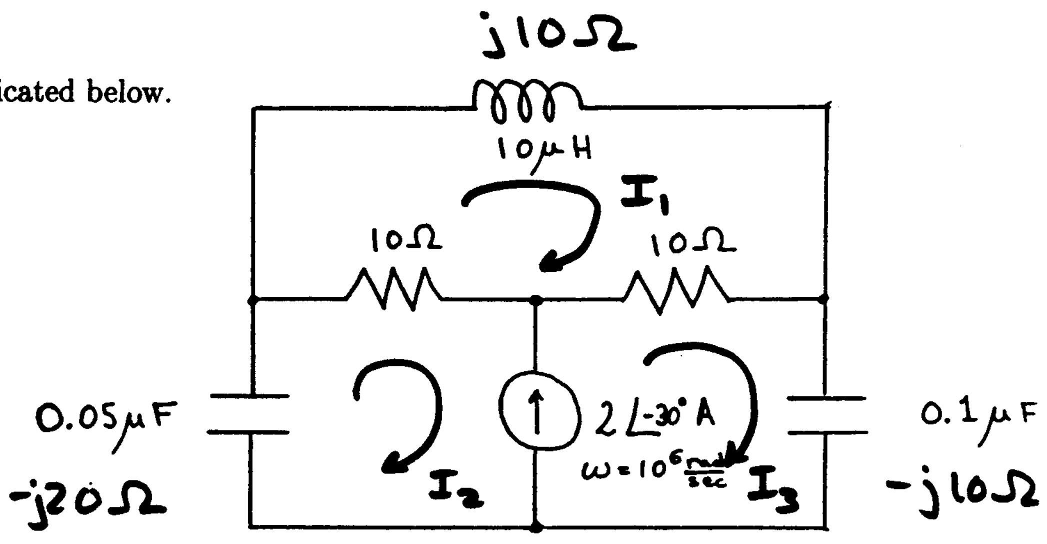
EECS 210, sec 001, F99 AH

Final Solutions

Problem 1 (16 pts):

Consider the circuit indicated below.



1.a (2 pts) Draw and label the mesh currents on the above circuit.

1.b (12 pts) Write down the mesh equations.

Mesh Equations :

Mesh 1 : $10(I_1 - I_2) + j10 I_1 + 10(I_1 - I_3) = 0$

Supermesh : $-j20 I_2 + 10(I_2 - I_1) + 10(I_3 - I_1) - j10 I_3 = 0$

Current branch: $I_3 - I_2 = 2\angle -30^\circ$

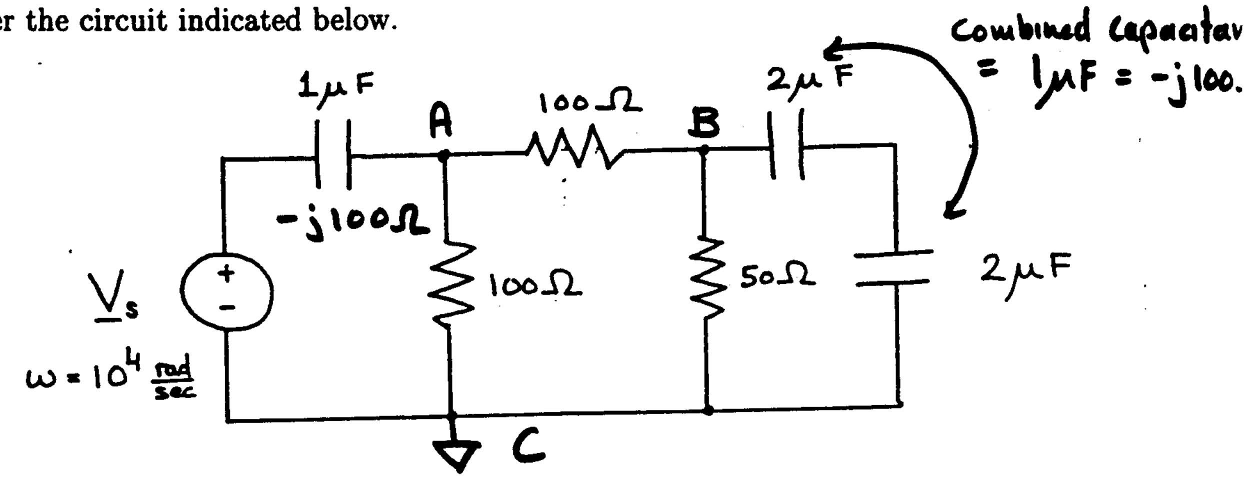
1.c (2 pts) Express the mesh equations in matrix form.

