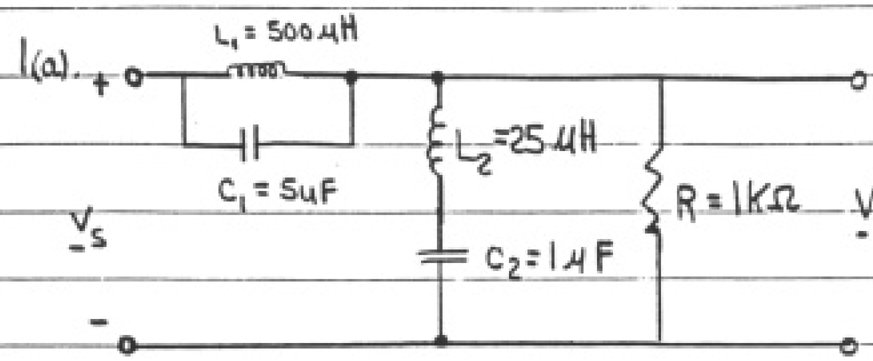
EECS 210 W97

HW #12 Solutions

1(a).



$s = j\omega$

$$Z_{L_1 \| C_1} = \frac{sL_1\left(\frac{1}{sC_1}\right)}{sL_1 + \frac{1}{sC_1}} = \frac{L_1 s}{1 + L_1 C_1 s^2}$$

$$Z_{L_2 + C_2} = \frac{1}{sC_2} + sL_2 = \frac{1 + L_2 C_2 s^2}{C_2 s}$$

$$Z_{(L_2+C_2)\|R} = \frac{Z_{L_2+C_2} \cdot R}{R + Z_{L_2+C_2}} = \frac{R(1 + L_2 C_2 s^2)}{L_2 C_2 s^2 + R C_2 s + 1}$$

Voltage divider

$$H(s) = \frac{V_o}{V_s} = \frac{Z_{(L_2+C_2)\|R}}{Z_{(L_2+C_2)\|R} + Z_{L_1\|C_1}}$$

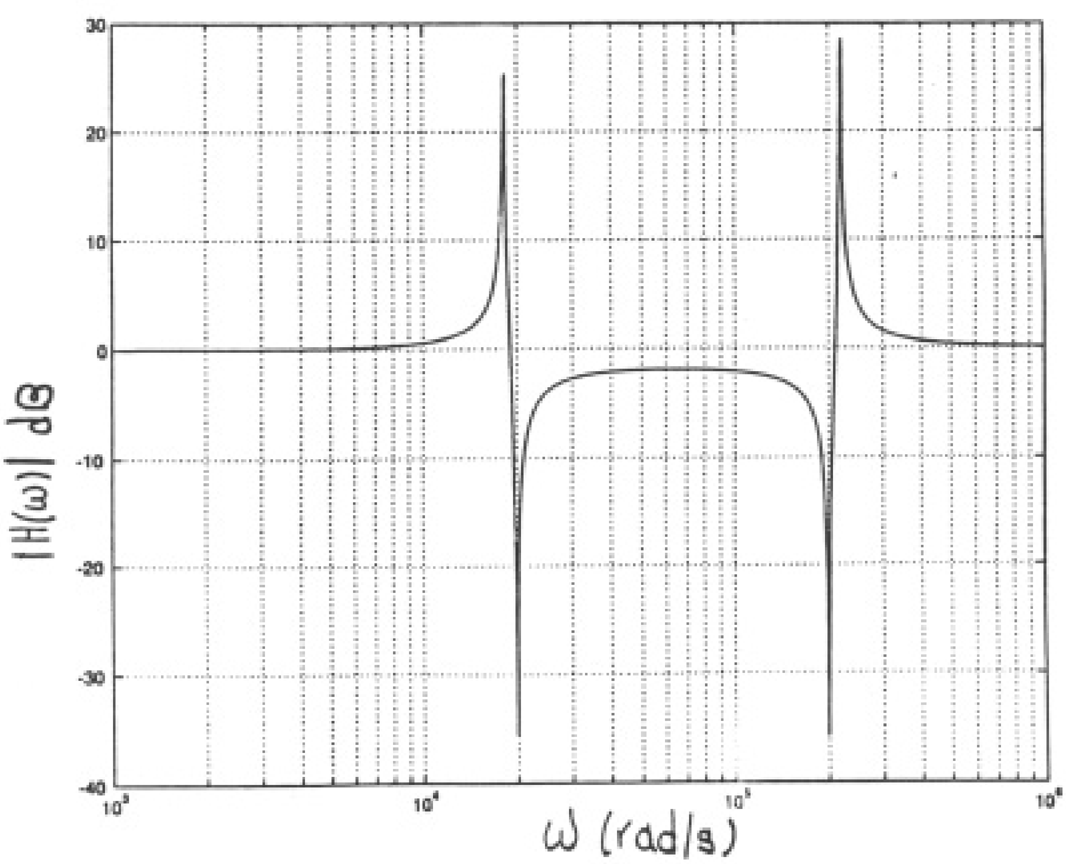
$$= \frac{R(1 + L_2 C_2 s^2)(1 + L_1 C_1 s^2)}{R(1 + L_2 C_2 s^2)(1 + L_1 C_1 s^2) + L_1 s(L_2 C_2 s^2 + R C_2 s + 1)}$$

