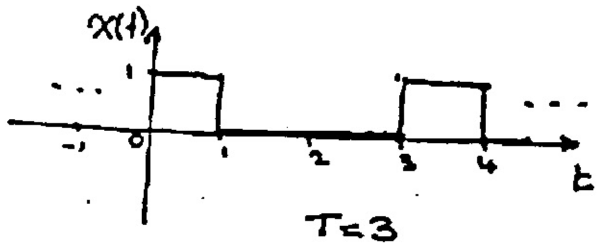
Problem 2



$T=3$

(a) $f = \frac{1}{T} = \frac{1}{3}\text{ Hz}$

(b) $\omega = 2\pi f = 2\pi \cdot \frac{1}{3} = \frac{2\pi}{3}\text{ rad/sec}$

(c) $f_3 = 3f = 3\cdot\frac{1}{3} = 1\text{ Hz}$

$f_5 = 5f = 5\cdot\frac{1}{3} = \frac{5}{3}\text{ Hz}$

(d) $a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt = \frac{1}{3}\int_0^1 1\cdot dt + \cancel{\frac{1}{3}\int_1^2 0\cdot dt}^{\,0} = \underline{\underline{\frac{1}{3}}}$

$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos\left(\frac{2\pi}{T}nt\right)dt = \frac{2}{3}\int_0^1 1\cdot\cos\left(\frac{2\pi}{3}nt\right)dt + 0$

$= \frac{2}{3}\cdot\frac{3}{2\pi n}\sin\left(\frac{2\pi n}{3}\right)\Big|_0^1 = \frac{1}{\pi n}\left[\sin\left(\frac{2\pi n}{3}\right) - \sin(0)\right] = \frac{1}{\pi n}\sin\left(\frac{2\pi n}{3}\right)$
$\underline{\underline{\text{for } n\neq 0}}$

$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin\left(\frac{2\pi}{T}nt\right)dt = \frac{2}{3}\int_0^1 1\cdot\sin\left(\frac{2\pi}{3}nt\right)dt + 0$

$= \frac{2}{3}\cdot\frac{3}{2\pi n}\left(-\cos\left(\frac{2\pi n}{3}\right)\right)\Big|_0^1 = -\frac{1}{\pi n}\left[\cos\left(\frac{2\pi n}{3}\right) - \cos(0)\right]$

$= \underline{\underline{\frac{1}{\pi n}\left[1 - \cos\left(\frac{2\pi n}{3}\right)\right],\ \text{for } n\neq 0}}$

(e) $$C_n = \sqrt{a_n^2 + b_n^2} = \sqrt{\left(\frac{1}{\pi n}\sin\left(\frac{2\pi n}{3}\right)\right)^2 + \left(\frac{1}{\pi n}\left[1 - \cos\left(\frac{2\pi n}{3}\right)\right]\right)^2}$$

$$= \frac{1}{\pi n}\sqrt{\sin^2\left(\frac{2\pi n}{3}\right) + 1 + \cos^2\left(\frac{2\pi n}{3}\right) - 2\cos\left(\frac{2\pi n}{3}\right)}$$

Use $\cos^2\theta + \sin^2\theta = 1$

$$\Rightarrow C_n = \frac{1}{\pi n}\sqrt{1 + 1 - 2\cos\left(\frac{2\pi n}{3}\right)}$$

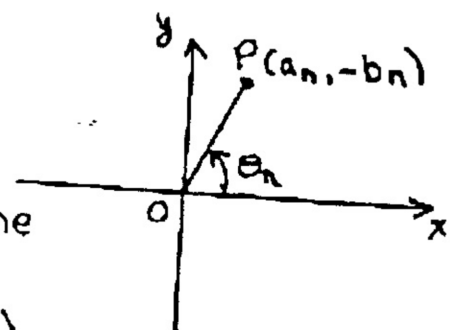
Use $1 - \cos 2\theta = 2\sin^2\theta$

$$\Rightarrow C_n = \frac{\sqrt{2}}{\pi n}\sqrt{1 - \cos\left(\frac{2\pi n}{3}\right)} = \frac{\sqrt{2}}{\pi n}\sqrt{2\sin^2\left(\frac{\pi n}{3}\right)} = \frac{2}{\pi n}\left|\sin\left(\frac{\pi n}{3}\right)\right|$$

$$C_0 = a_0 = \frac{1}{3}$$

$$\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

$$= \tan^{-1}\left(\frac{-\left[1 - \cos\left(\frac{2\pi n}{3}\right)\right]}{\sin\left(\frac{2\pi n}{3}\right)}\right)$$



To find θ_n, draw the line from $O(0,0)$ to $P(a_n, -b_n)$.

