EECS 210 Fall 99

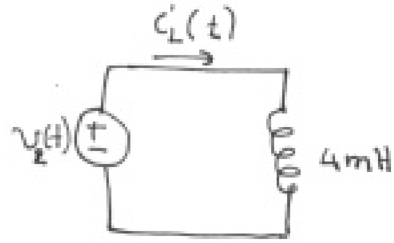
Homework # 8

2. Problem 6.2

$L = 4\,mH$

$i = 2.5\ A \qquad t \le 0$

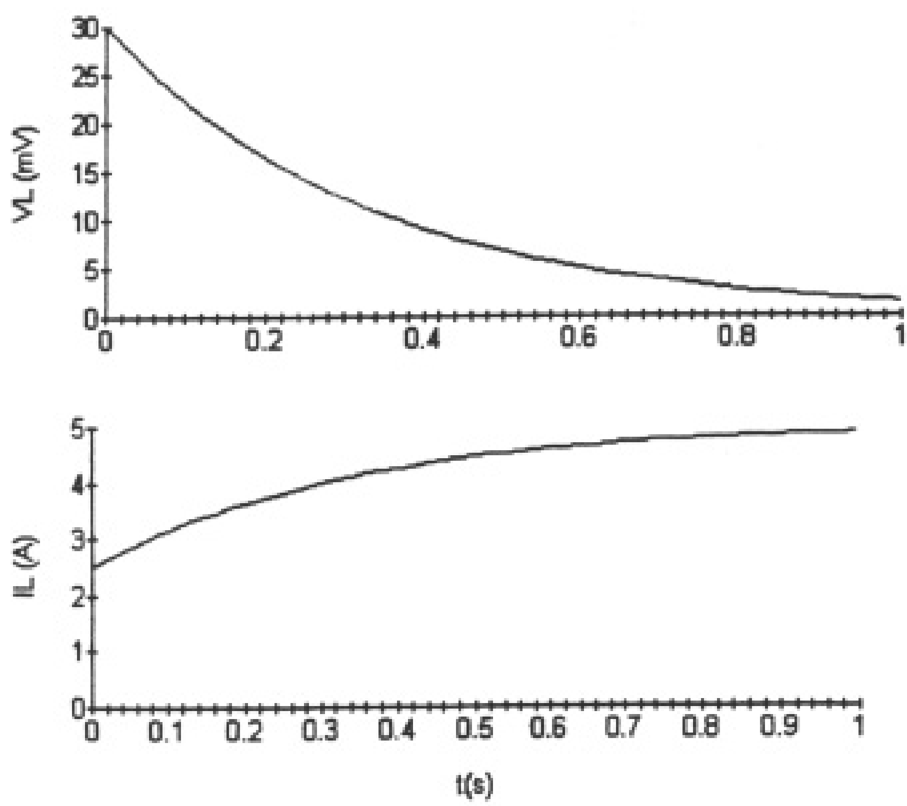
$v_L(t) = 30e^{-3t}\,mV\ , \quad 0^+ \le t < \infty$



$$i_L = \frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0) \Rightarrow i_L(t) = \frac{1}{4\times\cancel{10^{-3}}}\int_0^t 30e^{-3t}\times\cancel{10^{-3}}\,dt + 2.5$$

$$\Rightarrow i_L(t) = 7.5\left(\frac{1}{-3}e^{-3t}\right)\Big|_0^t + 2.5 = 2.5 + 2.5\left(1 - e^{-3t}\right)$$

$$= 5 - 2.5\times e^{-3t}\ A\ , \quad 0 < t < \infty$$



3. Problem 6.9

$L = 25mH$

$i_L = -10A \quad , \quad t \le 0$

$i_L = -[10\cos 400t + 5\sin 400t]e^{-200t} \; A \quad , \; t \ge 0$

(a) At what instant of time is the voltage across the inductor maximum?