Matrix Form :

$$\begin{bmatrix} 2+j & -1 & -1 \\ -2 & 1-2j & 1-j \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2\angle -30^\circ \end{bmatrix}$$

Problem 2 (16 pts):

Consider the circuit indicated below.



2.a (2 pts) Label the essential nodes. Choose C as ref. node.

2.b (12 pts) write the node equations.

Node Equations :

node A $(\underline{V}_A - \underline{V}_s)\, j.01 + \underline{V}_A .01 + (\underline{V}_A - \underline{V}_B).01 = 0$

node B $(\underline{V}_B - \underline{V}_A).01 + \underline{V}_B .02 + \underline{V}_B\, j.01 = 0$

2.c (2 pts) Express the node equations in matrix form.

Matrix Form :

$$\begin{bmatrix} 2+j & -1 \\ -1 & 3+j \end{bmatrix} \begin{bmatrix} \underline{V}_A \\ \underline{V}_B \end{bmatrix} = \begin{bmatrix} j\underline{V}_s \\ 0 \end{bmatrix}$$

Problem 3 (16 pts)

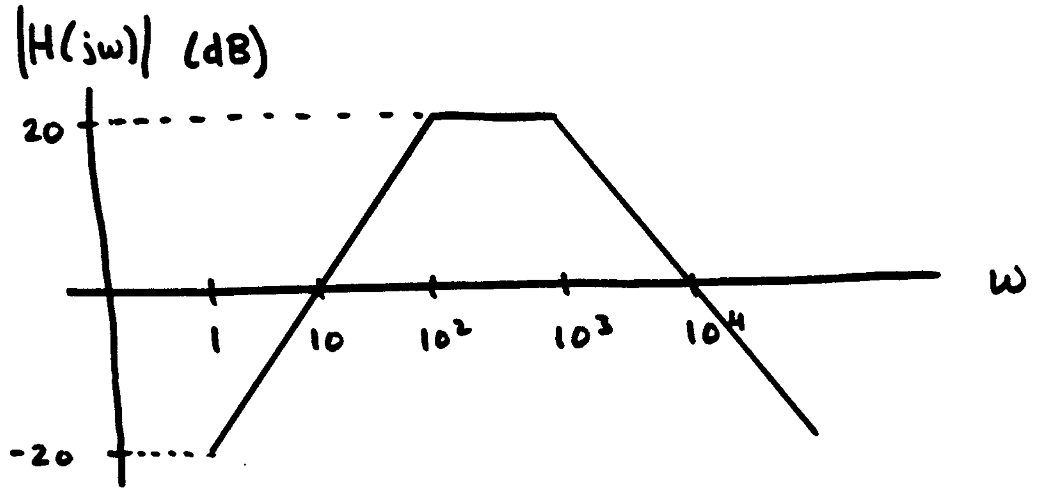
Consider the following system transfer function

$$H(s) = \frac{10^4 s}{s^2 + (10^2 + 10^3)s + 10^5}$$

3.a (12 pts): Sketch the asymptotic magnitude Bode plot of $|H(jw)|$ over the range $1 \leq \omega \leq 10^4$.

First put into standard factored form:

$$H(s) = \frac{10^4 s}{(s+10^2)(s+10^3)} = \frac{1}{10} \frac{s}{(1+s/10^2)(1+s/10^3)}$$