θ_n is the anticlockwise angle between the x-axis and the line OP as shown above.

$$\because \quad \underbrace{\tan^{-1}\left(\frac{-2}{-2}\right)}_{= \pi + \pi/4 = \frac{5\pi}{4}} \neq \underbrace{\tan^{-1}\left(\frac{2}{2}\right)}_{= \pi/4}$$

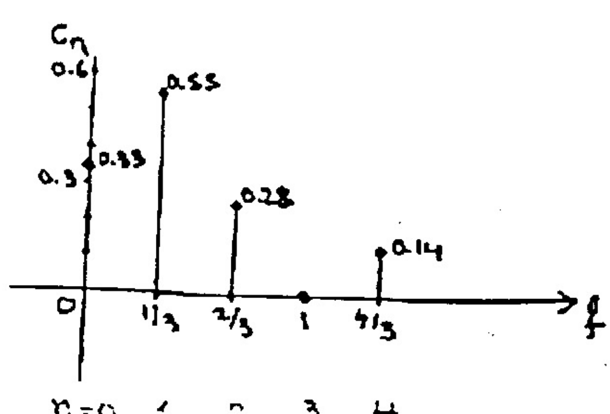
(f) Magnitude Spectra:

$n=0$: $C_0 = a_0 = \frac{1}{3} = 0.33$

$n=1$: $C_1 = \left|\frac{2}{\pi}\sin\frac{\pi}{3}\right| = 0.55$

$n=2$: $C_2 = \left|\frac{2}{2\pi}\sin\frac{2\pi}{3}\right| = 0.28$

$n=3$: $C_3 = \left|\frac{2}{3\pi}\sin\frac{3\pi}{3}\right| = 0$



Phase Spectra

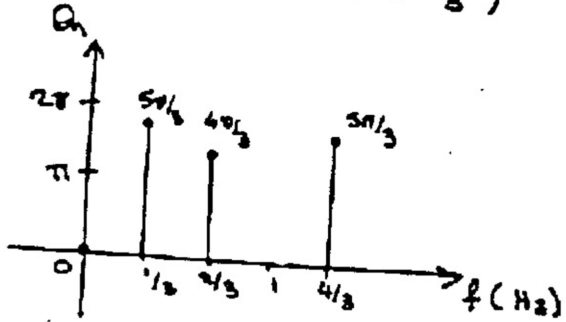
$n=0 \qquad \theta_0 = 0$

$n=1 \qquad \theta_1 = \tan^{-1}\left(\dfrac{-[1-\cos(\frac{2\pi}{3})]}{\sin(\frac{2\pi}{3})}\right) = \tan^{-1}\left(\dfrac{-1.5}{0.8660}\right) = \dfrac{5\pi}{3}$

$n=2 \qquad \theta_2 = \tan^{-1}\left(\dfrac{-[1-\cos(\frac{2\pi \cdot 2}{3})]}{\sin(\frac{2\pi \cdot 2}{3})}\right) = \tan^{-1}\left(\dfrac{-1.5}{-0.8660}\right) = \dfrac{4\pi}{3}$

$n=3 \qquad \theta_3 =$ undefined since $c_3 = 0$

$n=4 \qquad \theta_4 = \tan^{-1}\left(\dfrac{-[1-\cos(\frac{2\pi \cdot 4}{3})]}{\sin(\frac{2\pi \cdot 4}{3})}\right) = \tan^{-1}\left(\dfrac{-1.5}{0.8660}\right) = \dfrac{5\pi}{3}$