$$v_L = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = -\frac{d}{dt}\left[10\cos 400t\, e^{-200t} + 5\sin 400t\, e^{-200t}\right]$$

$$= 10\times 400\times \sin 400t\, e^{-200t} - \cancel{10\cos 400t\times(-200)}\, e^{-200t}$$

$$-5\times \cancel{400\cos 400t}\, e^{-200t} + 5\times 200\, \sin 400t\cdot e^{-200t}$$

$$= 5000\, \sin 400t\, e^{-200t}$$

$$v_L = L\frac{di_L}{dt} = 25\times \cancel{10^{-3}} \times 5000\, \sin 400t\, e^{-200t}$$

$$= 125 \sin 400t\, e^{-200t} \quad V$$

$$\frac{dv_L}{dt} = \cancel{12}5\times 400 \cos 400t\, e^{-200t} - \cancel{12}5\times \sin 400t\, e^{-200t}\times 200 \equiv 0$$

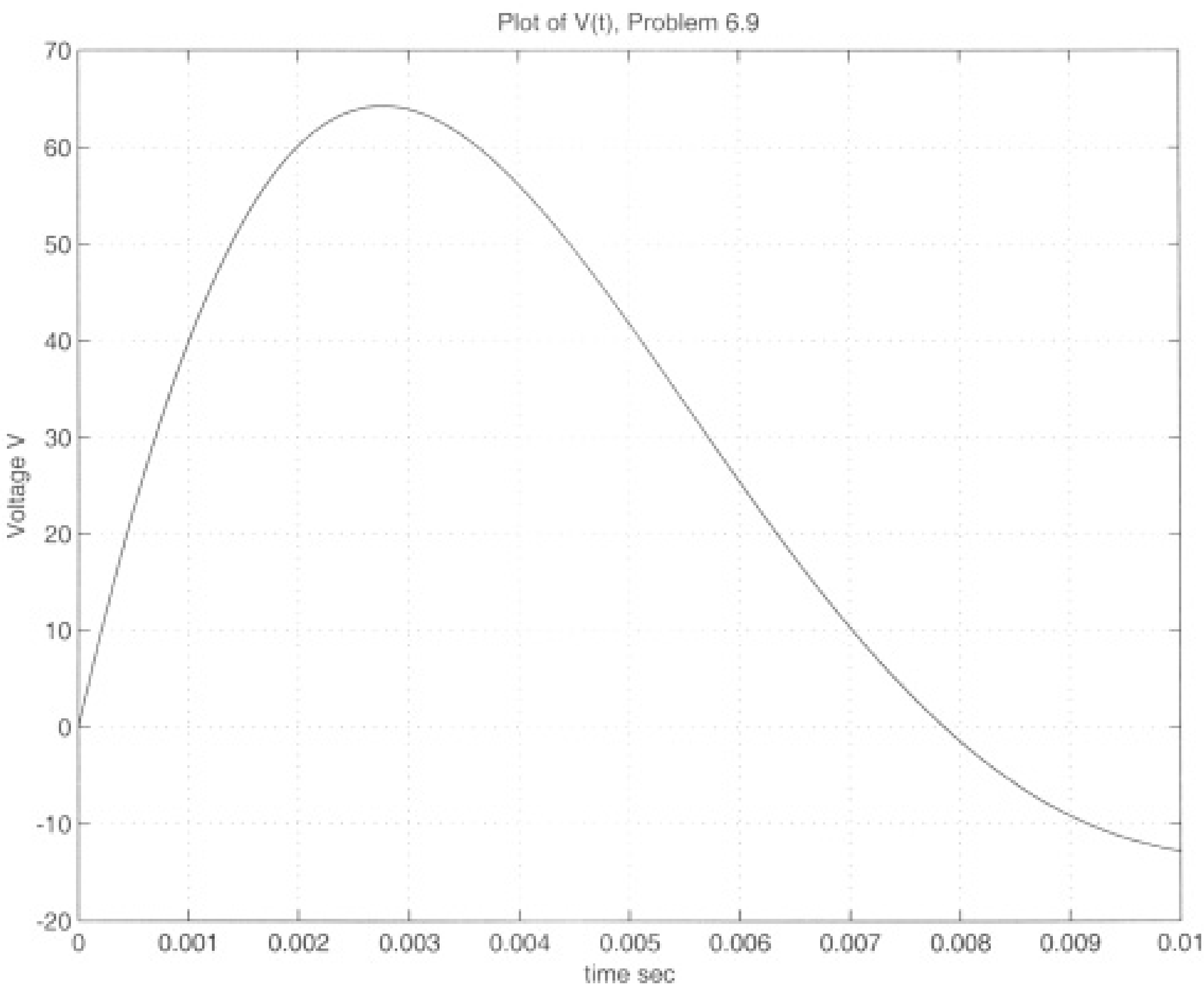
$\Rightarrow \; 2\cos 400t - \sin 400t = 0 \Rightarrow \sin 400t = 2\cos 400t$

