EECS 210

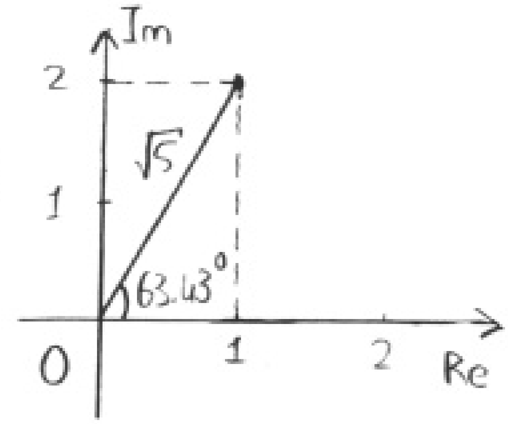
Homework #9 Solutions

1) a) $Z_1 = 1+2j$

Modulus $= \sqrt{1+2^2} = \sqrt{5} = 2.236$

Phase $= \tan^{-1}\left(\frac{2}{1}\right) = 63.43^\circ$

$\Rightarrow Z_1 = \sqrt{5} \angle 63.43^\circ$

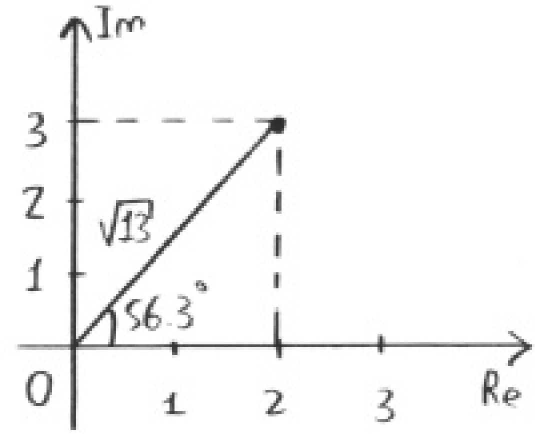


b) $Z_2 = 2+3j$

Modulus $= \sqrt{2^2+3^2} = \sqrt{13} = 3.605$

Phase $= \tan^{-1}\left(\frac{3}{2}\right) = 56.3^\circ$

$\Rightarrow Z_2 = \sqrt{13} \angle 56.3^\circ$

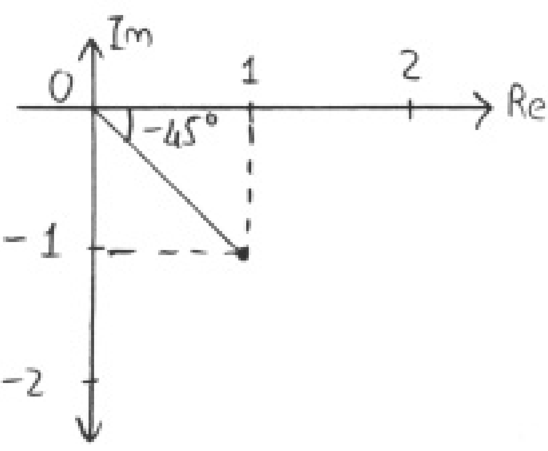


c) $Z_3 = 1-j$

Modulus $= \sqrt{1+1} = \sqrt{2} = 1.414$

Phase $= \tan^{-1}\left(-\frac{1}{1}\right) = -45^\circ$

$\Rightarrow Z_3 = \sqrt{2} \angle -45^\circ$



② a) $Z_1 . Z_2 . Z_3 = (Z_1 . Z_2) . Z_3 = [(1+2j)(2+3j)](1-j)$

$= (2+3j+4j-6)(1-j) = (-4+7j)(1-j)$

$= -4+4j+7j+7 = 3+11j = \sqrt{130}\ \underline{/74.74^\circ} = 11.4\ \underline{/74.74^\circ}$ //

b) $Z_1 . Z_2 . Z_3 = \left(\sqrt{5}\ \underline{/63.43^\circ}\right)\left(\sqrt{13}\ \underline{/56.3^\circ}\right)\left(\sqrt{2}\ \underline{/-45^\circ}\right)$

$= \sqrt{5 \cdot 13 \cdot 2}\ \underline{/63.43^\circ + 56.3^\circ - 45^\circ} = \sqrt{130}\ \underline{/74.74^\circ}$

$= 11.4\ \underline{/74.74^\circ}$ //

③ a) $\dfrac{Z_1}{Z_2} = \dfrac{1+2j}{2+3j} = \dfrac{\sqrt{5}\ \underline{/63.43^\circ}}{\sqrt{13}\ \underline{/56.3^\circ}} = \left(\dfrac{\sqrt{5}}{\sqrt{13}}\right)\underline{/63.43^\circ - 56.3^\circ}$

$= 0.62\ \underline{/7.13^\circ} = 0.615 + j0.0769$

b) $\dfrac{Z_1}{Z_2} = \dfrac{1+2j}{\underset{(2-3j)}{2+3j}} = \dfrac{(1+2j)(2-3j)}{4+9} = \dfrac{2-3j+4j+6}{13}$