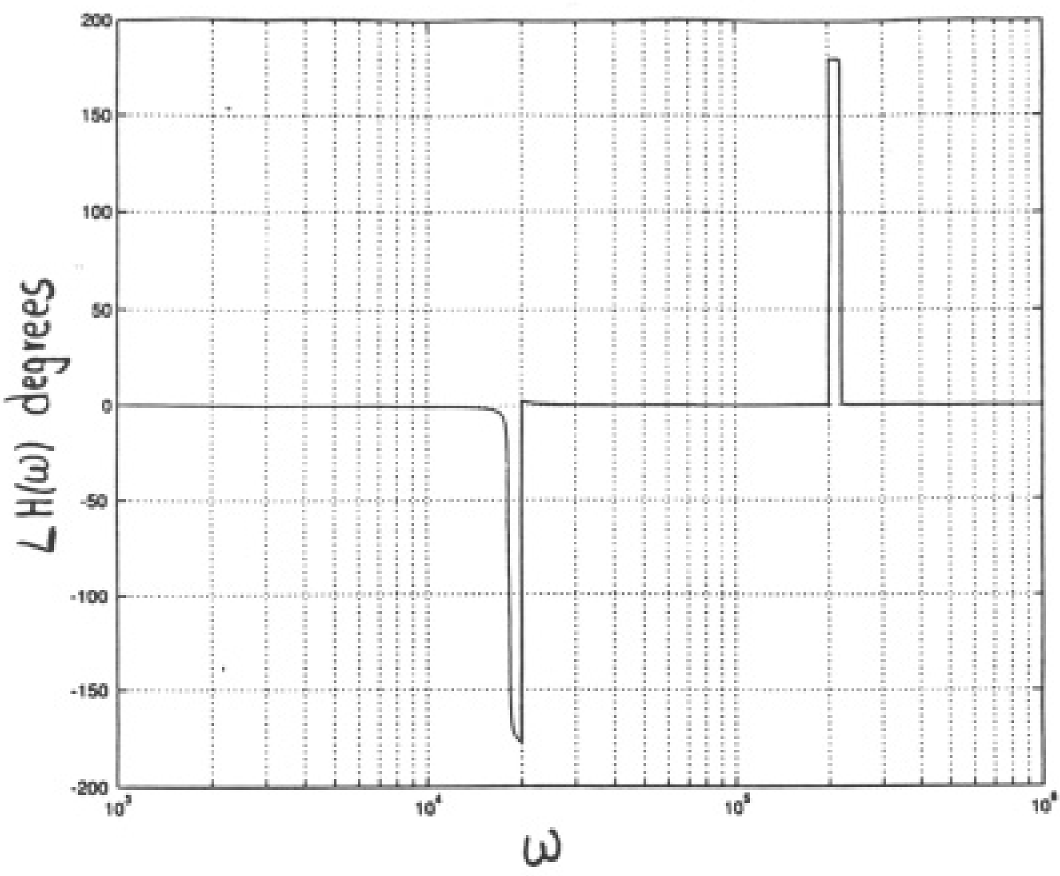
$$= \frac{R L_1 C_1 L_2 C_2 s^4 + R(L_1 C_1 + L_2 C_2)s^2 + R}{R L_1 C_1 L_2 C_2 s^4 + L_1 L_2 C_2 s^3 + [R(L_1 C_1 + L_2 C_2) + R L_1 C_2]s^2 + L_1 s + R}$$

$$\therefore \quad \boxed{H(s) = \frac{6.25\times10^{-17}s^4 + 2.5250\times10^{-6}s^2 + 10^3}{6.25\times10^{-17}s^4 + 1.25\times10^{-14}s^3 + 3.025\times10^{-6}s^2 + 5\times10^{-4}s + 10^3}}$$

(b) matlab program

```
num = [6.25x10^-17  0  2.525x10^-6  0  10^3];
denom = [6.25x10^-17  1.25x10^-14  3.025x10^-6  5x10^-4  10^3];
w = logspace(3, 6, 1000);
[mag, phase, w] = bode(num, denom, w);
magdb = -20*log10(mag);
semilogx(w, mag);
grid
semilogx(w, phase);
grid
```



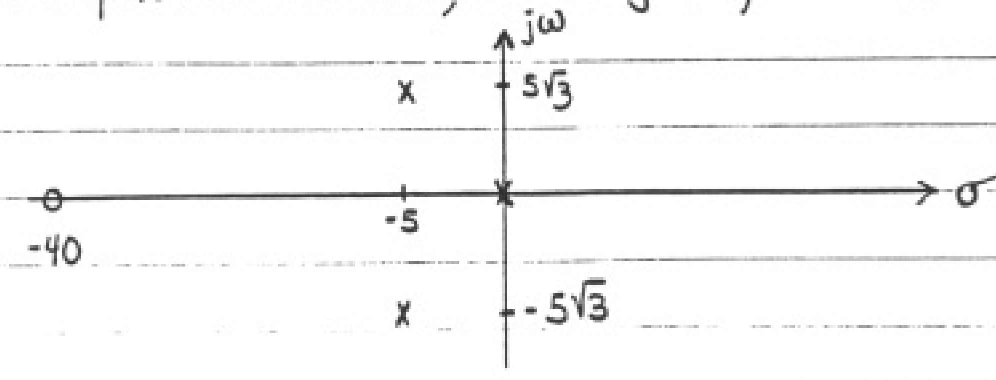


(c) Frequencies rejected are $\underline{2\times10^4 \text{ rad/s}}$ & $\underline{2\times10^5 \text{ rad/s}}$

2. $$H(s) = \frac{10s+400}{s^3+10s^2+100s}$$

zeros : $s=-40$

poles : $s=0$, $s=-5-j5\sqrt{3}$, $s=-5+j5\sqrt{3}$



HW #12 EECS 210 F99 Problem 2 (Bode plot solution)