$\Rightarrow \; \tan 400t = 2 \quad (\text{when } \sin 400t \ne 0)$

$\Rightarrow \; 400t = \arctan(2) \Rightarrow t = \frac{1}{400}\arctan 2 = \frac{1.107 + \pi n}{400}$ sec
$n = 0, 1, \ldots$

Notice that at $t=0$, $\frac{dv_L}{dt} > 0 \Rightarrow$ curve has positive slope until it reaches a maximum at $\underline{t = 0.0028 \text{ sec}}$

(b) $v_L(0.0028) = 125 \sin\left(400\times\frac{1.107}{400}\right) e^{-200\times 0.0028} = \underline{64.27 \; V}$



Plot of V(t), Problem 6.9
Voltage V
time sec
70
60
50
40
30
20
10
0
-10
-20
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.01

4. Problem 6.14

$C = 0.5\mu F \Rightarrow \frac{1}{C} = 2\times10^{6}$

$V = 20V$ at $t = 0$

(a) $0 \le t \le 50\mu s$

$$v(t) = \frac{1}{C}\int_0^t 20\times10^{-3}\,dt + v(0) = 2\times10^{6}\int_0^t 20\times10^{-3}\,dt + 20$$

$$= \underline{40\times10^{3}\,t + 20\ V} \quad \Rightarrow \quad v(50) = 40\times10^{3}\times50\times10^{-6} + 20 = 22V$$

(b) $50\mu s \le t \le 200\mu s$

$$v(t) = \frac{1}{C}\int_{50\times10^{-6}}^{t}(-40\times10^{-3}\,dt) + v(50) = 2\times10^{6}\int_{50\times10^{-6}}^{t}(-40\times10^{-3}\,dt) + 22$$

$$= -2\times10^{6}\times40\times10^{-3}\,(t - 50\times10^{-6}) + 22$$

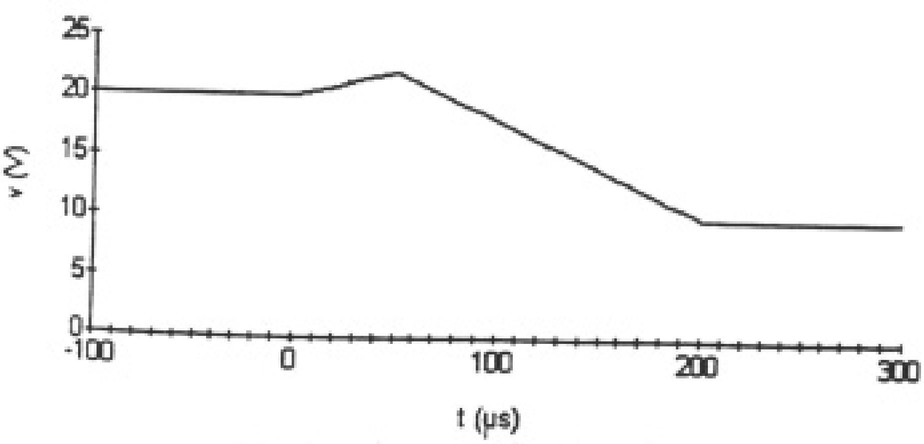
$$= -8\times10^{4}t + 4 + 22 = \underline{26 - 8\times10^{4}t}$$

$$v(200) = 26 - 8\times10^{4}\times200\times10^{-6} = 26 - 16 = 10V$$

(c) $200\mu s \le t < \infty$

$$v(t) = \frac{1}{C}\int_{200\mu sec}^{t}(0)\,dt + v(200) = \underline{10V}$$

(d)



s. Problem 6.15

$c = 0.8\mu F \qquad v_c(t) = \begin{cases} 20t^3\ V & 0 \le t \le 1\,sec \\ 2.5(3-t)^3\ V & 1 \le t \le 3\,sec \\ 0 & elsewhere \end{cases}$

