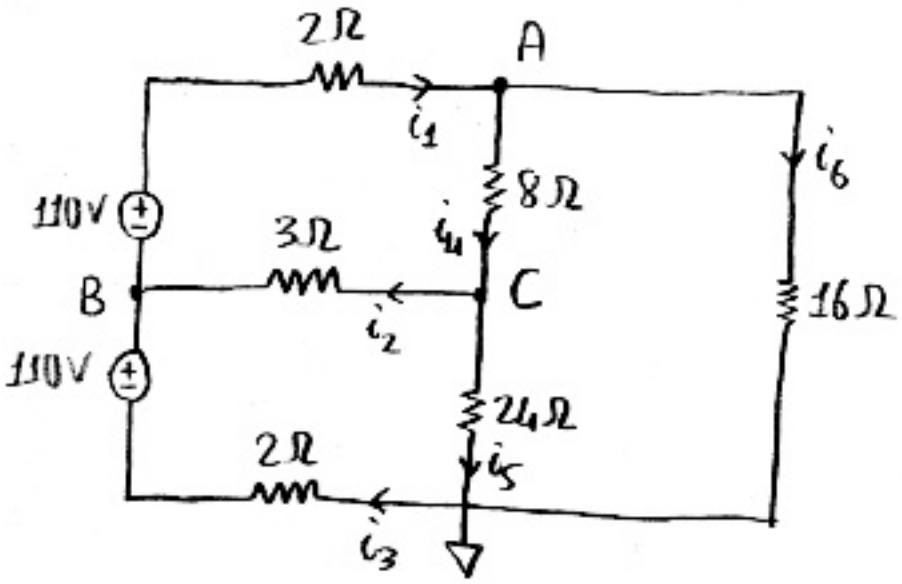
10/11/99

EECS 210

Homework #5 Solutions

② Problem 4.6 (a)



Node A → $\frac{V_A-(V_B+110)}{2} + \frac{V_A-V_C}{8} + \frac{V_A}{16} = 0$

Node B → $\frac{V_B-V_C}{3} + \frac{V_B-110}{2} + \frac{V_B+110-V_A}{2} = 0$

Node C → $\frac{V_C-V_B}{3} + \frac{V_C-V_A}{8} + \frac{V_C}{24} = 0$

$$\Rightarrow \begin{aligned} 11V_A - 8V_B - 2V_C &= 880 \\ -3V_A + 8V_B - 2V_C &= 0 \\ -3V_A - 8V_B + 12V_C &= 0 \end{aligned} \Rightarrow \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 157.14 \\ 82.5 \\ 94.285 \end{bmatrix} V$$

$$i_1 = \frac{V_B + 110 - V_A}{2} = 17.68\ A$$

$$i_2 = \frac{V_c - V_B}{3} = 3.93\ A$$

$$i_3 = -\frac{(V_B - 110)}{2} = 13.75\ A$$

$$i_4 = \frac{V_A - V_c}{8} = 7.86\ A$$

$$i_5 = \frac{V_c}{24} = 3.93\ A$$

$$i_6 = 9.82\ A$$

Check KCL:

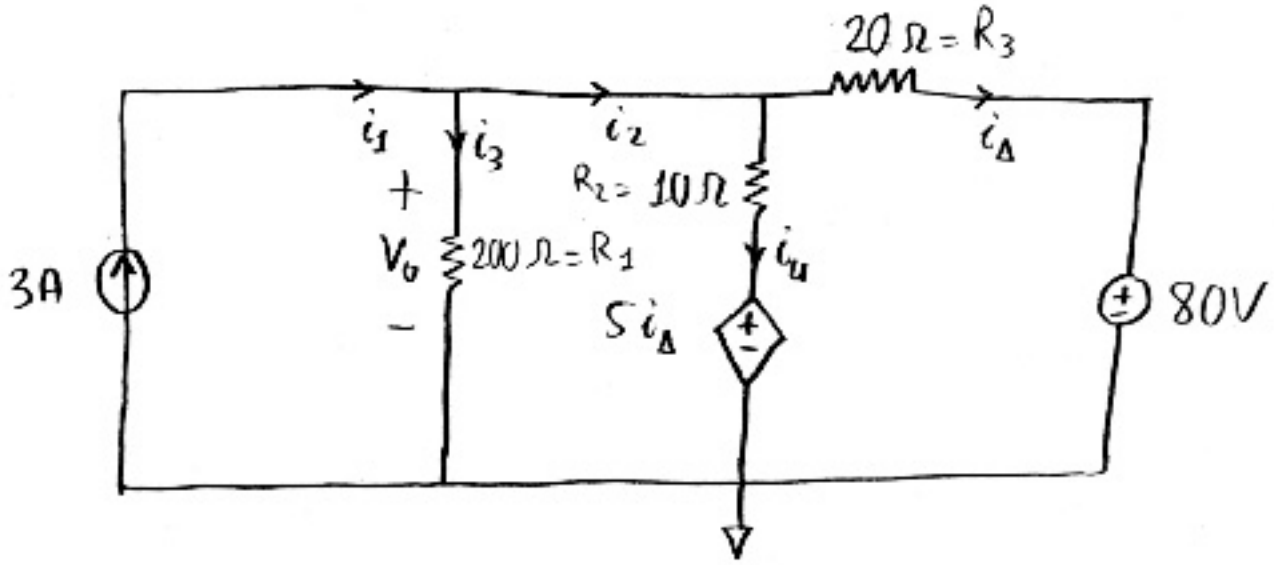
$i_1 \stackrel{?}{=} i_4 + i_6$ ✓

$i_4 \stackrel{?}{=} i_2 + i_5$ ✓

$i_3 \stackrel{?}{=} i_5 + i_6$ ✓

$i_1 \stackrel{?}{=} i_2 + i_3$ ✓

(3) Problem 4.22



a) Find V_o.

$$i_\Delta = \frac{V_o - 80}{20}$$

$$-3 + \frac{V_o}{200} + \frac{V_o - 5i_\Delta}{10} + \frac{V_o - 80}{20} = 0$$

Inserting i_Δ in this equation yields:

$$V_o = 38.46 \text{ V}$$

b) $$i_\Delta = \frac{V_o - 80}{20} = -2.077 \text{ A}$$

$$i_4 = \frac{V_o - 5i_\Delta}{10} = 4.8845 \text{ A} \Rightarrow P = i_4 \cdot 5i_\Delta = -50.7255 \text{ W}$$

(source is delivering power)

c) $P_V = -80.\ i_\Delta = -166.16\ W$ (source is delivering)

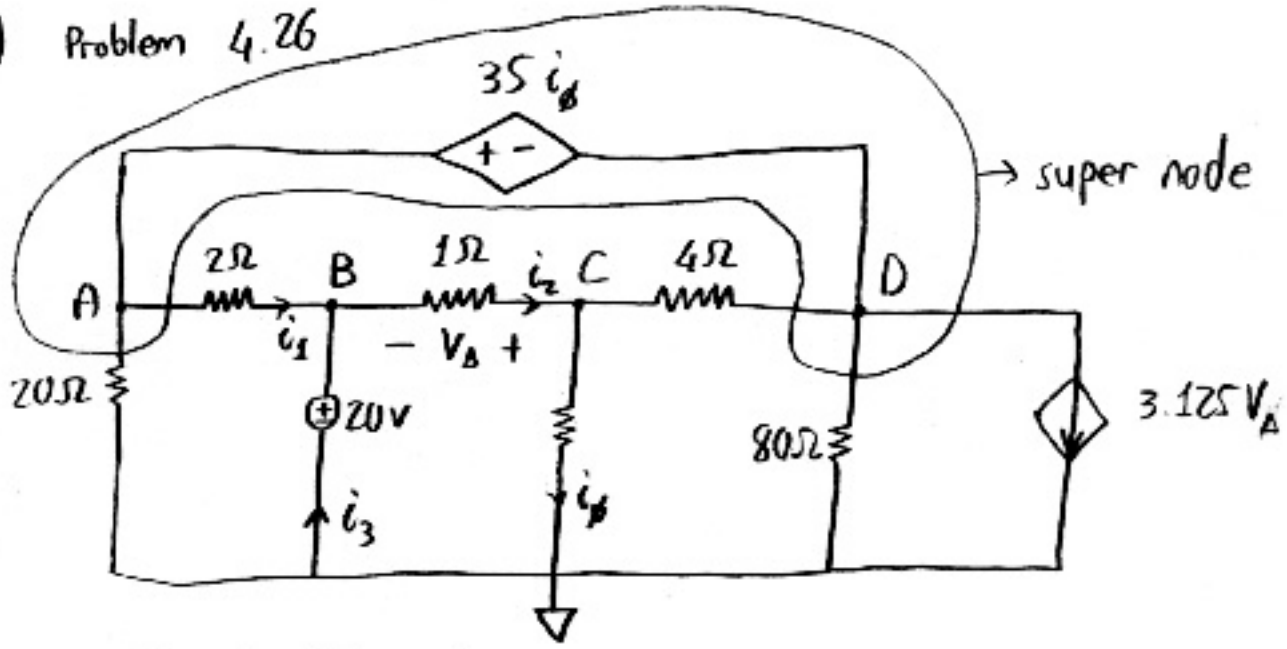
$P_C = -V_0 . 3 = -115.2\ W$ (source is delivering)