Problem 3

(a) $T = 8\,\mu sec$

(b) $f = \frac{1}{T} = 125\ KHz$

$\omega = 2\pi f = 2\pi * 125 KHz = 785.4\ Krad/sec$

(c) $f_4 = 4f = 500 KHz$

(d)
$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt = \frac{1}{8}\left[\int_{-2}^{-1} 25\,dt + \int_{-1}^{1} 50\,dt + \int_{1}^{2} 25\,dt\right]$$
$$= \frac{1}{8}\left[25(1) + 50(2) + 25(1)\right] = \frac{150}{8} = \underline{\underline{18.75}}$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos\left(\frac{2\pi}{T}nt\right)dt, \quad n \neq 0$$
$$= \frac{2}{8}\left[\int_{-2}^{-1} 25\cos\left(\frac{2\pi nt}{8}\right)dt + \int_{-1}^{1} 50\cos\left(\frac{2\pi nt}{8}\right)dt + \int_{1}^{2} 25\cos\left(\frac{2\pi nt}{8}\right)dt\right]$$
$$= \frac{2}{8}\left[\frac{8}{2\pi n}\cdot 25\sin\left(\frac{2\pi nt}{8}\right)\Big|_{-2}^{-1} + \frac{8}{2\pi n}\,50\sin\left(\frac{2\pi nt}{8}\right)\Big|_{-1}^{1} + \frac{8}{2\pi n}\,25\sin\left(\frac{2\pi n}{8}\right.\right.$$
$$= \frac{25}{\pi n}\left[\sin\left(-\frac{2\pi n}{8}\right) - \sin\left(-\frac{2\pi n\cdot 2}{8}\right) + \sin\left(\frac{2\pi n\,2}{8}\right) - \sin\left(\frac{2\pi n}{8}\right)\right]$$
$$+ \frac{50}{\pi n}\left[\sin\left(\frac{2\pi n}{8}\right) - \sin\left(-\frac{2\pi n}{8}\right)\right]$$
$$= \frac{25}{\pi n}\left[2\sin\left(\frac{\pi n}{2}\right) - 2\sin\left(\frac{\pi n}{4}\right)\right] + \frac{50}{\pi n}\left[2\sin\left(\frac{\pi n}{4}\right)\right]$$
$$= \frac{50}{\pi n}\sin\left(\frac{\pi n}{2}\right) + \frac{50}{\pi n}\sin\left(\frac{\pi n}{4}\right) = \frac{50}{\pi n}\left[\sin\left(\frac{\pi n}{2}\right) + \sin\left(\frac{\pi n}{4}\right)\right]$$

$b_n = 0$ by observation (even function).

To show this:
$$b_n = \frac{2}{8}\int_{-2}^{-1} 25\sin\left(\frac{2\pi nt}{8}\right)dt + \frac{2}{8}\int_{1}^{2} 25\sin\left(\frac{2\pi nt}{8}\right)dt + \frac{2}{8}\int_{-1}^{1} 50\sin\left(\frac{2\pi nt}{8}\right)d\ldots$$
$n \neq \ldots$
$$= +\frac{2}{8}\,\frac{8}{2\pi n}\,25\left[\cancel{\cos\left(\frac{2\pi n}{8}\right)} + \cos\left(\frac{2\pi n2}{8}\right) - \cos\left(\frac{2\pi n2}{8}\right) + \cancel{\cos\left(\frac{2\pi n}{8}\right)} - \right.$$
$$\left. - 2\cos\left(\frac{2\pi n}{8}\right) + 2\cos\left(\frac{2\pi n}{8}\right)\right]$$
$= 0$

(e)

$$C_n = \sqrt{a_n^2 + b_n^2} = a_n \quad n \neq 0 \quad \text{since } b_n = 0$$

$$\Rightarrow C_n = \frac{50}{\pi n}\left|\left[\sin\left(\frac{\pi n}{2}\right) + \sin\left(\frac{\pi n}{4}\right)\right]\right| \quad , n \neq 0$$

and $c_0 = a_0$

$$\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right) = \tan^{-1}\left(\frac{-b_n=0}{a_n}\right) = 0 \text{ or } \pi \text{ depending on the sign of } a_n$$

(f)