$i_c = c\frac{dv}{dt}$

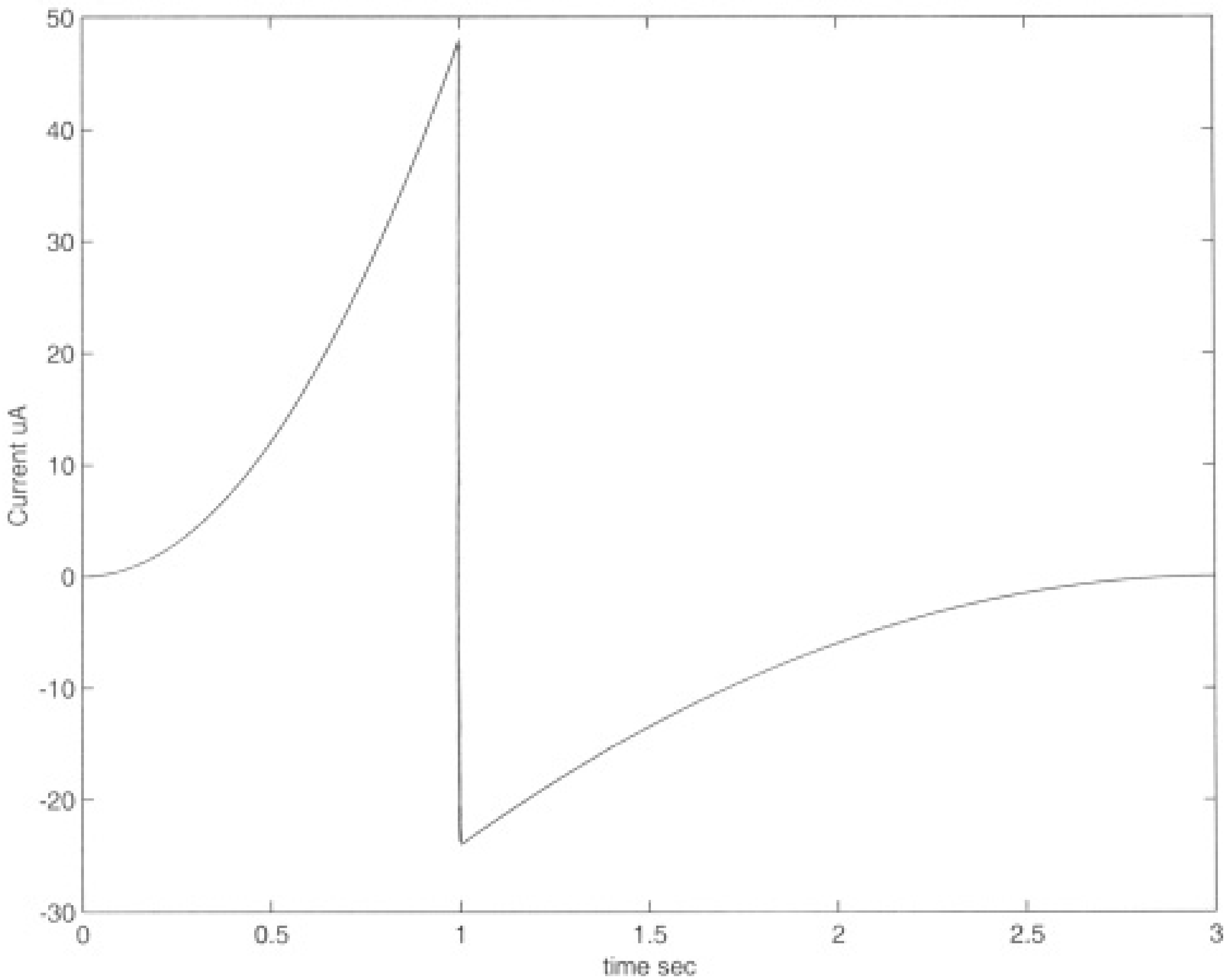
for $0 \le t \le 1$ $\qquad i_c = 0.8\times10^{-6}\frac{d}{dt}(20t^3) = 0.8\times10^{-6}\times20\times3\,t^2$