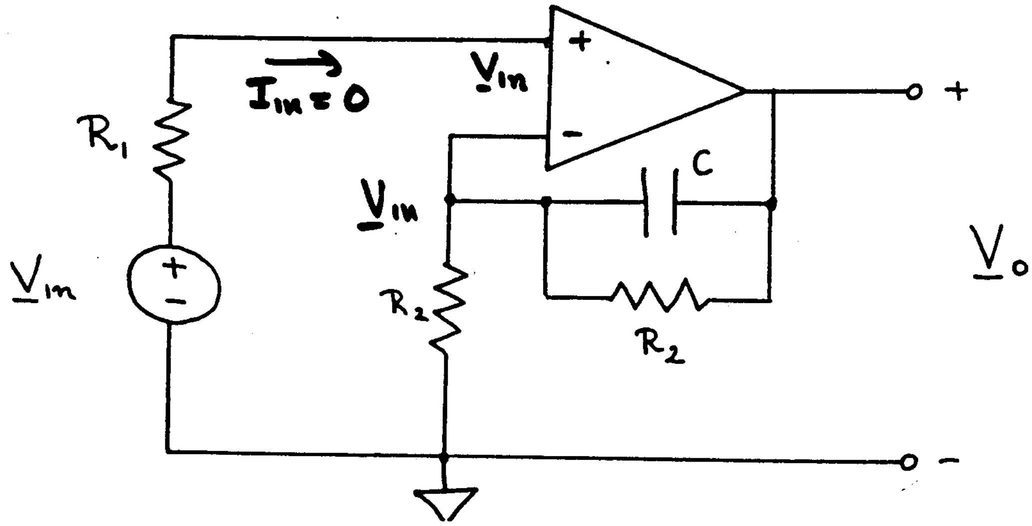


3.b (4pts) What kind of filter is this (Lowpass, highpass, bandpass, bandreject, none of the above).

Kind of filter : $\boxed{\text{BPF}}$

Problem 4 (16 pts):

Consider the following Op-Amp circuit (Assume that Op-Amp is ideal and not in saturation):



4.a (12 pts) Use the Golden rules and circuit analysis to derive the transfer function $H(s) = \underline{V}_o/\underline{V}_{in}$ $(s = j\omega)$.

GR#2 : "+" terminal of op amp draws 0 current $\Rightarrow \underline{V}_+ = \underline{V}_{in}$

GR#1 : $\underline{V}_+ = \underline{V}_- \Rightarrow \underline{V}_- = \underline{V}_{in}$

Hence, applying GR#2 again to "−" terminal of op-amp, by Voltage divider formula: $\underline{V}_{in} = \dfrac{R_2}{R_2 + C \| R_2} \underline{V}_o$

$$\Rightarrow H(s) = \frac{R_2 + C\|R_2}{R_2} = \frac{R_2 + R_2/(1+sR_2C)}{R_2} = \frac{2 + R_2Cs}{1 + R_2Cs}$$

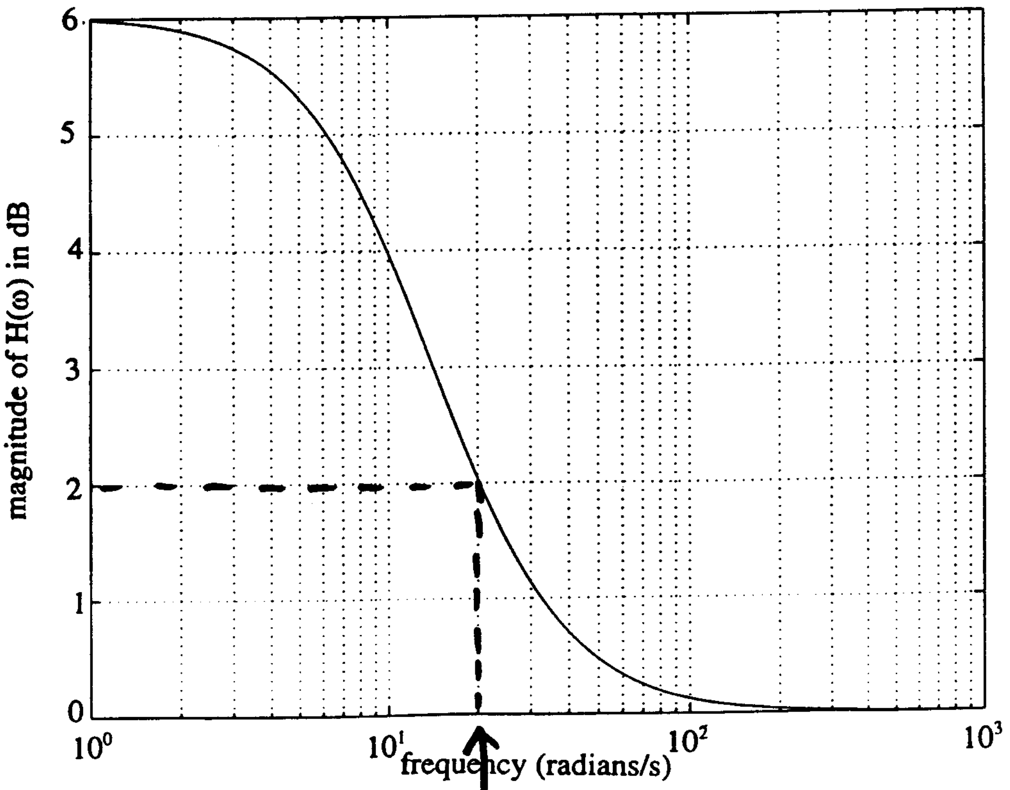
$$H(s) = \boxed{\frac{2 + R_2Cs}{1 + R_2Cs}}$$

