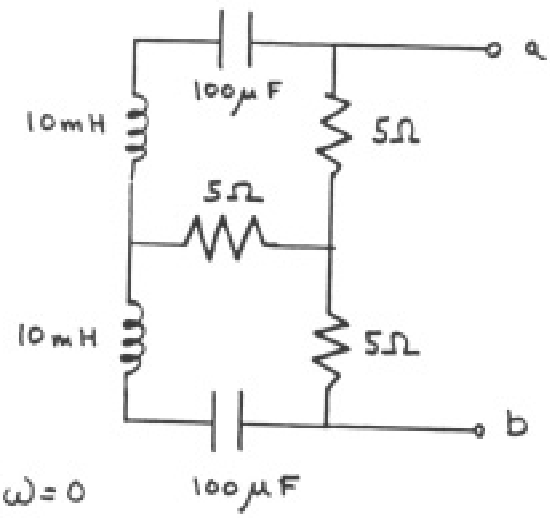
F99 EECS 210
Midterm #2 Solutions

Problem: 1 (20 pts)

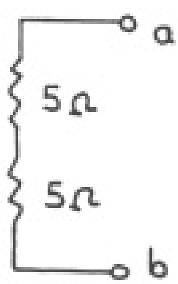
1.a. (5 pts) Consider the circuit indicated below. Find the impedance across terminals a, b of the circuit for $\omega = 0$ (D.C.) and the impedance for $\omega = \infty$.



$Z_c = \frac{1}{j\omega C} = -j\infty$ @ $\omega = 0$

$Z_L = j\omega L = j\infty$ @ $\omega = \infty$

$\therefore$ at either $\omega = 0$ or $\omega = \infty$ series combination of L & C has $|Z_{L+C}| = \infty$ $\Rightarrow$ open-circuit. Thus the above circuit reduces to



$Z_{\omega=0}$ = 10 Ω , $Z_{\omega=\infty}$ = 10 Ω

1.b. (5 pts) Express the complex variable W below in both polar and rectangular form

$$W = \frac{e^{j\pi/3} - 2e^{j\pi/4}}{1+j}$$

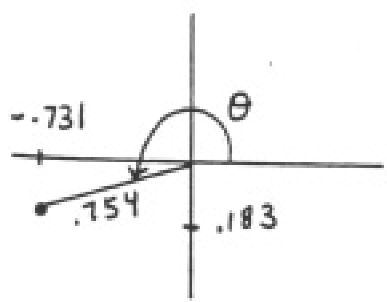
$$= \frac{e^{j\pi/3} - 2e^{j\pi/4}}{\sqrt{2}\, e^{j\pi/4}}$$

$$= \frac{1}{\sqrt{2}} e^{j\pi/12} - \sqrt{2}$$

$$= \frac{1}{\sqrt{2}}\left(\cos \pi/_{12} + j\sin \pi/_{12}\right) - \sqrt{2}$$

$$= .683 + j.183 - \sqrt{2}$$

$$= -.731 + j.183$$



$$r = \left[(0.731)^2 + (.183)^2\right]^{1/2} = .754$$

$$\theta = \tan^{-1}\left(\frac{.183}{-.731}\right) = 194.055^\circ$$

or -165.945°

$W =$ $0.754 \angle 194.055^\circ$ (polar form),

$W =$ $-0.731 + j.183$ (rectangular form)

1.c. (5 pts) Use phasors to find the quantities A and θ defined by

$$\cos(10t + \pi/4) - \sin(10t + \pi/6) = A\cos(10t + \theta)$$

$$= \cos(10t + \pi/4) + \cos\left(10t + \frac{\pi}{6} + \frac{\pi}{2}\right)$$

$$= e^{j\pi/4} + e^{j2\pi/3}$$

$$= \frac{1}{\sqrt{2}}(1+j) + \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$

$$= 0.207 + j1.573$$

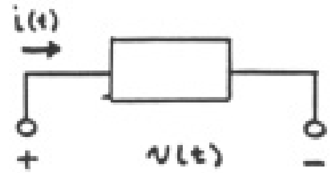
$$\therefore r = \left[(0.207)^2 + (1.573)^2\right]^{1/2} = 1.587$$

$$\theta = \tan^{-1}\left[\frac{1.573}{0.207}\right] = 1.44\,\mathrm{rad}$$

(82.5°)

$A=$ 1.587, $\theta=$ 82.5° (1.44 rad)

1.d. (5 pts) Consider the device below:



where the voltage $v(t)$ and the current $i(t)$ are

$$\begin{aligned} v(t) &= 2\cos(\omega t + \pi/3) \\ i(t) &= \cos(\omega t + \pi/4) \end{aligned}$$

Find the impedance Z of the device and the average power P dissipated by the device for $\omega = 20$ rad/sec.