$$H(s) = \frac{10s+400}{s^3+10s^2+100s}$$

$$= 4\,\frac{1+s/40}{s[(s/10)^2+s/10+1]}$$

$$= 4\,\frac{1+s/40}{s[(s/\omega_o)^2+2\zeta(s/\omega_o)+1]}$$

where $\omega_o = 10$ rad/s & $\zeta = .5$

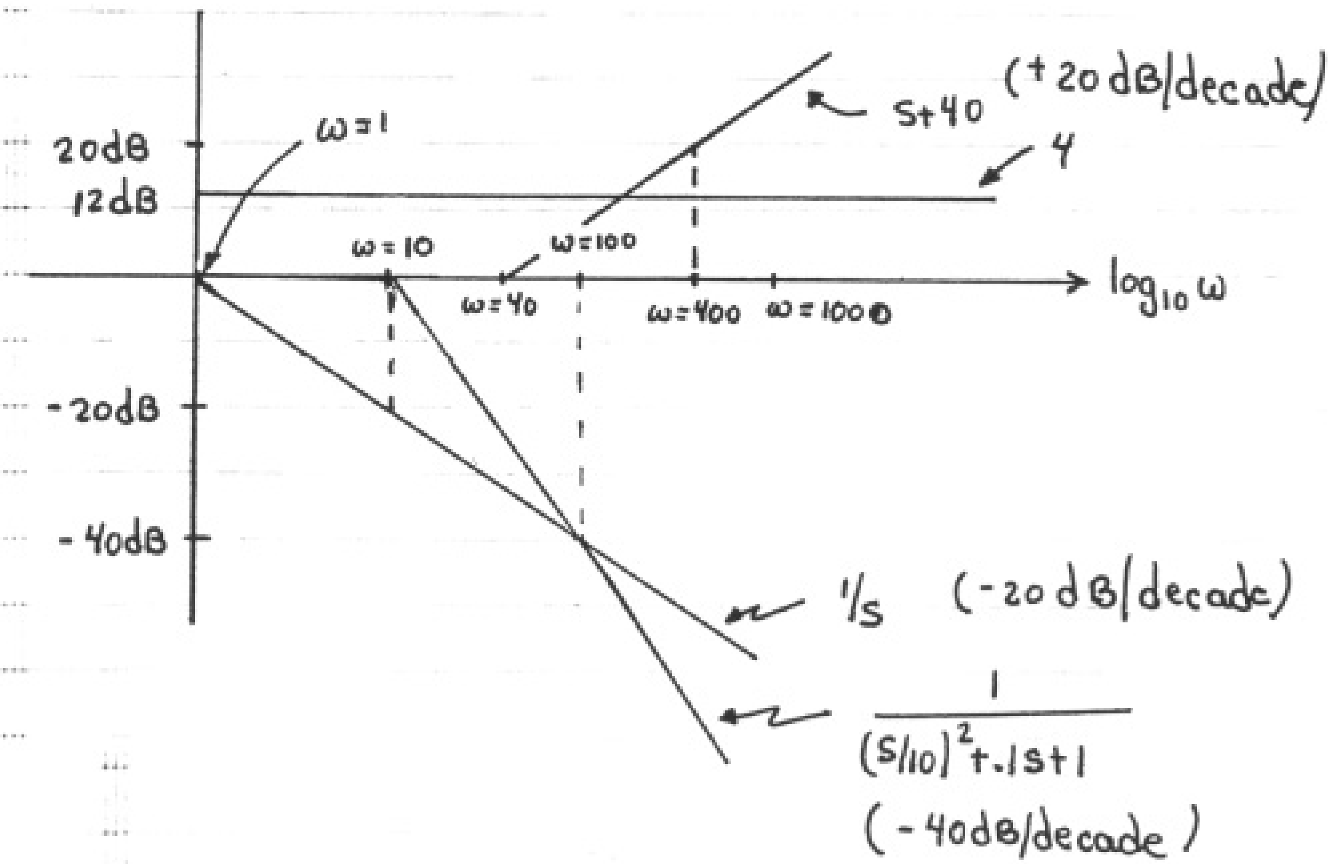
4 terms	asymptotic behavior
4	12 dB
s+40	0 dB over $0 \le \omega \le 40$ (corner freq.) & slope +20dB/decade over $\omega \ge 40$
s	0 dB @ $\omega = 1$ & slope -20dB/decade
$(s/\omega_o)^2 + 2\zeta(s/\omega_o)+1$	0 dB for $\omega \le \omega_o$; -40dB/decade slope for $\omega > \omega_o$

More accurately

$$\omega_p = \omega_o\sqrt{1-2\zeta^2} = .707\omega_o = 7.07 \text{ rad/s}$$

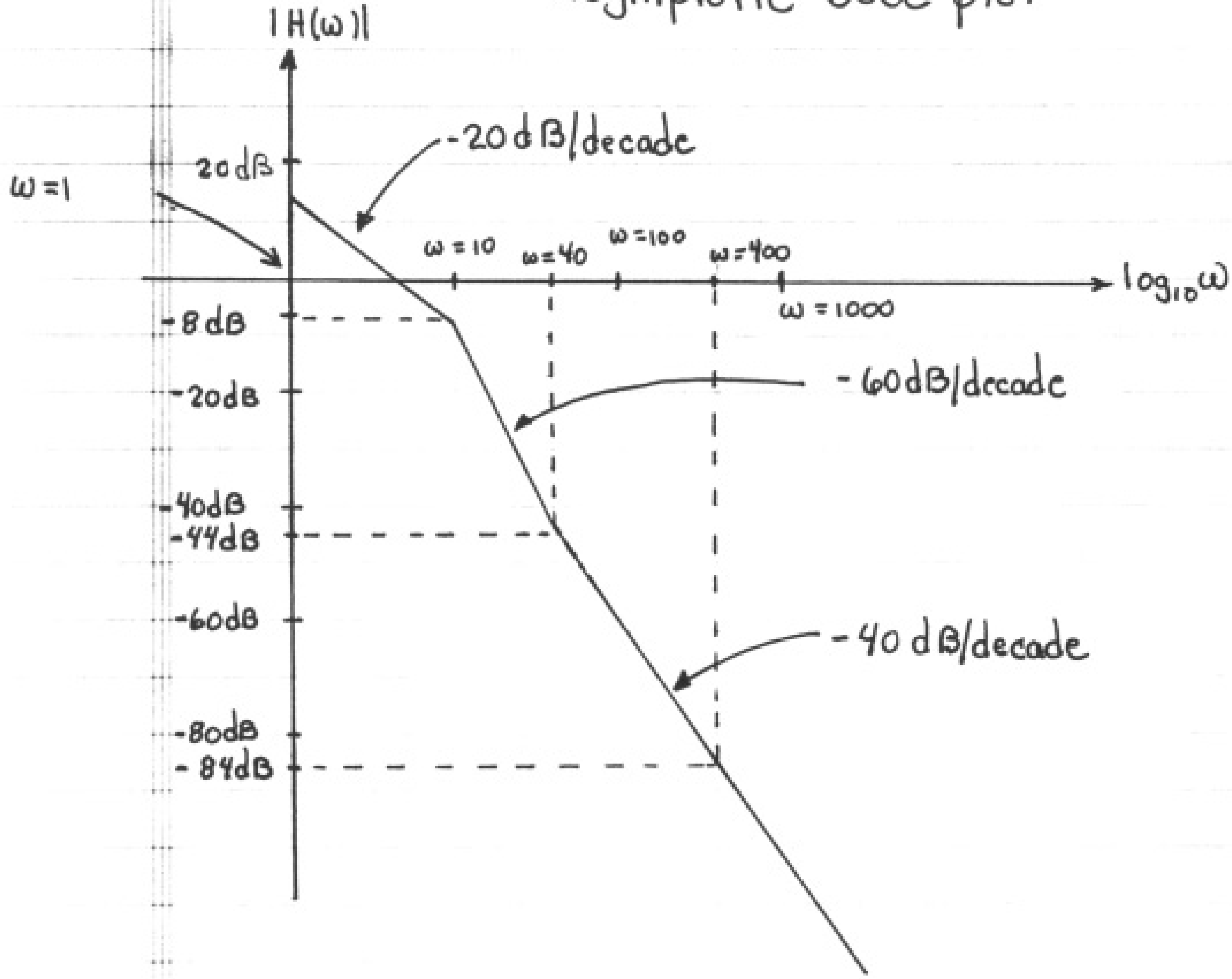
$$\max|H(\omega)| = \frac{1}{2\zeta[1-\zeta^2]} = 1.33 \;(2.5\text{ dB})$$