$\Rightarrow P_V + P_C = -231.36\ W$ //

Power check: $P_{delivered} = -50.72 - 166.16 - 115.2 = -332.08\ W$ //

$P_{absorbed} = \underbrace{P_{R_1} + P_{R_2} + P_{R_3}}_{Use\ \frac{V_i^2}{R_i}} = +332.08\ W$ // ✓

(4) Problem 4.26



$V_B = V_C - 20$ $\quad i_\phi = \frac{V_C}{40}$

Super Node → $\frac{V_A}{20} + \frac{V_A - V_B}{2} + \frac{V_D - V_C}{4} + \frac{V_D}{80} + 3.125\,V_A = 0$

From voltage source across super node:

$V_A - V_D = 35\,i_\phi \Rightarrow V_D = V_A - 35\,i_\phi$

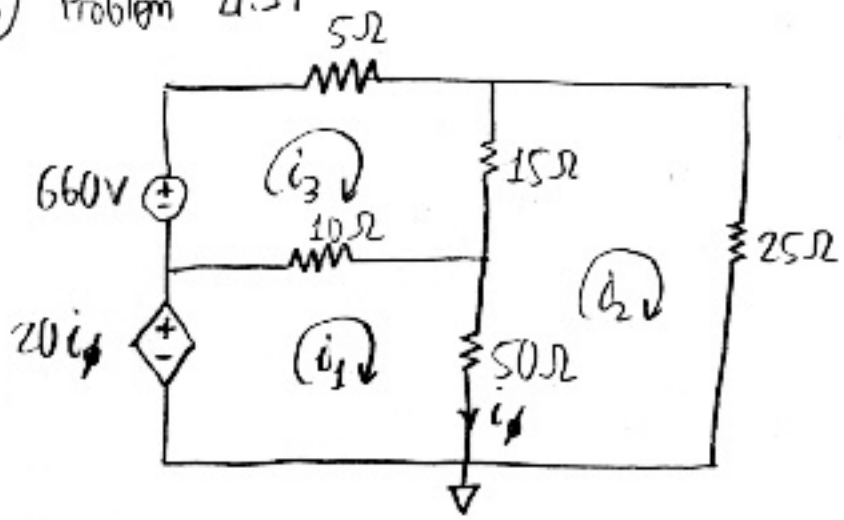
Node C → $\frac{V_C - V_B}{1} + \frac{V_C}{40} + \frac{V_C - V_D}{4} = 0$

$\Rightarrow$ $\begin{aligned} 2600\,V_A + 8465\,V_C &= 232\,000 \\ -10\,V_A + 59.75\,V_C &= 800 \end{aligned}$ $\Rightarrow$ $\begin{aligned} V_A &= 29.5416\ V \\ V_C &= 18.\overline{3}\ V \end{aligned}$

$$i_3 = i_2 - i_1 = \frac{V_B - V_C}{1} - \left(\frac{V_A - V_B}{2}\right) = -3.1\ A$$

$$\Rightarrow P_{=} = -20 \cdot i_3 = 62\,W_{//} \quad \text{(source is absorbing power)}$$