4.b (4 pts) When $R_1 = 1K\Omega$, $R_2 = 100K\Omega$ and $C = 1\mu F$, the circuit's magnitude transfer function $|H(j\omega)|$ has the Bode plot shown below. If R_1 is replaced by $10K\Omega$, R_2 is replaced by $R_2 = 1M\Omega$ and C is replaced by $0.01\mu F$ what is the frequency ω_1 (in rad/sec) for which the new circuit has $|H(j\omega_1)| = 2$dB?

1. Magnitude (impedance) scaling: $R_1' = 10 R_1$, $R_2' = 10 R_2 \Rightarrow k_m = 10$
To obtain equiv. $H(j\omega)$: $C' = \frac{1}{k_m} C = \frac{1}{10} 1\mu F = 0.1 \mu F$

2. Frequency scaling: $C'' = 0.01\mu F = \frac{1}{k_f} C' \Rightarrow k_f = 10$
Thus $\omega_1^{new} \rightarrow 10\, \omega_1^{old}$
$= 10(20)$
$= 200$

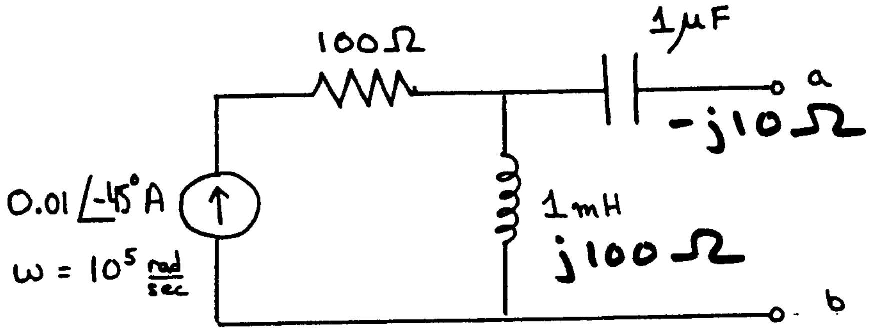
$\omega_1 = \boxed{200 \ \frac{rad}{sec}}$



$\omega_1 = 20 \ \frac{rad}{sec}$

Problem: 5 (16 pts)

5.a (8 pts) Consider the circuit below. Find the Thevenin equivalent voltage V_{th} and Thevenin equivalent impedance Z_{th}. Draw the Thevenin equivalent circuit.



$$V_{th} = V_{ab}^{oc} = (0.01\angle{+45°})(j100) = 1\angle{+45°}$$

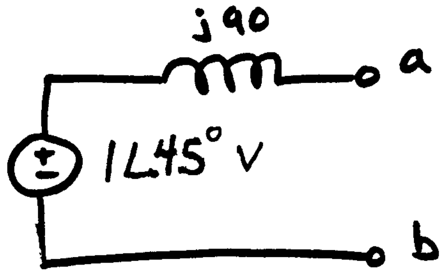
(Note: 0 current thru 1 μF capacitor)

$$Z_{th} = -j10 + j100 = j90\ \Omega$$

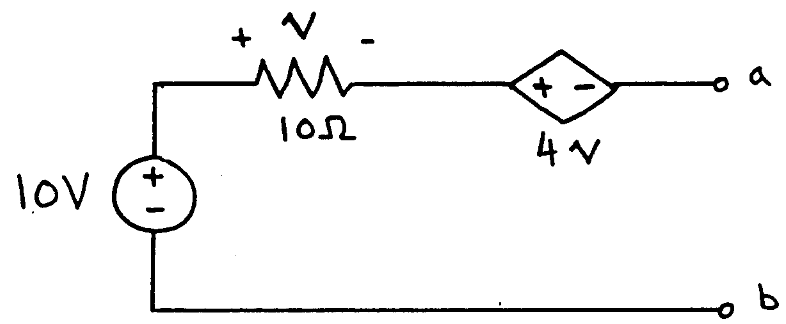
(open current source removes 100Ω from Z_{th}