freq. where max of $|H(\omega)|$ occurs



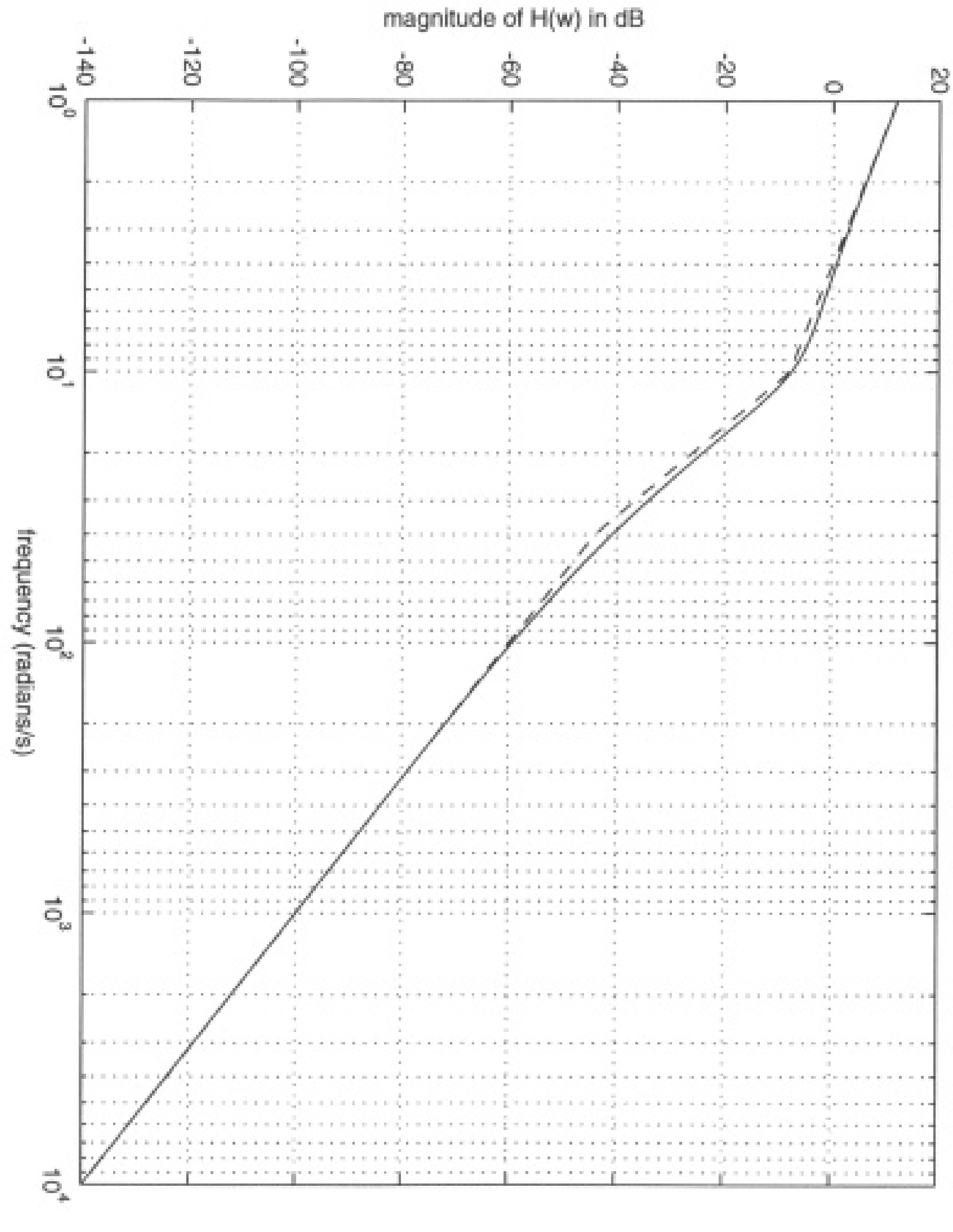
20dB
12dB
-20dB
-40dB
ω=1
ω=10
ω=100
ω=40
ω=400
ω=1000
$\log_{10}\omega$
s+40 (+20dB/decade)
4
1/s (-20dB/decade)
$\frac{1}{(s/10)^2+.1s+1}$
(-40dB/decade)