(5) Problem 4.37



$i_\phi = i_1 - i_2$

$$-20i_\phi + 10(i_1 - i_3) + 50(i_1 - i_2) = 0$$

$$50(i_2 - i_1) + 15(i_2 - i_3) + 25i_2 = 0$$

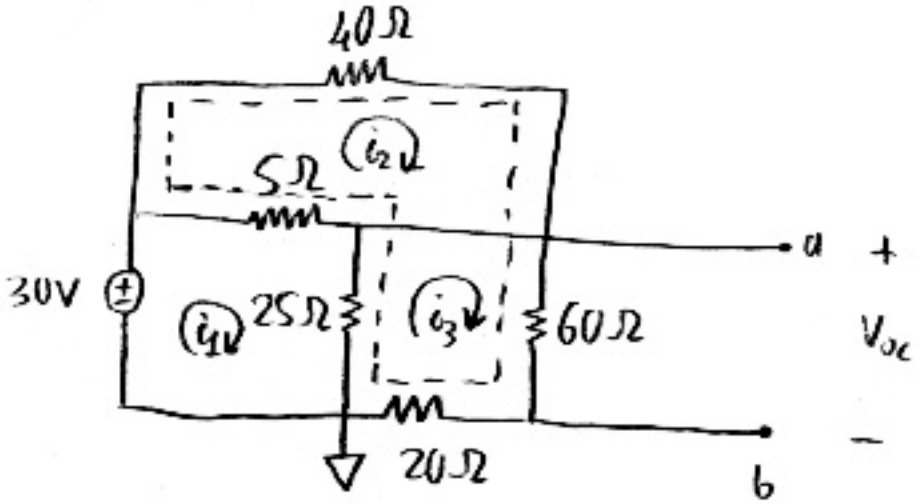
$$5i_3 + 15(i_3 - i_2) + 10(i_3 - i_1) - 660 = 0$$

$$\Rightarrow \quad \begin{aligned} 40i_1 - 30i_2 - 10i_3 &= 0 \\ -50i_1 + 90i_2 - 15i_3 &= 0 \\ -10i_1 - 15i_2 + 30i_3 &= 660 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 27 \\ 22 \\ 42 \end{bmatrix} \text{ A}$$

$$\Rightarrow \quad P = -20i_\phi \cdot i_1 = -20(i_1 - i_2)\, i_1 = -2700 \text{ W} //$$

(source is delivering power)

⑥ Problem 4.58



Supermesh : ① $25(i_3-i_1)+5(i_2-i_1)+40i_2+60i_3+20i_3=0$

② $i_3-i_2=1.5$

③ $-30+5(i_1-i_2)+25(i_1-i_3)=0$

From ① and ② : $-30i_1+45i_2+105i_3=0$

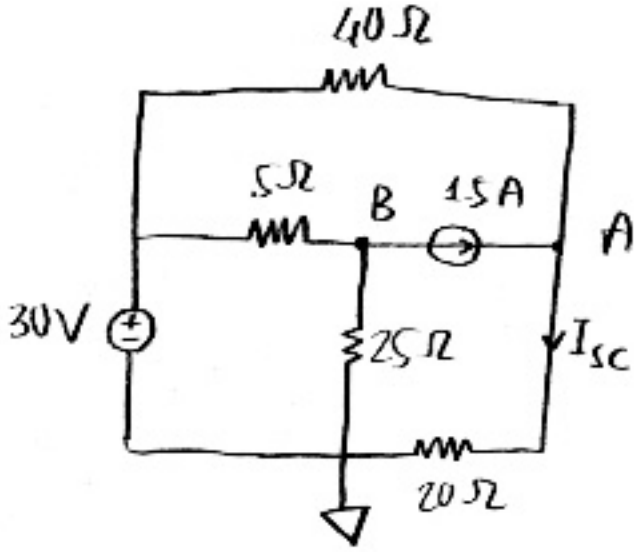
$\Rightarrow \boxed{-30i_1+150i_2=-157.5}$