$= 48t^2\ \mu A$

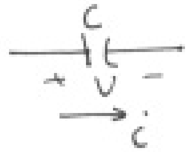
for $1 \le t \le 3$ $\qquad i_c = 0.8\times10^{-6}\frac{d}{dt}\left(2.5\,(3-t)^3\right)$

$\Rightarrow i_c = 2\times10^{-6}\times3\times(-1)\times(3-t)^2 = -6\times10^{-6}(3-t)^2$

$= -6\,(3-t)^2\ \mu A$



6. Problem 6.19



$$v = \begin{cases} -60\,\text{V} & t \le 0 \\ 15 - 15e^{-500t}(5\cos 2000t + \sin 2000t)\ \text{V} & t \ge 0 \end{cases}$$

$C = 0.4\,\mu F$

(a) $i = 0 \quad , t \le 0$ (capacitor acts as open circuit for DC

$i = C\dfrac{dv}{dt} = 0 \ , \ t \le 0$

(b) $$\frac{dv}{dt} = -15\left[(5\cos 2000t + \sin 2000t)(-500)e^{-500t} + e^{-500t}(-10000\sin 2000t + 2000\cos 2000t)\right)$$

$$= 15e^{-500t}\left[500\cos 2000t + 10500\sin 2000t\right)$$

$$i = C\frac{dv}{dt} = 0.4\times 10^{-6}\times 15\times 500\times e^{-500t}\left[\cos 2000t + 21\sin 2000t\right]$$

$$= 3\times 10^{-3}(\cos 2000t + 21\sin 2000t)$$

$$= 3(\cos 2000t + 21\sin 2000t)\ \text{mA}$$

(c) No; The voltage cannot change instantaneously across the terminals of a capacitor. Also observe from $v(t)$ specified in this problem that $V(0^-) = -60V$ and $V(0^+) = -6$

(d) From (b), find i for $t = 0^+$:

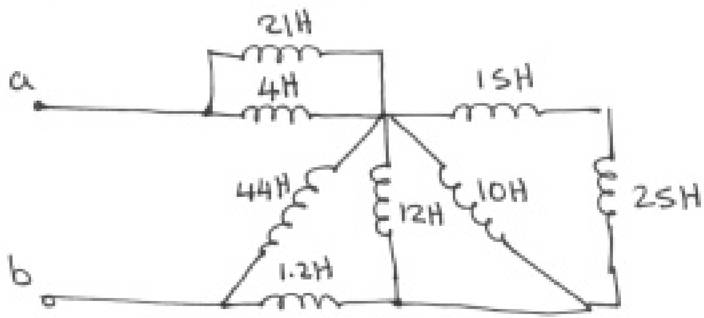
$$i = 3(1+0) = 3\,\text{mA}$$

$\Rightarrow$ Instantaneous change from 0

(e) $$v(\infty) = 15 - 15e^{-\infty} = 15 - 0 = 15\,V$$

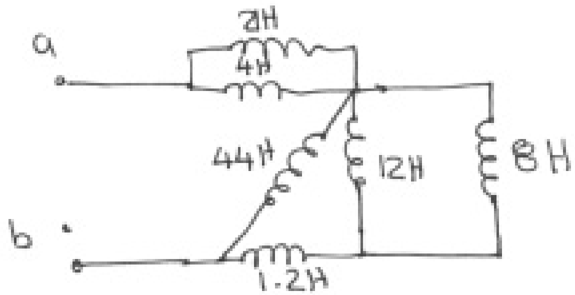
$$W = \frac{1}{2}Cv^2 = \frac{1}{2}\times 0.4\times 10^{-6}\times (15^2) = 45\,\mu J$$