$= \dfrac{8+j}{13} = 0.615 + j0.0769 = 0.62\ \underline{/7.13^\circ}$

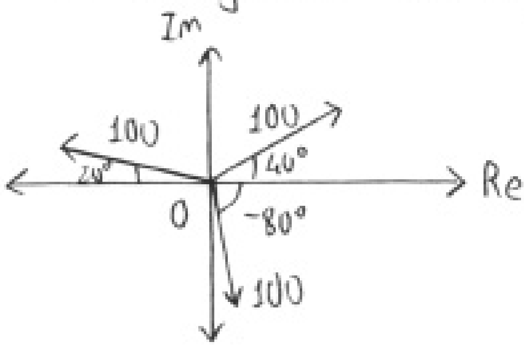
(4) a) $\frac{Z_1}{Z_2} = 0.62\angle 7.13^\circ \Rightarrow$ Modulus $= 0.62$ (from (3) part b)

b) $\left|\frac{Z_1}{Z_2}\right| = \frac{|Z_1|}{|Z_2|} = \frac{\sqrt{5}}{\sqrt{13}} = 0.62$

(5) Problem 9.6 (d): $y = 100\cos(\omega t + 40^\circ) + 100\cos(\omega t + 160^\circ) + 100\cos(\omega t - 80^\circ)$

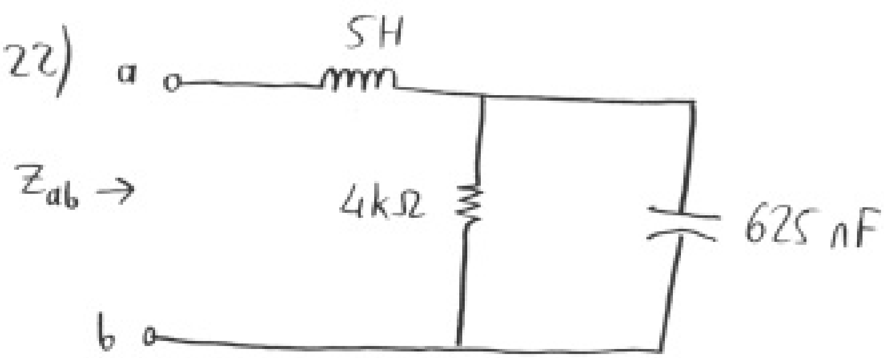
$\Rightarrow Y = 100\angle 40^\circ + 100\angle 160^\circ + 100\angle -80^\circ$

$= 76.6 + j64.27 - 93.96 + j34.2 + 17.36 - j98.4 = 0 \Rightarrow y = 0$



The three phasors exactly cancel, because they have the same magnitude and they are 120° apart.

(6) Problem 9.22)



a) $Z_{ab} = j5\omega + \dfrac{4\cdot 10^3 \cdot \frac{1}{j\omega 625\cdot 10^{-9}}}{4\cdot 10^3 + \frac{1}{j\omega 625\cdot 10^{-9}}} = j5\omega + \dfrac{4\cdot 10^3(1 - j\omega 2500\cdot 10^{-6})}{1 + (2500\cdot 10^{-6}\omega)^2}$

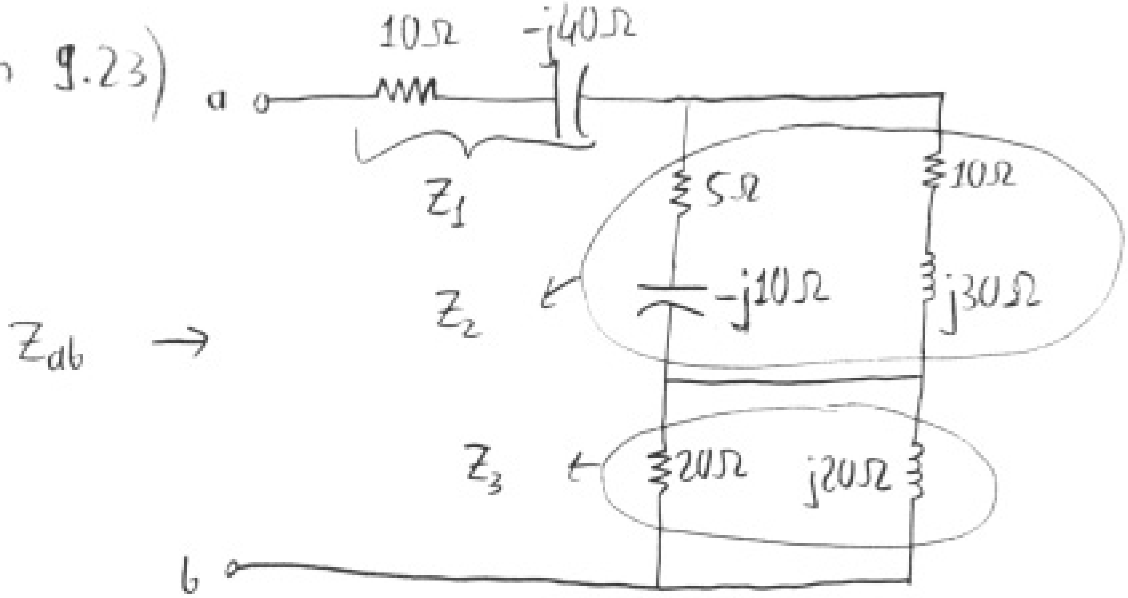
$$\Rightarrow Z_{ab} = j5w + \frac{4.10^3 - j10w}{1+6.25\times10^{-6}w^2} = \frac{4\,10^3}{1+6.25\times10^{-6}w^2} + j\left(5w - \frac{10w}{1+6.25\times10^{-6}w^2}\right)$$