From ② and ③ : $30i_1-5i_2-25i_3=30$, $i_3=i_2+1.5$

$\Rightarrow \boxed{30i_1-30i_2=67.5}$

$\Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1.5 \\ -0.75 \end{bmatrix}$ $\Rightarrow V_{oc}=60i_3=60(1.5+i_2)=45V$ //

ii) I_{sc}:



Node A → $-1.5 + \frac{V_A - 30}{40} + \frac{V_A}{20} = 0$

Node B → $\frac{V_B - 30}{5} + \frac{V_B}{25} + 1.5 = 0$

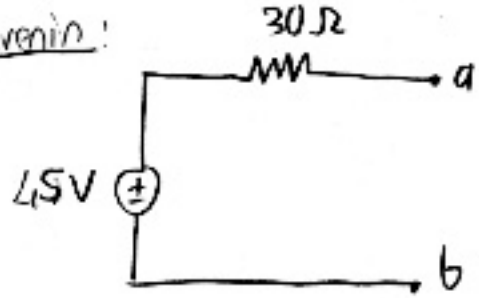
$\Rightarrow \quad V_A = 30V$

$V_B = 31.25V$

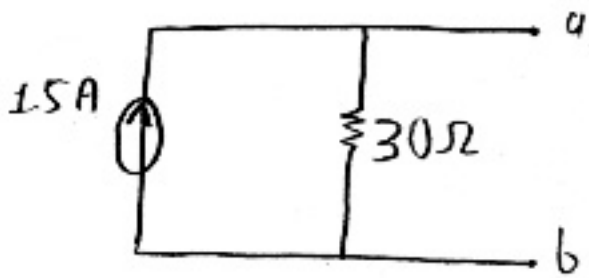
$\Rightarrow \quad I_{sc} = \frac{V_A}{20} = 1.5\,A$

$R_T = \frac{V_{oc}}{I_{sc}} = \frac{45}{1.5} = 30\,\Omega$

Thevenin:

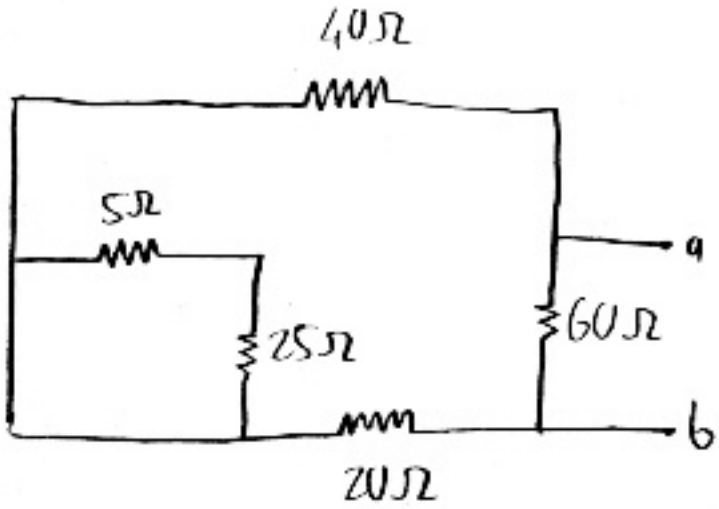


Norton:



b) Voltage source → O.C.

Current source → S.C.