7. Problem 6.20



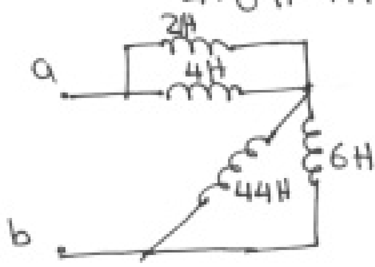
15H and 25H in series : 15 + 25 = 40 H

40H in parallel with 10H : 40//10 = 8 H



8H in parallel with 12H : 8//12 = 4.8H

4.8H in series with 1.2H : 4.8 + 1.2 = 6H

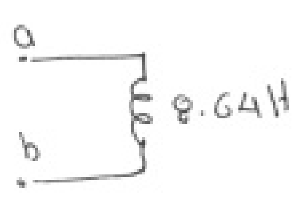


6H in parallel with 44H ⇒ 6//44 = 5.28 H

21 in parallel with 4H : 21//4 = 3.36H

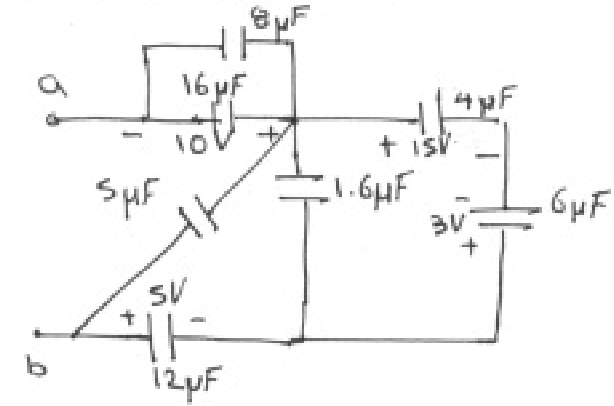
in series

3.36 + 5.28
= 8.64H



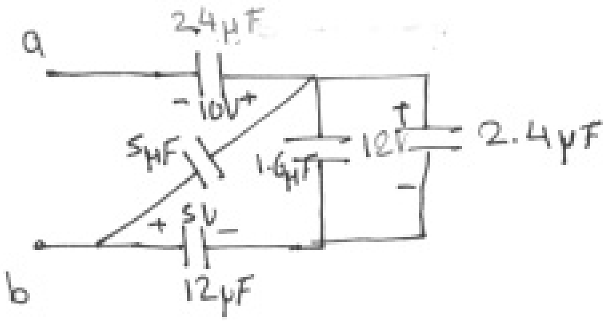
⇒ $L_{eq} = 8.64\text{ H}$

8. Problem 6.24



4 series 6 : $\frac{1}{4}+\frac{1}{6}=\frac{1}{c} \Rightarrow c=2.4\mu F$

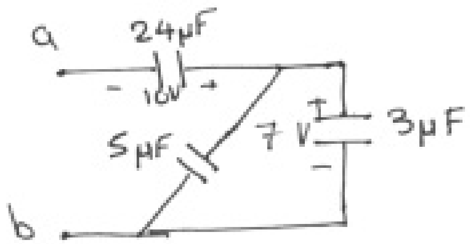
16 parallel 8 : $16+8=24\mu F$



2.4 parallel 1.6 : $2.4+1.6=4\mu F$

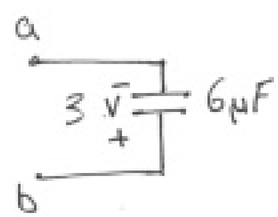
4μF in series 12μF : $\frac{1}{4}+\frac{1}{12}=\frac{1}{c}$