asymptotic bode plot

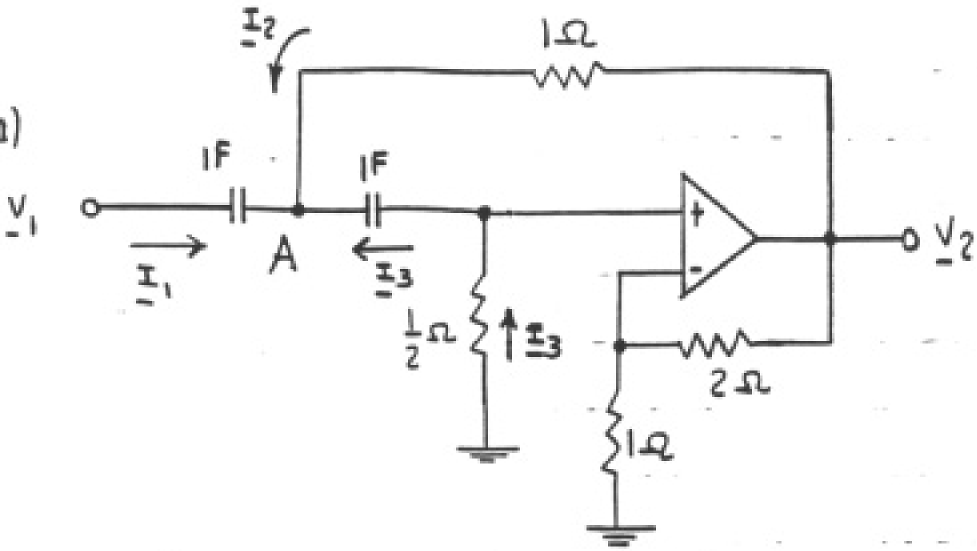


see next page for exact bode plot (solid curve) and asymptotic bode plot (-- curve) shown together

4


magnitude of H(w) in dB
frequency (radians/s)
20
0
-20
-40
-60
-80
-100
-120
-140
10^0
10^1
10^2
10^3
10^4

3(a)



$V_{(-)} = V_2/3 \quad (1) \qquad$ (voltage divider)

$V_{(+)} = V_{(-)} = V_2/3 \quad (2)$

$\therefore \; I_3 = \dfrac{0 - V_{(+)}}{1/2} = \dfrac{-2}{3} V_2 \quad (3)$

$V_A = V_{(+)} - I_3 Z_{1F} \quad (4)$

$= \dfrac{V_2}{3} + \dfrac{2}{3} V_2 \dfrac{1}{j\omega} \quad (5)$

$I_2 = \dfrac{V_2 - V_A}{1\Omega} = \dfrac{2}{3} V_2 - \dfrac{2}{3} V_2 \dfrac{1}{j\omega} \quad (6)$

$I_1 + I_2 + I_3 = 0 \quad (7)$

$\therefore \; I_1 = \dfrac{-2}{3} V_2 + \dfrac{2}{3} V_2 \dfrac{1}{j\omega} + \dfrac{2}{3} V_2 = \dfrac{2}{3} V_2 \dfrac{1}{j\omega} \quad (8)$

$V_1 - V_A = I_1 Z_{1F} \quad (9)$

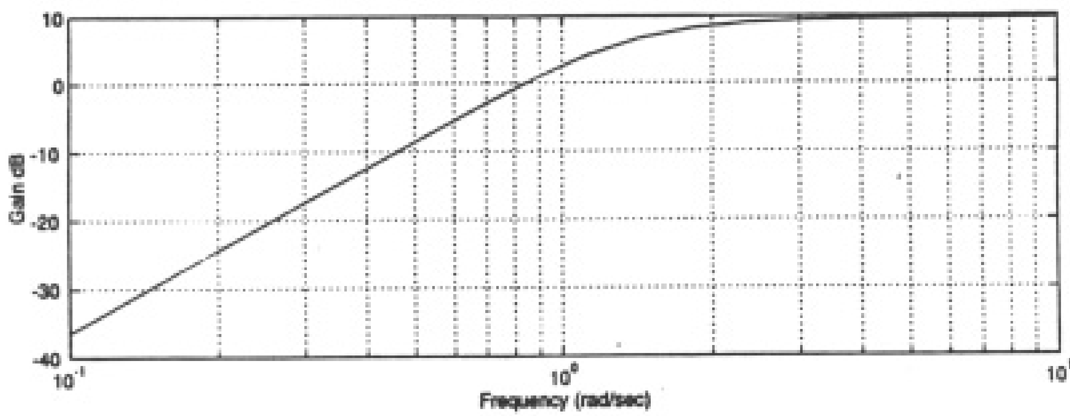
$= \dfrac{2}{3} V_2 \dfrac{1}{j\omega} \dfrac{1}{j\omega} = \dfrac{-2}{3\omega^2} V_2 \quad (10)$

$\therefore \; V_1 = \dfrac{V_2}{3} + \dfrac{2}{3} V_2 \dfrac{1}{j\omega} - \dfrac{2}{3\omega^2} V_2 \quad (11)$

$$\therefore \quad H(\omega) = \frac{3}{1 + \frac{2}{j\omega} + \frac{2}{(j\omega)^2}} \qquad (12)$$

$$\boxed{H(s) = \frac{3s^2}{s^2 + 2s + 2}} \qquad (13)$$

A <u>second order high pass filter</u>.



from Eq. (13)

(b) $|H(\omega = \infty)| = 3 \qquad (14)$

$|H(\omega_c)| = .707\,|H(\omega = \infty)| \qquad (15)$

$$|H(\omega_c)| = \frac{3\omega_c^2}{\sqrt{(2-\omega_c^2)^2 + 4\omega_c^2}} = \frac{3\omega_c^2}{\sqrt{4+\omega_c^4}} \qquad (16)$$

Combining Eqs. (14)-(16) yields

$$.707 = \frac{\omega_c^2}{\sqrt{4+\omega_c^4}}$$

$$\Longrightarrow \quad \omega_c = \sqrt{2} \ \text{rad/s}$$

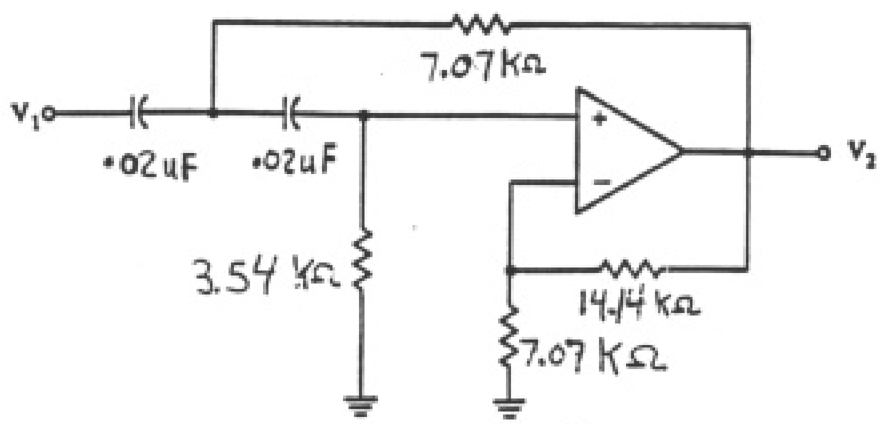
$$K_f = \frac{10^4}{\omega_c} = 7.072 \times 10^3$$

$$C_{new} = 0.02 \times 10^{-6} = \frac{C_{old}}{K_m K_f} = \frac{1}{7.07 \times 10^3 K_m}$$