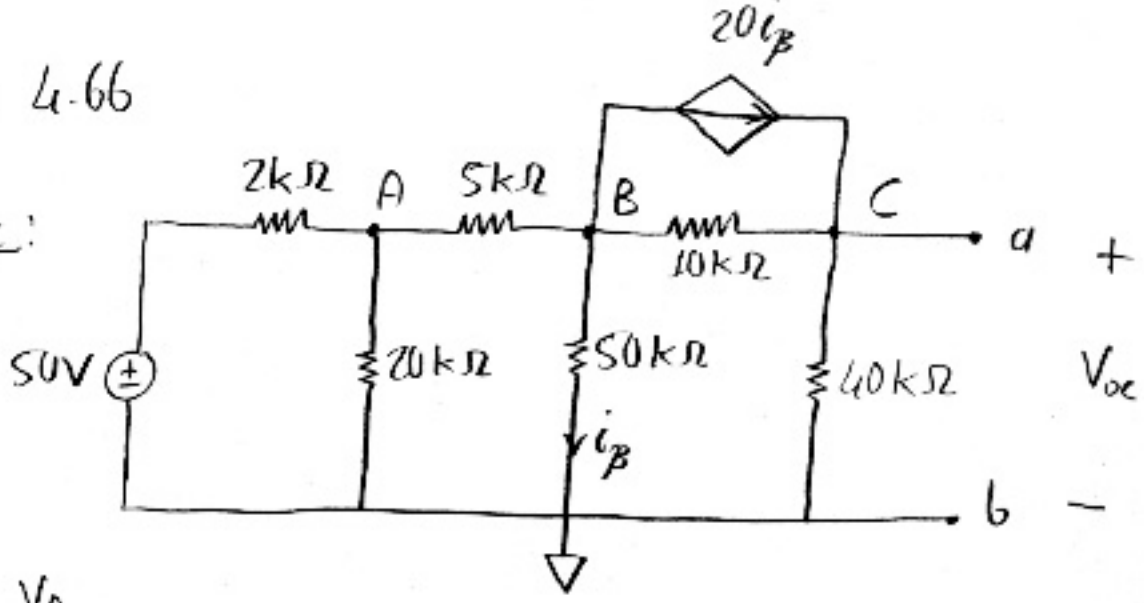


← $R_T = 60 // 60 = 30\,\Omega //$

Same result.

(7) Problem 4.66

a) i) V_{oc}:



$$i_\beta = \frac{V_B}{50k}$$

Node A → $\frac{V_A - 50}{2k} + \frac{V_A - V_B}{5k} + \frac{V_A}{20k} = 0$

Node B → $\frac{V_B - V_A}{5k} + \frac{V_B}{50k} + \frac{V_B - V_{oc}}{10k} + 20 i_\beta = 0$

Node C → $\frac{V_{oc} - V_B}{10k} - 20 i_\beta + \frac{V_{oc}}{40k} = 0$

$$15V_A - 4V_B = 500$$

$$\Rightarrow \quad -10V_A + 36V_B - 5V_{oc} = 0 \quad \Rightarrow$$

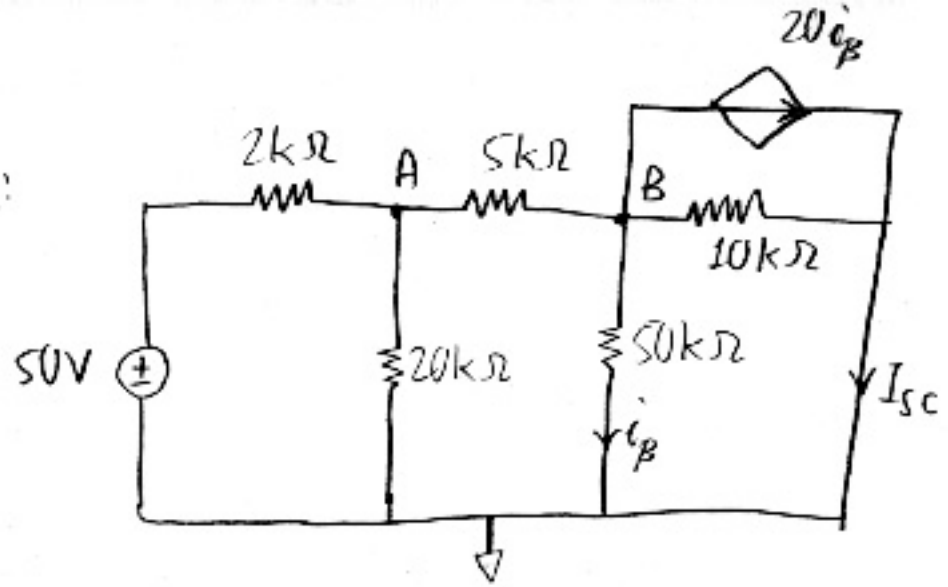
$$-100V_B + 25V_{oc} = 0$$

$V_A = 40V$

$V_B = 25V$

$V_{oc} = 100V$ //

ii) I_{SC}:



Node A → $\frac{V_A-50}{2k} + \frac{V_A-V_B}{5k} + \frac{V_A}{20k} = 0$

Node B → $\frac{V_B-V_A}{5k} + \frac{V_B}{50k} + \frac{V_B}{10k} + 20\,i_B = 0$

$i_B = \frac{V_B}{50k}$

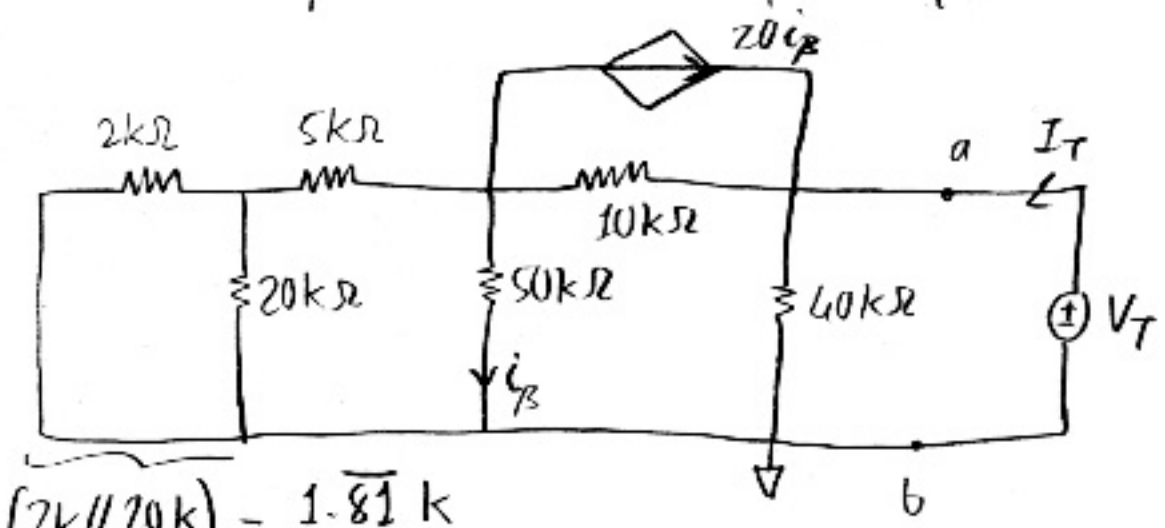
$\Rightarrow \quad 15V_A - 4V_B = 500$
$-10V_A + 36V_B = 0$
$\Rightarrow \quad V_A = 36V$
$V_B = 10V$

$I_{SC} = \frac{V_B}{10k} + 20\,i_B = 5\,mA$

$i_B = \frac{V_B}{50k}$

$\Rightarrow R_T = \frac{V_{OC}}{I_{SC}} = \frac{100}{5m} = 20\,k\Omega //$

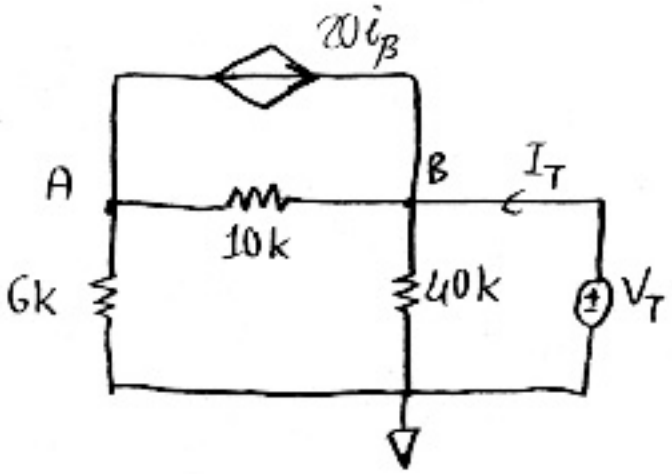
b) Turn off independent sources to find R_T:



$(2k // 20k) = 1.8\overline{1}\ k$

$1.8\overline{1}\ k + 5k = 6.8\overline{1}\ k$ $\qquad$ $6.8\overline{1}\ k // 50k = 6\ k\Omega$