$\underline{V} = 2 \angle \pi/3$

$\underline{I} = 1 \angle \pi/4$

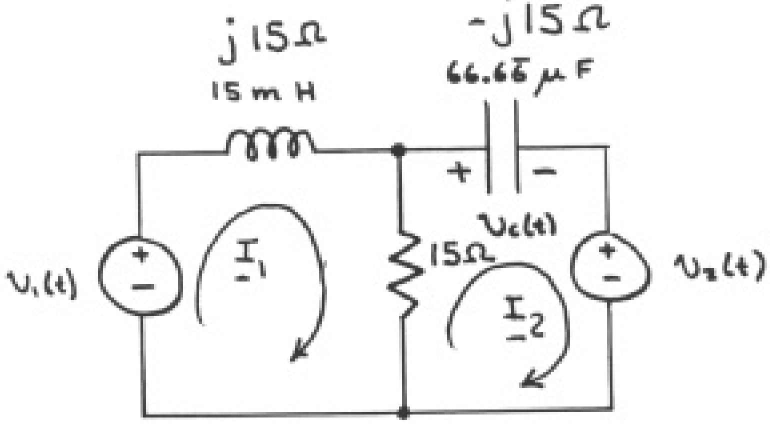
$Z = \dfrac{\underline{V}}{\underline{I}} = 2 \angle \pi/12 \quad = 1.932 + j0.518\ \Omega$

$P = \frac{1}{2} \mathrm{Re}[\underline{V} \cdot I^*] = \frac{1}{2} \mathrm{Re}[2 \angle \pi/12] = 0.966\ \mathrm{W}$

$Z=$ $\underline{1.932 + j0.518\,\Omega}$, $\quad P=$ $\underline{0.966\ \mathrm{W}}$

Problem: 2 (20 pts)

Consider the following circuit where $v_1(t) = 30\cos(1000t)$ V and $v_2(t) = 30\cos(1000t - 90^o)$ V:



2.a (5 pts): Draw and label the mesh currents on the circuit diagram.

2.b (10pts) Write down the mesh equations.

mesh 1 $\quad -30 + j15\underline{I}_1 + 15(\underline{I}_1 - \underline{I}_2) = 0$

$$15(\underline{I}_2 - \underline{I}_1) - j15\underline{I}_2 - 30j = 0$$

$$\begin{bmatrix} 15(1+j) & -15 \\ -15 & 15(1-j) \end{bmatrix} \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix} = \begin{bmatrix} 30 \\ 30j \end{bmatrix}$$

2.c (5 pts) Solve the mesh equations and find $v_c(t)$.

$$\therefore (1+j)\underline{I}_1 - \underline{I}_2 = 2$$

$$\underline{I}_1 - (1-j)\underline{I}_2 = -2j$$

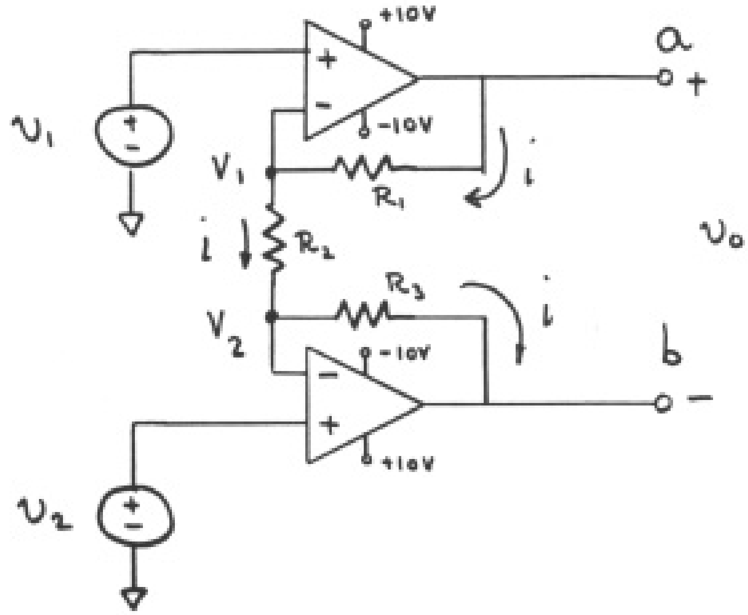
$$\therefore \underline{I}_1 = 2A \ \& \ \underline{I}_2 = 2j \ A$$

$$\underline{V}_c = (-j15)(\underline{I}_2) = 30 V$$

$v_c(t) =$ 30 cos(1000t) V

Problem: 3 (20 pts)

Consider the following circuit:



3.a (10 pts) Assume that ˜ Op-Amp is ideal and not in saturation. Find the output voltage v_o in terms of the input voltages v_1, v_2, and resistances R_1, R_2 and R_3.