$\Rightarrow c=3\mu F$



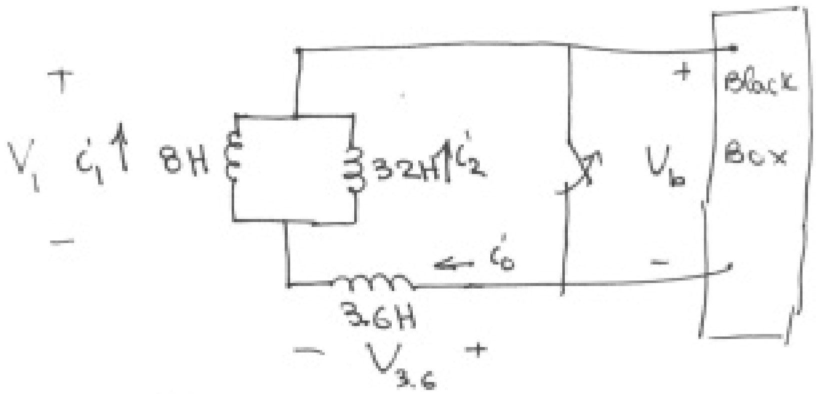
3μF parallel 5μF : $3+5=8\mu F$

8μF series 24μF : $\frac{1}{c}=\frac{1}{8}+\frac{1}{24} \Rightarrow c=6\mu F$



$\Rightarrow \underline{\underline{C_{eq}=6\mu F}}$

9. Problem 6.26



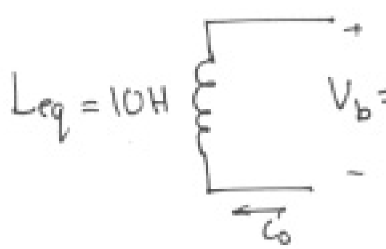
$v_b = 1250 e^{-25t}$ V , $t \geq 0$

$i_1(0) = 10$A

$i_2(0) = -5$A

(a) $i_0(0) = i_1(0) + i_2(0) = 10 - 5 = \underline{5\text{ A}}$

(b) $L_{eq} = 3.6H + (8H // 32H) = 3.6H + 6.4H = 10H$



$$\Rightarrow i_0 = -\frac{1}{10}\int_0^t 1250 e^{-25t}\,dt + 5 = 5 + \frac{(-125)}{(-25)} e^{-25t}\Big|_0^t$$

$$= 5 + 5(e^{-25t} - 1) = \underline{\underline{5 e^{-25t}\text{ A}}} \quad , \; t \geq 0$$

(c) $v_{3.6} = 3.6 \dfrac{d}{dt}(i_0(t)) = 3.6 \times 5 \times (-25) e^{-25t} \underline{\underline{= -450 e^{-25t}\quad V}}$

$$\Rightarrow V_1(t) = V_{3.6}(t) + V_b(t) = 1250 e^{-25t} - 450 e^{-25t}$$

$$\underline{\underline{= 800 e^{-25t}\text{ V}}}$$

$$i_1(t) = -\frac{1}{8}\int_0^t 800 e^{-25t}\,dt + i_1(0)$$

$$= -\frac{1}{8} \cdot 800(-25)^{-1}(e^{-25t} - 1) + 10 = \underline{\underline{4 e^{-25t} + 6\text{ A}}} \; , t \geq 0$$

(d) $i_2(t) = i_0(t) - i_1(t)$

$$= 5e^{-25t} - 4e^{-25t} - 6 = \underline{\underline{e^{-25t} - 6 \quad A}}$$

(e) $$W(0) = \frac{1}{2} L_1 (i_1(0))^2 + \frac{1}{2} L_2 (i_2(0))^2 + \frac{1}{2} L_0 (i_0(0))^2$$

$$= \frac{1}{2} 8 (10)^2 + \frac{1}{2}(32)(5)^2 + \frac{1}{2}(3.6)(5)^2 = \underline{\underline{845 \ J}}$$

(f) $i_0(\infty) = 0, \ i_1(\infty) = 6A, \ i_2(\infty) = -6A$

$$\Rightarrow W_{Final} = \frac{1}{2} L_1 (i_1(\infty))^2 + \frac{1}{2} L_2 (i_2(\infty))^2 = \frac{1}{2} 8(6)^2 + \frac{1}{2}(32)(6^2) = 720 \ J$$

$$W_{del} = W(0) - W(\infty) = 845 - 720 = \underline{\underline{125 \ J}}$$

<u>or</u> (equivalent):

$$W_{del} = \int_0^\infty i_0(t)\, v_0(t)\, dt = \int_0^\infty 5e^{-25t} \times 1250 e^{-25t}\, dt$$

$$= 5 \times 1250 \int_0^\infty e^{-50t}\, dt = -\frac{5 \times 1250}{50} \cdot e^{-50t} \Big|_0^\infty$$

$$= -125 \left(e^{-\infty} - e^{0}\right) = -125 (0 - 1) = \underline{\underline{125 \ J}}$$

(g) $W_{trapped} = W_{Final}$ that was calculated in part (f)

$$\Rightarrow W_{trapped} = \underline{\underline{720 \ J}}$$