V_{th} = $\boxed{1\angle45°\ \text{V}}$, Z_{th} = $\boxed{j90\ \Omega}$

Equivalent circuit

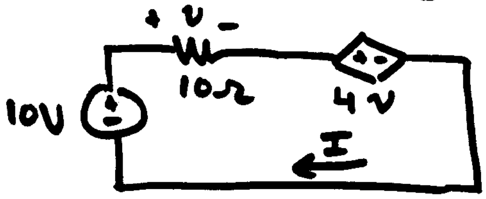


5.b (8 pts) Consider the circuit below. Find the Thevenin equivalent voltage V_{th} and Thevenin impedance Z_{th}. Draw the Thevenin equivalent circuit.



i). $V_{th} = V_{ab}^{oc} = 10V$ (no current flows through 10Ω so $v = 0$)

ii). $Z_{th} = V_{ab}^{oc} / I_{ab}^{sc}$. $I_{ab}^{sc} = I$ found by KVL around loop

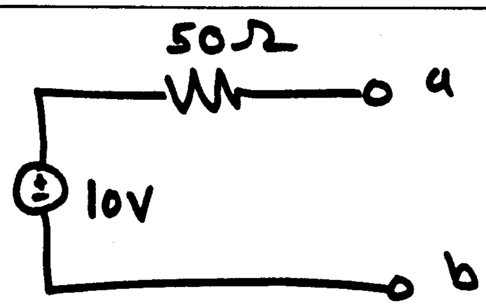


$-10 + 5v = 0$, $v = IR = 10I$

$\Rightarrow I = \frac{1}{5} = 0.2\,A$. $\therefore Z_{th} = \frac{10}{1/5} = 50$

$V_{th} =$ $\boxed{10\,V}$, $Z_{th} =$ $\boxed{50\Omega}$

Equivalent circuit



Problem: 6 (16 pts)

A voltage waveform $x(t)$ has the following Fourier series representation

$$x(t) = 2 + \sum_{n=1}^{\infty} \frac{1+(-1)^n}{2n} \cos(\overset{100}{\cancel{20}}\pi n t + \pi/n) \quad (V)$$

{ note: odd harmonics are zero.

6.a (2 pts) What is the fundamental frequency (in Hz) of $x(t)$?

$100\pi n t = \underbrace{50}_{f}\, 2\pi n t$

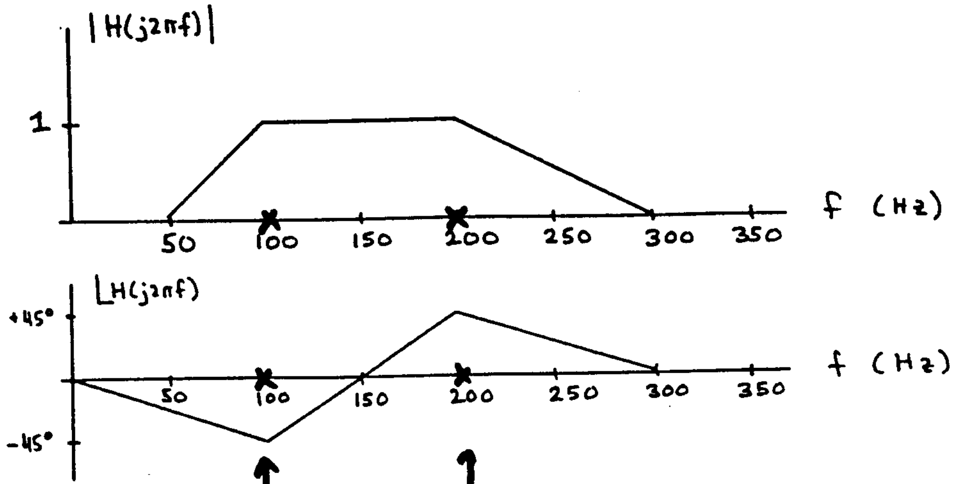
$f = \boxed{50 \text{ Hz}}$

6.b (2 pts) What is the D.C. value of $x(t)$?

$V_{D.C.} = \boxed{2}$

6.b (12 pts) A filter has the magnitude and phase response shown in the plot below. Find the output $y(t)$ when the input to the filter is $x(t)$ above.

$y(t) = \boxed{\frac{1}{2}\cos(200\pi t + 45^\circ) + \frac{1}{4}\cos(400\pi t + 90^\circ)}$



Harmonic : $n = 2$ $\quad n = 4$

Freq. component at input of H : $\frac{1}{2}\cos(200\pi t + 90^\circ)$ $\quad \frac{1}{4}\cos(400\pi t + 45^\circ)$