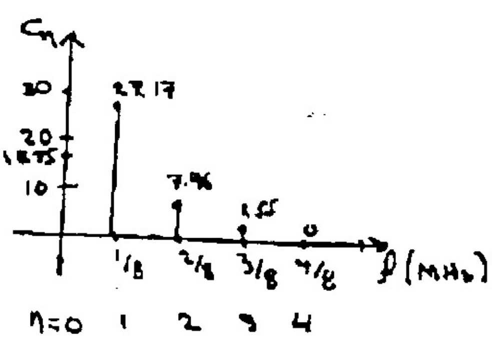
$C_0 = 18.75$

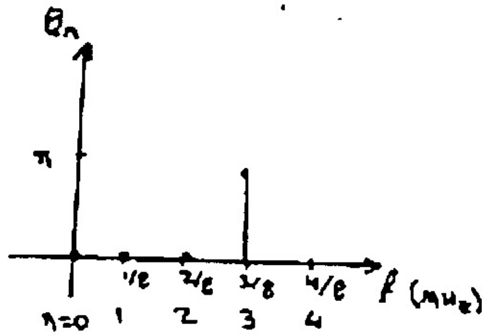
$n=1 \quad C_1 = \frac{50}{\pi}\left|\left[\sin\frac{\pi}{2} + \sin\frac{\pi}{4}\right]\right| = 27.17 \quad \Rightarrow \theta_1 = 0 \quad (a_n > 0)$

$n=2 \quad C_2 = \frac{50}{\pi 2}\left|\left[\sin(\pi) + \sin\left(\frac{\pi 2}{4}\right)\right]\right| = 7.96 \quad \Rightarrow \theta_2 = 0 \quad (a_n > 0)$

$n=3 \quad C_3 = \frac{50}{\pi 3}\left|\left[\sin\left(\frac{3\pi}{2}\right) + \sin\left(\frac{3\pi}{4}\right)\right]\right| = 1.55 \quad \Rightarrow \theta_3 = \pi \quad (a_n < 0)$

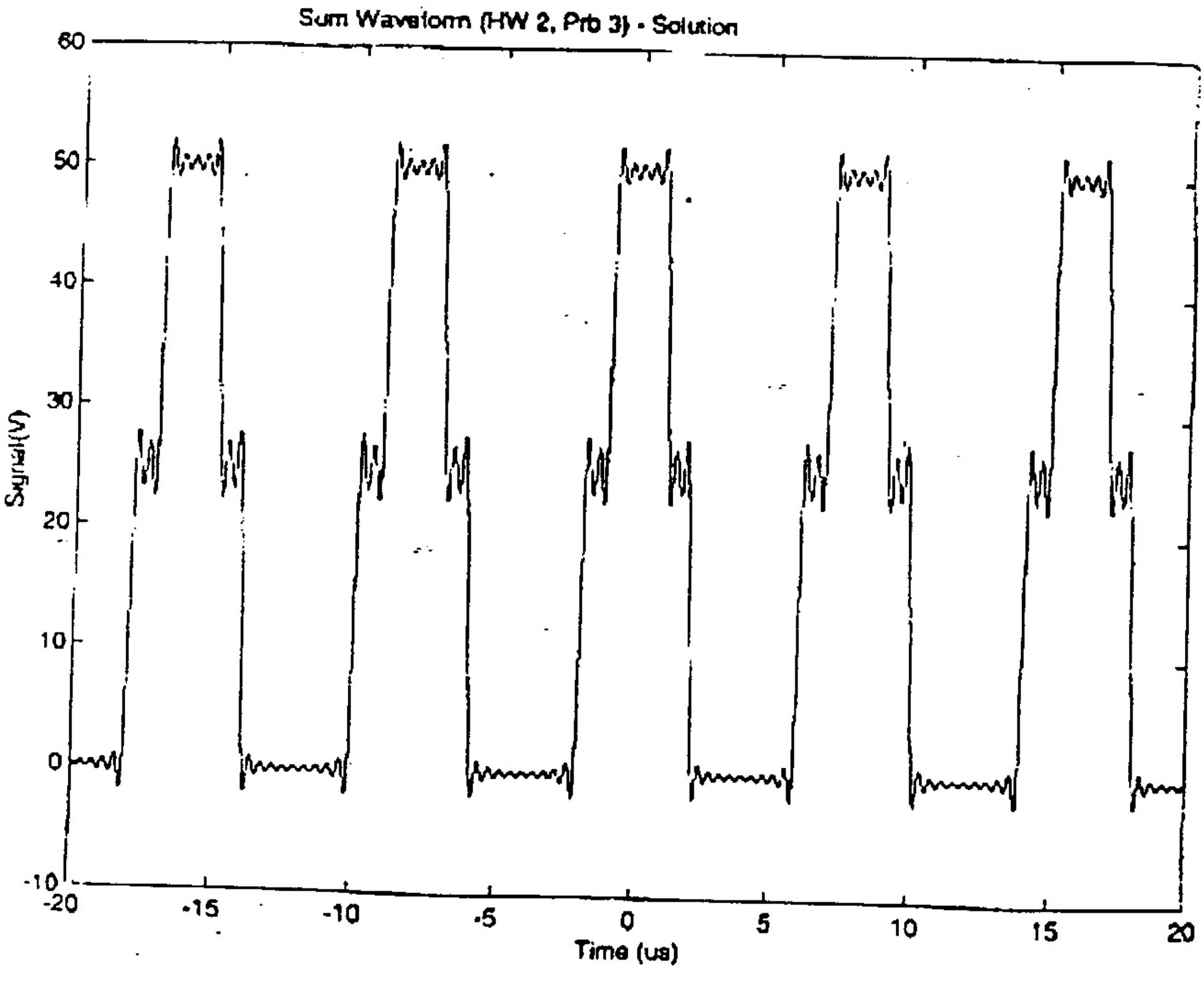
$n=4 \quad C_4 = \frac{50}{\pi 4}\left|\left[\sin 2\pi + \sin\pi\right]\right| = 0 \quad \Rightarrow \theta_4$ is undefined.





