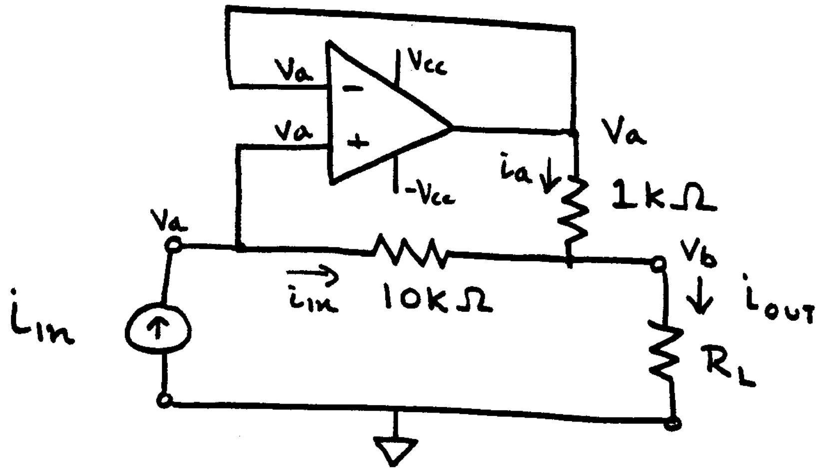
V_{th} = $2\sqrt{13} \cdot \angle 123°$ V, Z_{th} = 12.5 + 2.5 j Ω

4.b (10 pts) Draw the Thevenin equivalent circuit using a voltage source, a resistor and an inductor or capacitor.



Problem: 5 (20 pts)

Consider the following current amplifier circuit.



5.a (10 pts) Find the output current i_{out} in terms of the input current i_{in} assuming the op-amp is not in saturation.

Soln: Define i_a, V_a, V_b as indicated. Then $i_{OUT} = i_{in} + i_a$.

$i_a = \frac{V_a - V_b}{10^3}$ or $V_a - V_b = 10^3 i_a$

But also: $V_a - V_b = 10^4 i_{in}$

$\Rightarrow 10^3 i_a = 10^4 i_{in}$

$i_a = 10 i_{in}$

$\therefore$ $i_{OUT} = i_{in} + 10 i_{in} = 11 i_{in}$

$i_{out} =$ $11 i_{in}$

5.b (10 pts) Find the maximum value of $|i_{in}|$ such that the op-amp is not in saturation when the load resistor is $R_L = 1K\Omega$ and $V_{cc} = 10V$.

Soln: For no saturation $|V_a| \leq V_{cc} = 10V$

But we have: $V_a = i_{OUT} \cdot R_L + i_a 10^3 =$

$= 11 i_{in} \cdot 10^3 + 10 i_{in} \cdot 10^3 = 21 \cdot 10^3 i_{in}$

$\therefore$ $|V_a| \leq 10V \Leftrightarrow |i_{in}| \leq \frac{10}{21 \cdot 10^3} = \frac{10}{21} 10^{-3} \approx 0.5 mA$

$\max\{|i_{in}|\} =$ $10/21$ mA