By Golden Rule # 1 $i = (V_1 - V_2)/R_2$

By Golden Rule # 2 $i = V_o/(R_1+R_2+R_3)$

$$\therefore \frac{V_1 - V_2}{R_2} = \frac{V_o}{R_1+R_2+R_3} \Longrightarrow V_o = \frac{R_1+R_2+R_3}{R_2}(V_1 - V_2)$$

$v_o = \frac{R_1+R_2+R_3}{R_2}(V_1 - V_2)$

3.b (10 pts) If $v_2 = 0$ and if $R_1 = R_2 = R_3 = 1000\Omega$ determine how large v_1 can be before the top Op-Amp saturates.

Note: $V_2 = 0$. Thus by a voltage divider $V_1 = \frac{R_2}{R_1+R_2} V_a$

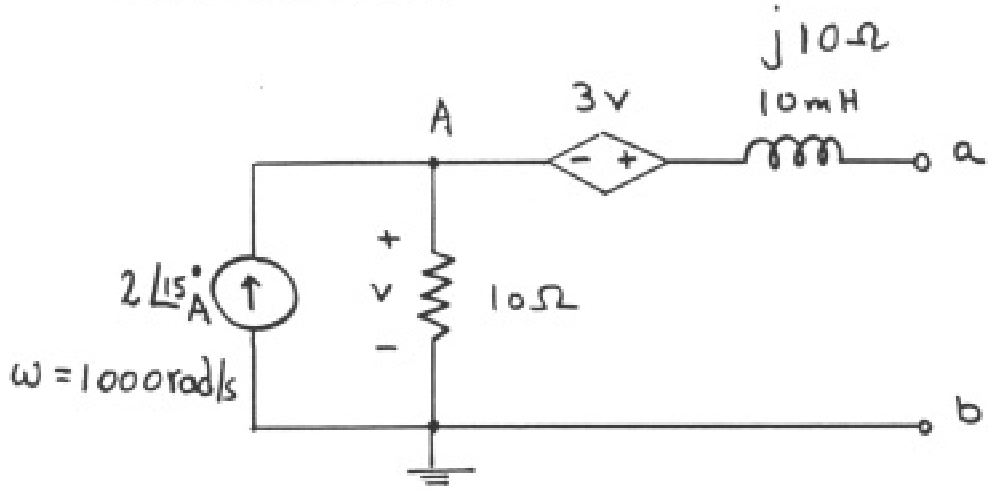
saturation occurs for $V_a > 10\,V$ $= 2V_a$

$\therefore$ max $V_1 = 5\,V$

max $v_1 =$ 5 V

Problem: 4 (20 pts)

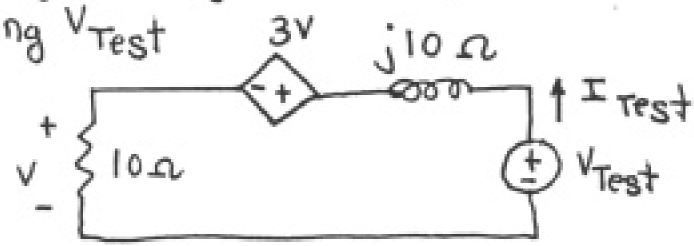
Consider the following circuit:



4.a (15 pts) Find the Thevenin equivalent voltage V_{th} and impedance Z_{th}.

$V_T = V_{oc} = V_A + 3V = 4V_A = (4 \times 2\angle 15^\circ) \times 10 = 80\angle 15^\circ$ V

Find Z_T by turning off independent current source & applying V_{Test}



V_{th} = $80\angle 15^\circ$, Z_{th} = $40 + j10\Omega$

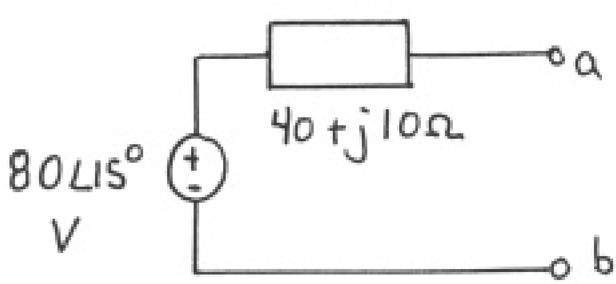
4.b (5 pts) Draw the Thevenin equivalent circuit.