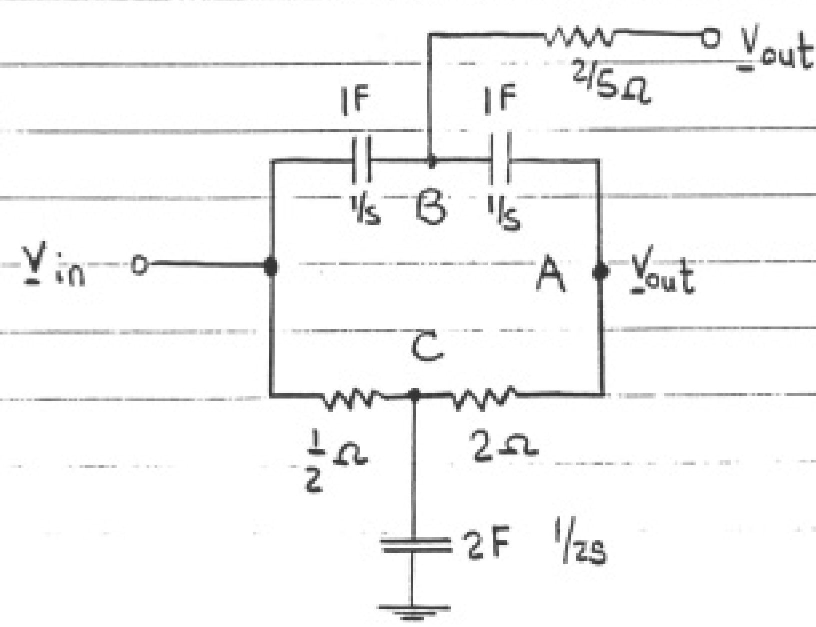
$$\therefore K_m = 7.07 \times 10^3$$

$$R_{new} = K_m R_{old} = 7.07 \times 10^3 R_{old}$$

Thus the scaled circuit becomes



4(a)



Write node equations @ nodes A, B & C.

@ A: $(\underline{V}_B - \underline{V}_{out})s + (\underline{V}_C - \underline{V}_{out})\frac{1}{2} = 0$ (1)

$\therefore (s + \frac{1}{2})\underline{V}_{out} - s\underline{V}_B - \frac{1}{2}\underline{V}_C = 0$ (2)

@ B: $(\underline{V}_{in} - \underline{V}_B)s + (\underline{V}_{out} - \underline{V}_B)5/2 + (\underline{V}_{out} - \underline{V}_B)s = 0$ (3)

$\therefore (2s + \frac{5}{2})\underline{V}_B - (s + \frac{5}{2})\underline{V}_{out} = s\underline{V}_{in}$ (4)

@ C: $(\underline{V}_{in} - \underline{V}_C)2 + (0 - V_C)2s + (\underline{V}_{out} - \underline{V}_C).5 = 0$ (5)

$\therefore (2s + 5/2)\underline{V}_C - \frac{1}{2}\underline{V}_{out} = 2\underline{V}_{in}$ (6)

From (4)

$$\underline{V}_B = \frac{(2s+5)\underline{V}_{out} + 2s\underline{V}_{in}}{4s+5} \quad (7)$$

From (6)

$$\underline{V}_C = \frac{\underline{V}_{out} + 4\underline{V}_{in}}{4s+5} \quad (8)$$

Substituting Eqs. (7) & (8) into Eq. (1) yields

$$\boxed{H(s) = \frac{\underline{V}_{out}}{\underline{V}_{in}} = \frac{s^2+1}{s^2+s+1}} \quad (9)$$

See Eq. (9.25) in your notes for a bandstop filter

$$H(s) = \frac{s^2 + \omega_0^2}{s^2 + (\Delta\omega)s + \omega_0^2} \quad (10)$$

Comparing Eqs. (9) & (10) we see that

$\omega_o = 1$ rad/s

$\Delta\omega = 1$ rad/s

$$\Longrightarrow \quad Q = \frac{\omega_o}{\Delta\omega} = 1$$

(b) $K_f = \dfrac{2\pi \times 60}{\omega_o} = \dfrac{2\pi \times 60}{1} = 377$

$$1\,nF = C_{new} = \frac{C_{old}}{K_m K_f} = \frac{1\,F}{K_f K_m} = \frac{1\,F}{377\,K_m}$$

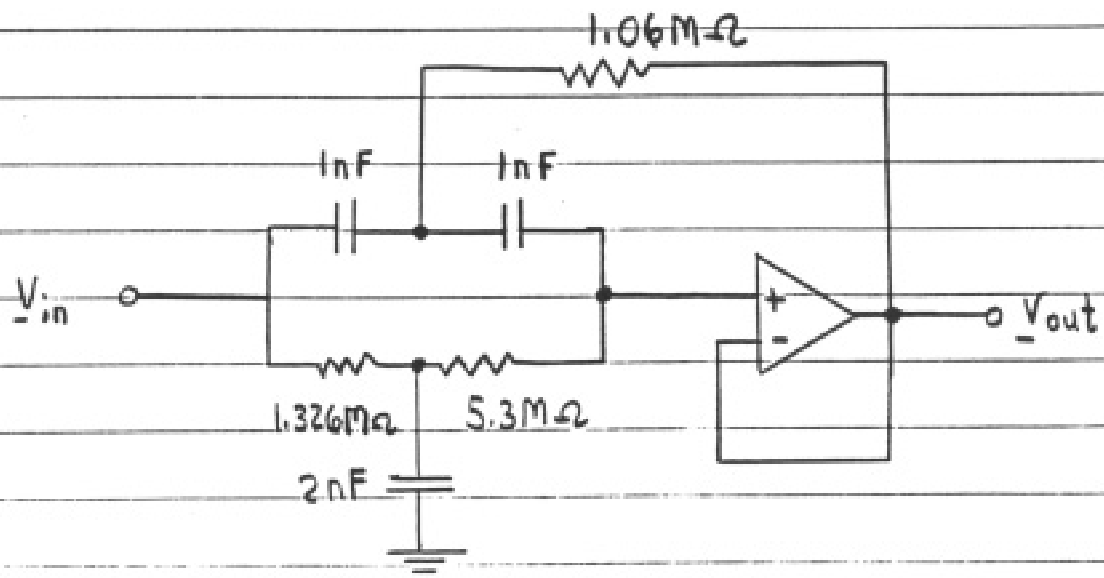
$$\Longrightarrow K_m = 2.652 \times 10^6$$