(g)

```
%Matlab Code for HW2 Problem 3/Solutions
%Generation of the signal
t=linspace(-20,20,10000);
n=1;
x=0;
for n=1:1:20
   cn=(50/(pi*n))*(sin(pi*n/2)+sin(pi*n/4));
   xn=cn*cos(2*pi*n*t/8);
   x=x+xn;
end
c0=18.75;
x=x+c0;
plot(t,x);
xlabel('Time (us)')
ylabel('Signal(y)')
title('S[illegible] 2, [illegible] - Solution, September [illegible]')
```



Sum Waveform (HW 2, Prb 3) - Solution
60
50
40
30
20
10
0
-10
Signal(V)
-20
-15
-10
-5
0
5
10
15
20
Time (us)

Problem 4

$$y(t) = x(t) - x(t-1)$$

(a) Let $x_1(t) \to y_1(t) \Rightarrow y_1(t) = x_1(t) - x_1(t-1)$

$x_2(t) \to y_2(t) \Rightarrow y_2(t) = x_2(t) - x_2(t-1)$

$$x(t) = a x_1(t) + b x_2(t)$$

$$x(t-1) = a x_1(t-1) + b x_2(t-1)$$

$$y(t) = x(t) - x(t-1) = [a x_1(t) + b x_2(t)] - [a x_1(t-1) + b x_2(t-1)]$$

$$= a[x_1(t) - x_1(t-1)] + b[x_2(t) - x_2(t-1)]$$

$$= a\, y_1(t) + b\, y_2(t)$$

$\Rightarrow$ system is linear

(b) Let $x(t) = x_1(t) \Rightarrow y_1(t) = x_1(t) - x_1(t-1)$

$x(t) = x_2(t) = x_1(t-t_0) \Rightarrow y_2(t) = x_2(t) - x_2(t-1)$

$= x_1(t-t_0) - x_1(t-t_0-1)$

$\Rightarrow y_2(t) = y_1(t-t_0) \Rightarrow$ system is time invariant.

(c) $|H(f)| = \left|\dfrac{A_{out}}{A_{in}}\right| = \dfrac{|2A \sin \omega/2|}{A} = 2|\sin \omega/2|$, $\omega = 2\pi f$.

$$\text{angle } H(f) = \phi - \theta = \left(\frac{\pi}{2} + \theta - \frac{\omega}{2}\right) - \theta \left\{ \begin{array}{ll} = \frac{\pi}{2} - \frac{\omega}{2}, & \text{provided } \sin\frac{\omega}{2} > \\ = \frac{3\pi}{2} - \frac{\omega}{2} & \text{if } \sin\frac{\omega}{2} < 0 \end{array} \right.$$

(d) $x(t) = \sin(2\pi 50 t + \pi/3)$

In this case $\omega_1 = 50\,Hz$, $A = 1$, $\theta = \pi/3$

$A_{out} = B = |H(f)| \cdot A = 2\sin \omega/2 = 2\sin(2\pi 50/2) = 0.$

No need to find ϕ since the output is 0.