The circuit beromes:



$$i_\beta = \frac{V_A}{50k}$$

Node A $\rightarrow$ $\frac{V_A}{6k} + \frac{V_A - V_T}{10k} + 20\, i_\beta = 0$

$\Rightarrow$ $\boxed{V_A = \frac{3}{20} V_T}$

KCL at node B → $-20i_\beta - \frac{V_A - V_T}{10k} - I_T + \frac{V_T}{40k} = 0$

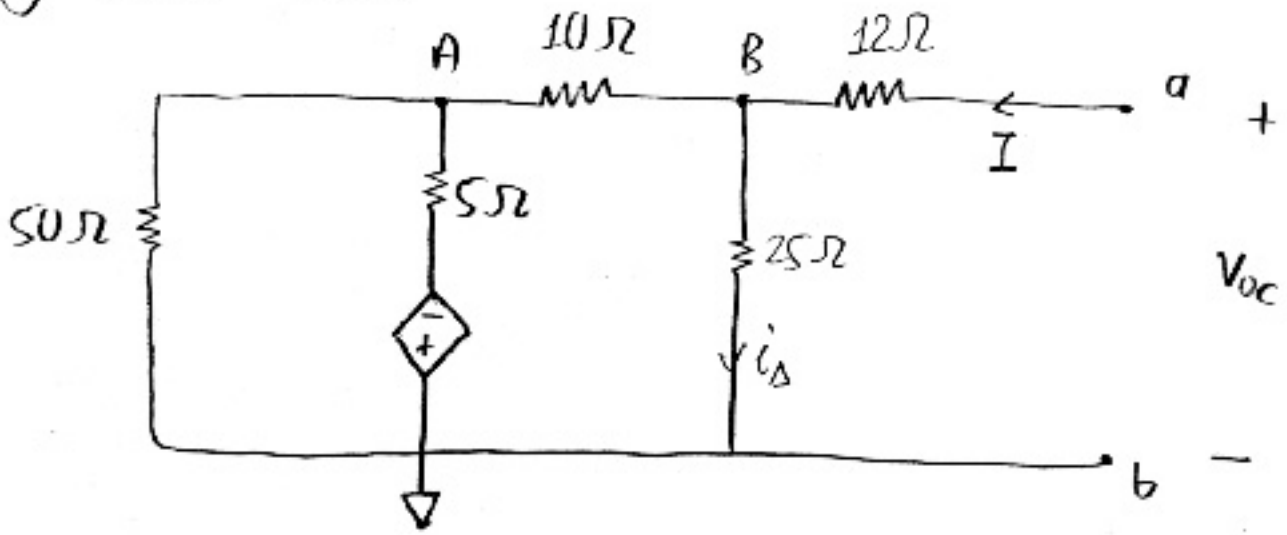
$$\Rightarrow -\frac{2V_A}{5k} - \frac{V_A}{10k} + \frac{V_T}{10k} + \frac{V_T}{40k} = -I_T$$

Using $V_A = \frac{3}{20} V_T$, we get:

$$\left(\frac{2}{5k} \cdot \frac{3}{20} + \frac{3}{200k} - \frac{1}{10k} - \frac{1}{40k} \right) V_T = -I_T$$

$\Rightarrow R_T = \frac{V_T}{I_T} = 20\,k\Omega //$ Same result as before.

(8) Problem 4.70



Node A $\rightarrow \frac{V_A}{50} + \frac{V_A + 6.5 i_\Delta}{5} + \frac{V_A - V_B}{10} = 0$

Node B $\rightarrow \frac{V_B - V_A}{10} + \frac{V_B}{25} - I = 0$ $\qquad i_\Delta = \frac{V_B}{25}$

$\Rightarrow V_B = 6.\overline{6}\, V_A$, $I = 0.8\overline{3}\, V_A$

$V_{oc} = V_B + 12I = \frac{6.\overline{6}}{0.8\overline{3}} I + 12I = 20\, I$

$\Rightarrow \boxed{R_T = 20\,\Omega \text{ , } \quad V_{oc} = I_{sc} = 0}$

(The voltage for the Thevenin equivalent and the current for the Norton equivalent are zero)