$$\therefore R_{new} = K_m R_{old} = 2.652 \times 10^6 R_{old}$$

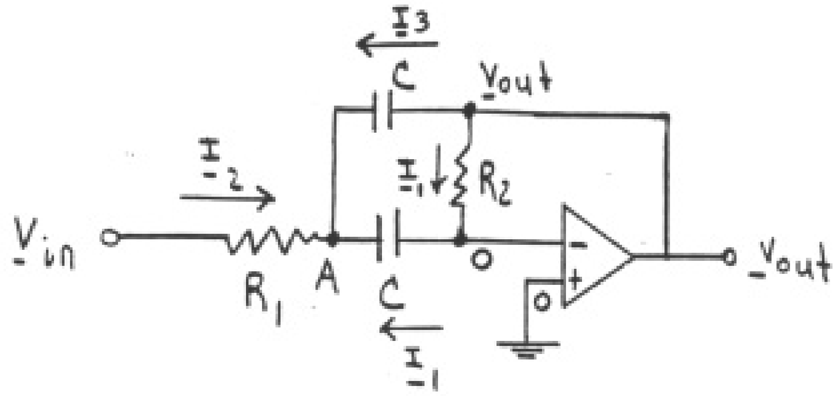
$$\Rightarrow \frac{1}{2}\,\Omega \longrightarrow 1.326\,M\Omega$$

$$2\,\Omega \longrightarrow 5.3\,M\Omega$$

$$\frac{2}{5}\,\Omega \longrightarrow 1.06\,M\Omega$$



5 (a) similar to Op Amp problem on exam #2



$$\underline{I}_1 = \underline{V}_{out}/R_2 \qquad (1)$$

$$\therefore \underline{V}_A = -\underline{I}_1 \frac{1}{sC} = \frac{-\underline{V}_{out}}{R_2 C s} \qquad (2)$$

$$\underline{I}_3 = (\underline{V}_{out} - \underline{V}_A) sC = \underline{V}_{out}\left[1 + \frac{1}{R_2 C s}\right] sC \qquad (3)$$

$$\underline{I}_2 = -\underline{I}_1 - \underline{I}_3 = -\underline{V}_{out}\left[\frac{2}{R_2} + sC\right] \qquad (4)$$

$$\underline{V}_{in} - \underline{V}_A = \underline{I}_2 R_1 \qquad (5)$$

$$\therefore \underline{V}_{in} = \frac{-\underline{V}_{out}}{R_2 C s} - \underline{V}_{out}\left[\frac{2R_1}{R_2} + sR_1 C\right] \qquad (6)$$

$$\Longrightarrow \boxed{H(s) = \frac{\underline{V}_{out}}{\underline{V}_{in}} = \frac{-R_2 C s}{R_1 R_2 C^2 s^2 + 2R_1 C s + 1}} \qquad (7)$$

(b) Recall from Eq. (9.24) in your notes that

$$H(s) = \frac{(\Delta\omega)s}{s^2 + (\Delta\omega)s + \omega_0^2} \qquad (8)$$

is a bandpass filter. Comparing Eqs. (7) & (8) yields

$$\boxed{\begin{aligned} \omega_o &= \frac{1}{\sqrt{R_1R_2}\,C} \\ \Delta\omega &= \frac{2}{R_2 C} \end{aligned}}$$

center freq. (9)

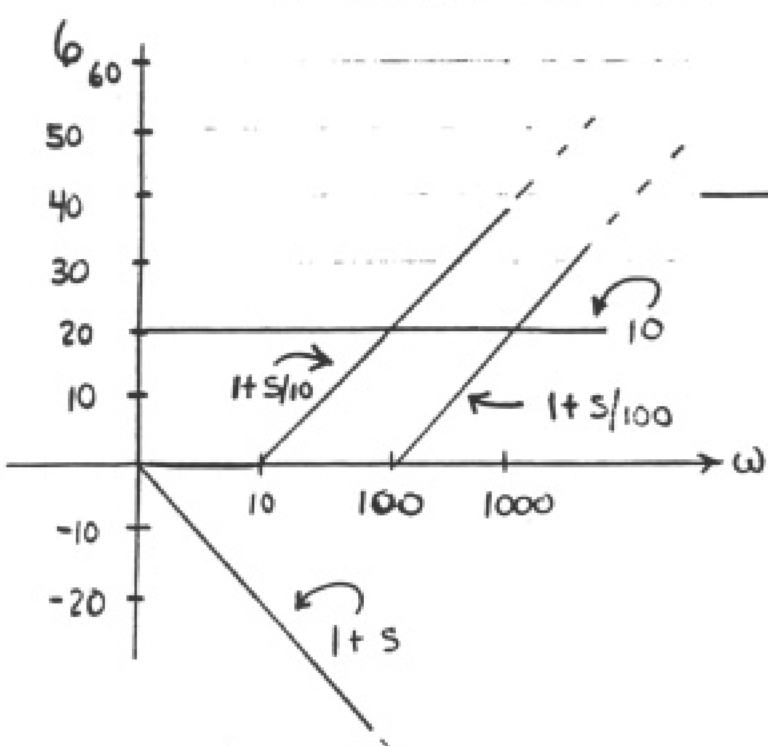
FWHM bandwidth (10)