Problem 5

(a) $x(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos\left(\frac{2\pi}{T} nt + \theta_n\right)$

$$y(t) = \frac{1}{3}|H(0)| + \sum_{n=1}^{\infty} |H(n/3)| \frac{2}{\pi n} \sin\frac{\pi n}{3} \cdot \cos\left(\frac{2\pi}{3} nt - \frac{\pi n}{3} + \angle\right.$$

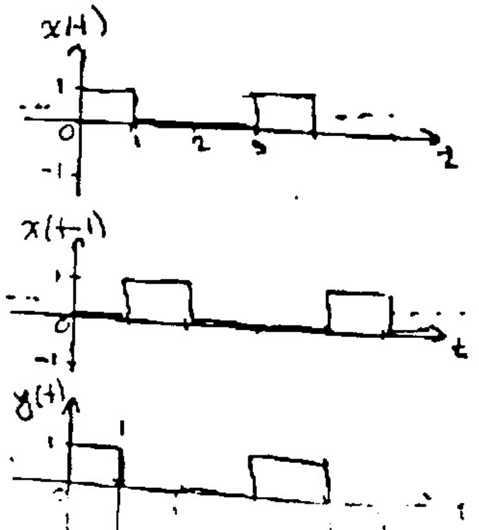
We can do this only because the system linear, time invariant.

$$y(t) = \sum_{n=1}^{\infty} \frac{4}{\pi n} \left|\sin\frac{\pi n}{3}\right| \cdot \sin\frac{\pi n}{3} \cos\left(\frac{2\pi n t}{3} + \frac{\pi}{2} - \frac{2\pi n}{3}\right)$$

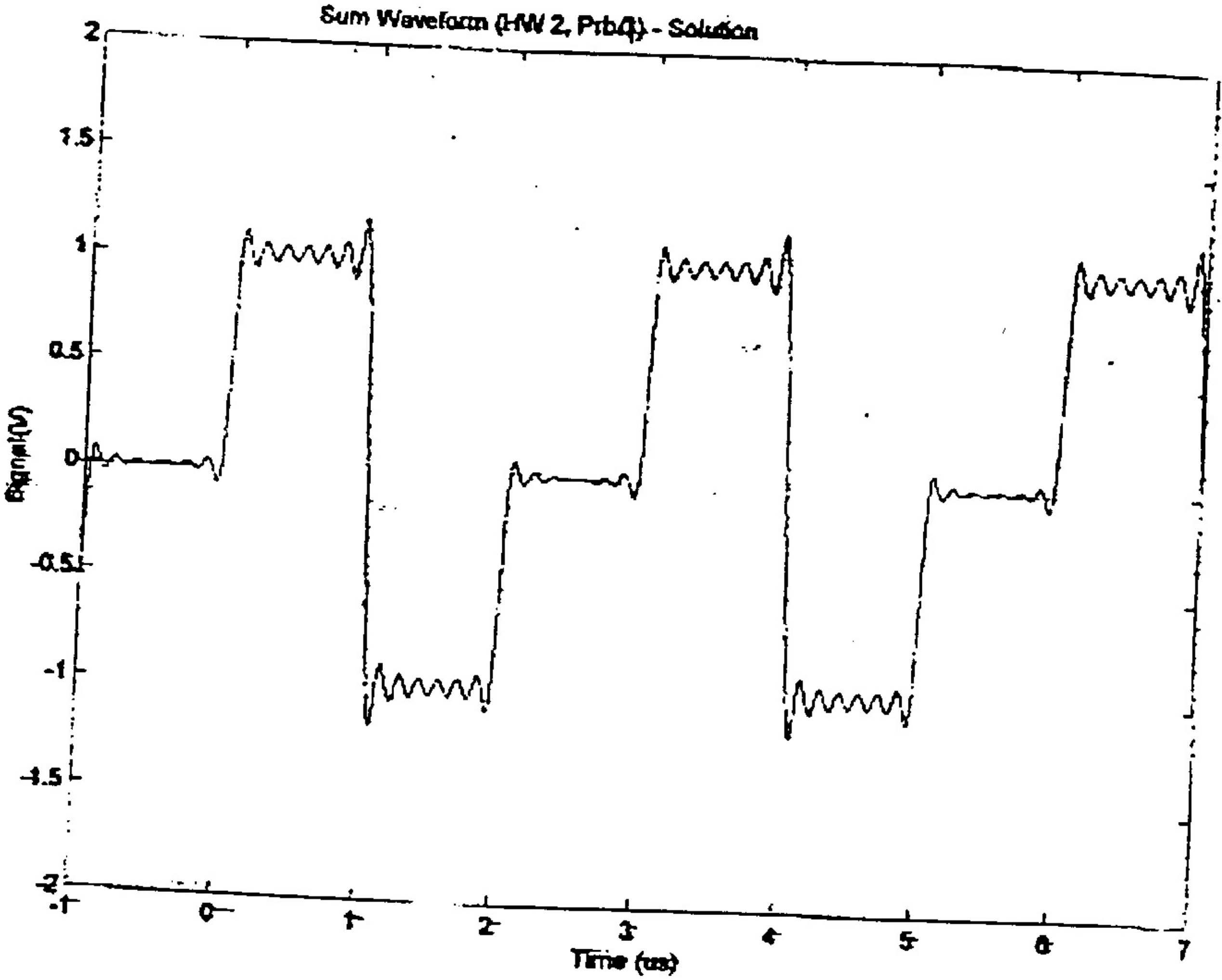
(b) Run the following MATLAB program:

```
%Matlab Code for HW2 Problem 4/Solutions
%Generation of the signal
t=linspace(-20,20,10000);
n=1;
x=0;
for n=1:1:20
    cn=((4/(pi*n))*sin(pi*n/3)*sin(pi*n/3));
    xn=cn*cos(2*pi*n*t/3+pi/2-2*pi*n/3);
    x=x+xn;
end
plot(t,x);
xlabel('Time (us)')
ylabel('Signal(V)')
title('Sum Waveform (HW 2, Prb 4) - Solution, September 19th, 1999')
axis ([-1 7 -2 2]);
```