Apply KVL $\therefore$ $-v - 3v - (j10)I_{Test} + V_{Test} = 0$

$v = 10\, I_{Test}$

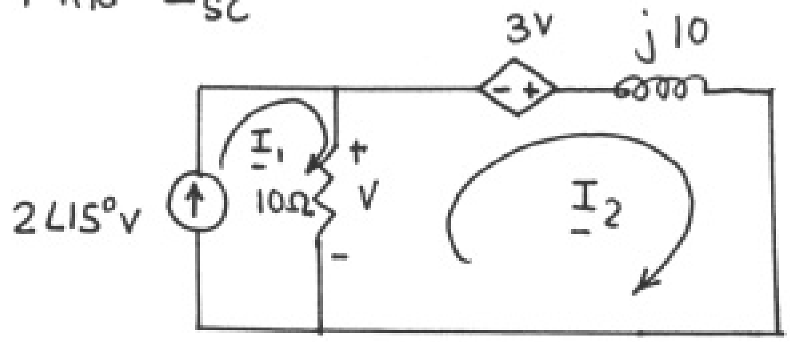
$\therefore -40 I_{Test} - j10 I_{Test} + V_{Test} = 0$

$Z_{th} = \dfrac{V_{Test}}{I_{Test}} = 40 + j10\Omega$



Alternate method for finding Z_T.

Find I_{sc}



mesh 1 $\quad I_1 = 2\angle 15°$

mesh 2 $\quad (I_2 - I_1)10 - 3V + j10 I_2 = 0$

$V = 10(I_1 - I_2)$

$\therefore \; 10(I_2 - I_1) - 30(I_1 - I_2) + j10 I_2 = 0$

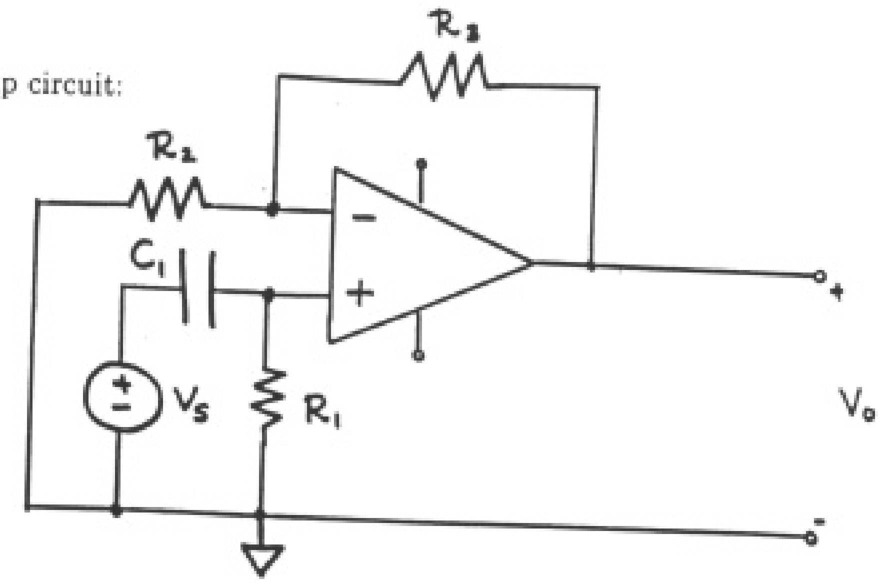
substitute $I_1 = 2\angle 15°$ in above equation to get

$$I_{sc} = I_2 = \frac{80\angle 15°}{40 + j10}$$

$$Z_T = \frac{V_{oc}}{I_{sc}} = \frac{80\angle 15°}{\frac{80\angle 15°}{40+j10}} = 40 + j10\,\Omega$$

Problem: 5 (20 pts)

Consider the following Op-Amp circuit:



Assume that the Op-Amp is ideal and not in saturation. Compute the transfer function $H(j\omega)$ for $v_o(t)/v_s(t)$

$$V_{(+)} = \frac{R_1}{R_1 + \frac{1}{j\omega C_1}} V_s = \frac{j\omega R_1 C_1}{1 + j\omega R_1 C_1} V_s \quad \text{(voltage divider)}$$

KCL @ (-) terminal

$$V_{(-)} = V_{(+)} \qquad \text{(Golden Rule \#1)}$$

$$\frac{0 - V_{(-)}}{R_2} + \frac{V_o - V_{(-)}}{R_3} = 0$$

$$\therefore \quad V_o = \left(1 + \frac{R_3}{R_2}\right) V_{(-)} = \left(1 + \frac{R_3}{R_2}\right) \frac{j\omega R_1 C_1}{1 + j\omega R_1 C_1}$$

$$H(j\omega) = \left(1 + \frac{R_3}{R_2}\right) \frac{j\omega R_1 C_1}{1 + j\omega R_1 C_1}$$