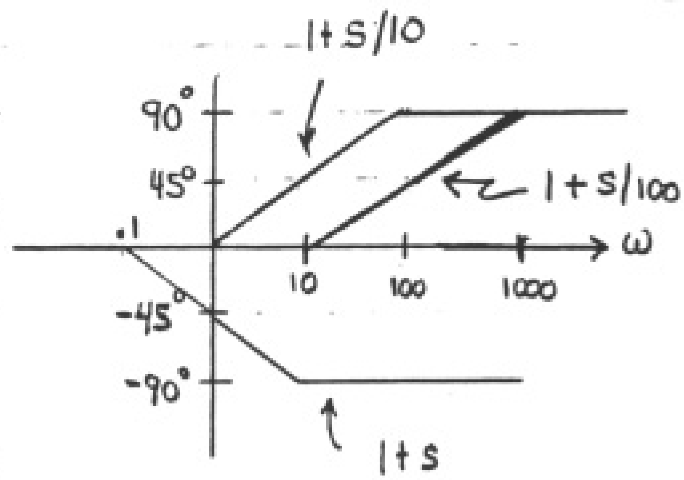
(c) $\omega_o = 2\pi \times 500 = \dfrac{1}{\sqrt{R_1R_2}\,10^{-8}}$

$\Delta\omega = 2\pi \times 50 = \dfrac{2}{R_2 10^{-8}}$

$\therefore$ $\underline{R_2 = 636.6\,\mathrm{k}\Omega}$ & $\underline{R_1 = 1.59\,\mathrm{k}\Omega}$

Also note $|H|_{max} = R_2/(2R_1) = 46\,\mathrm{dB}$





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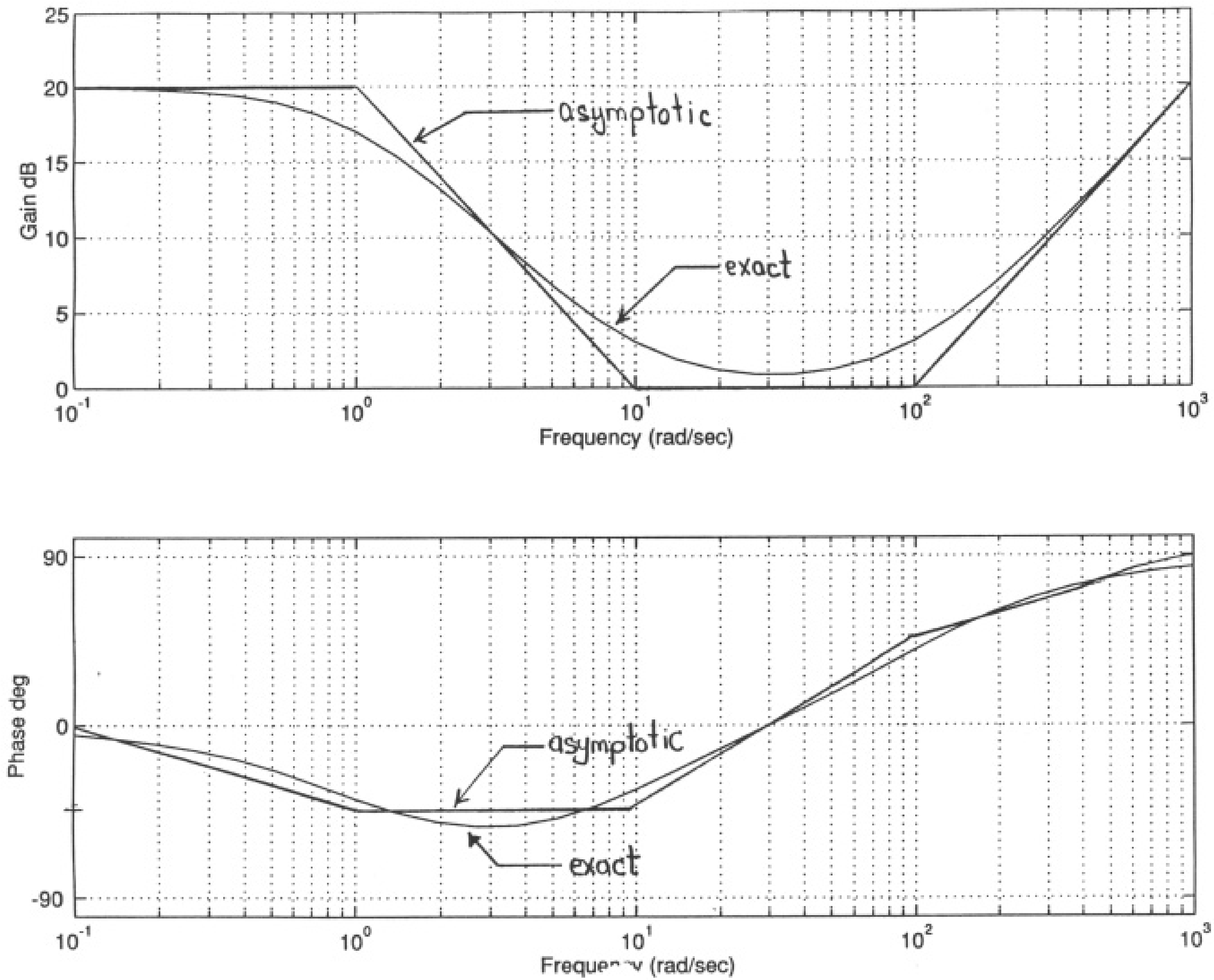

Gain dB
25
20
15
10
5
0
asymptotic
exact
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/sec)
Phase deg
90
0
-90
asymptotic
exact
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/sec)

7.

$$H(s) \approx .02\,\frac{(1+s/.1)^2}{(1+s/.7)^2}\,\frac{(1+s/10)}{(1+s/100)}\,\frac{1}{(1+s/600)^2}$$

Because:

$.02 \rightarrow -34\,dB$

$(1+s/.1)^2$ + 40dB/decade with corner freq. $\omega = .1$ rad/s

$(1+s/.7)^2$ - 40dB/decade with corner freq. $\omega = .7$ rad/s

$(1+s/10)$ + 20dB/decade with corner freq. $\omega = 10$ rad/s

$(1+s/100)$ -20dB/decade with corner freq. $\omega = 100$ rad/s

$(1+s/600)^2$ -40dB/decade with corner freq. $\omega = 600$ rad/s