c)



Result (b) (next page) agrees with result (c).



Sum Waveform (HW 2, Prb4) - Solution
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
Signal(V)
-1
0
1
2
3
4
5
6
7
Time (us)

Problem 6

Let $x_1(t) = 2\sin(2\pi 50t + \pi/2)$

$x_2(t) = 4\cos(2\pi 30t + \pi/3)$

$\Rightarrow x(t) = x_1(t) + x_2(t)$

Since the system is linear time-invariant, we can pass each component of $x(t)$ through the circuit separately and then add the two outputs to get $y(t)$

$x_1(t) \longrightarrow y_1(t)$

$x_2(t) \rightarrow y_2(t)$

$|H(f=50)| = 0 \Rightarrow y_1(t) = 0$

$|H(f=30)| = \frac{6}{3} = 2$

$\underline{/H(f=30)} = \pi/2 - \pi/6 = \pi/3$

$$\Rightarrow y_2(t) = |H(f=30)| \cdot 4\cos(2\pi 30t + \pi/3 + \underline{/H(f=30)})$$

$$= 8\cos(2\pi 30t + \pi/3 + \pi/3)$$

$$= 8\cos(2\pi 30t + 2\pi/3)$$

$$y(t) = y_1(t) + y_2(t)$$

$$= 0 + 8\cos(2\pi 30t + 2\pi/3)$$

$$= 8\cos(2\pi 30t + 2\pi/3)$$

Problem 7.

$$x(t) = 10\sin(2\pi 200t + \pi/6)$$

From magnitude plot at $f = 200\,Hz$, $|H(f)| = -15\,dB$

$$\Rightarrow \frac{A_{out}}{A_{in}} = 10^{-15/20} = 0.178$$

From phase plot at $f = 200\,Hz$, $\underline{\angle H(f)} = -1.1$

$$\Rightarrow \theta_{out} - \theta_{in} = -1.1$$

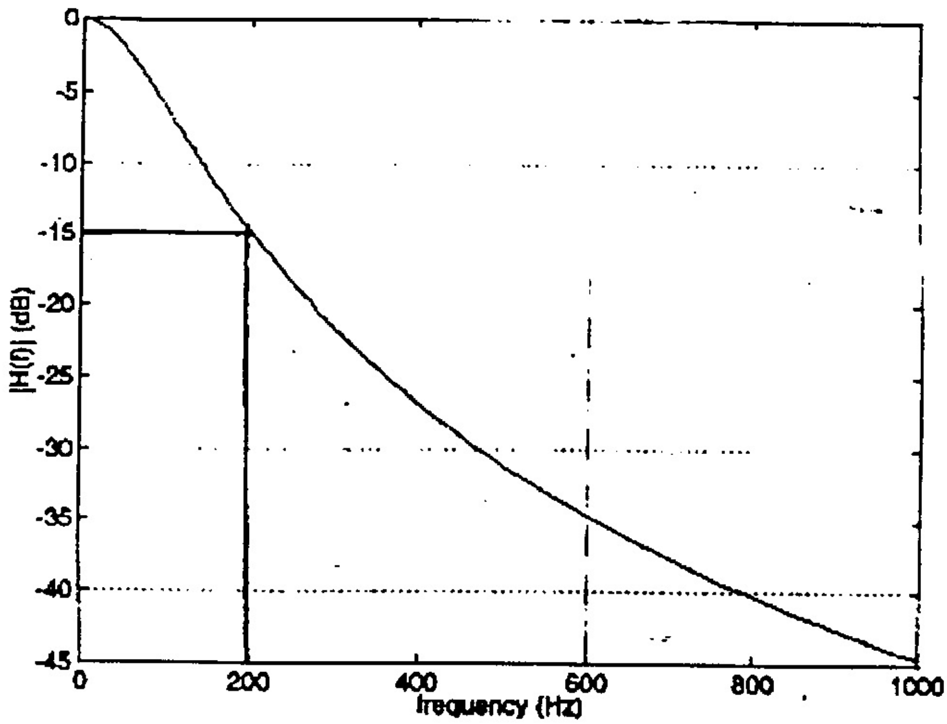
$A_{in} = 10$

$\theta_{in} = \pi/6$

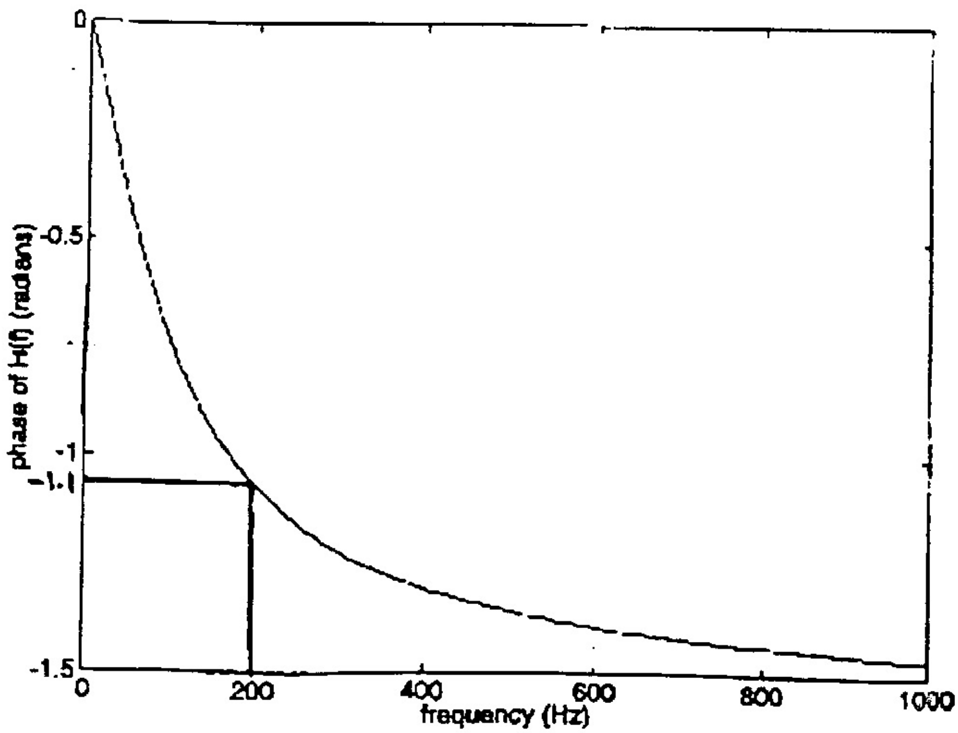
$$\Rightarrow A_{out} = 1.78$$

$$\theta_{out} = \pi/6 - 1.1 = -0.576$$

So, $\underline{y(t) = 1.78\sin(2\pi 200t - 0.576)}$



0
-5
-10
-15
-20
-25
-30
-35
-40
-45
|H(f)| (dB)
0
200
400
600
800
1000
frequency (Hz)



0
-0.5
-1
-1.1
-1.5
phase of H(f) (radians)
0
200
400
600
800
1000
frequency (Hz)