Purely resistive $\Rightarrow$ $Im(Z_{ab}) = 0$

$$\Rightarrow 5\cancel{w} = \frac{10\cancel{w}}{1+6.25\times10^{-6}w^2} \Rightarrow \boxed{w = 400 \text{ rad/sec}}$$

b) $$Z_{ab}(w=400) = Re(Z_{ab}) = \left.\frac{4.10^3}{1+6.25\times10^{-6}w^2}\right|_{w=400} = \frac{4000}{2} = 2k\Omega$$

(7) Problem 9.23)



$Z_1 = 10 - j40\,\Omega$

$$Z_2 = \frac{(5-j10)(10+j30)}{15+j20} = \frac{50+j150-j100+300}{15+j20}$$

$$= \frac{350+j50}{15+j20} = \frac{70+j10}{3+j4} = \frac{(70+j10)(3-j4)}{3^2+4^2}$$

$$= \frac{210-j280+j30+40}{25} = \frac{250-j250}{25} = 10-j10\,\Omega$$

$$Z_3 = \frac{20\cdot(j20)}{20+j20} = \frac{20j}{1+j} = \frac{20j(1-j)}{1+1} = \frac{20j+20}{2} = 10+j10\,\Omega$$

$Z_{ab} = Z_1 + Z_2 + Z_3 = 10 - j40 + 10 - j10 + 10 + j10 = 30 - j40\ \Omega$

$= 50\angle -53.15^\circ$ //

(8) Problem 9.29)

$v_g = 150\cos(8000\pi t + 20^\circ)\ V$

$i_g = 30\sin(8000\pi t + 38^\circ)\ A$

$= 30\cos(8000\pi t - 52^\circ)\ A$

a) $Z = \dfrac{V_g}{I_g} = \dfrac{150\angle 20^\circ}{30\angle -52^\circ} = 5\angle 20^\circ + 52^\circ = 5\angle 72^\circ$

$= 1.545 + j\,4.75\ \Omega$

b) $v_g = 150\cos(8000\pi t + 20^\circ)\ V = 150\cos(8000\pi(t - t_{v_g}))\ V$

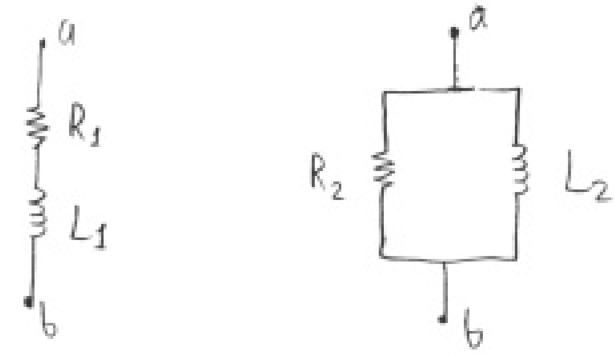
$i_g = 30\cos(8000\pi t - 52^\circ)\ A = 30\cos(8000\pi(t - t_{i_g}))\ A$

where $-8000\pi(t_{v_g}) = 20^\circ \cdot \left(\dfrac{2\pi}{360^\circ}\right)$ radians $\Rightarrow t_{v_g} = -13.8\bar{8}\ \mu sec$

$-8000\pi(t_{i_g}) = 52^\circ \cdot \left(\dfrac{2\pi}{360^\circ}\right)$ radians $\Rightarrow t_{i_g} = 36.1\bar{1}\ \mu sec$

$\Rightarrow \Delta t = t_{i_g} - t_{v_g} = 50\ \mu sec$ (i_g lags v_g by $50\ \mu sec$)

(9) Problem 9.25)



$$Z_1 = R_1 + j\omega L_1$$

$$Z_2 = \frac{R_2 (j\omega L_2)}{R_2 + j\omega L_2} = \frac{j\omega R_2 L_2 (R_2 - j\omega L_2)}{R_2^2 + (\omega L_2)^2}$$

$$= \frac{\omega^2 L_2^2 R_2 + j\omega L_2 R_2^2}{R_2^2 + \omega^2 L_2^2}$$

$$Z_1 = Z_2 \Rightarrow \quad R_1 = \frac{\omega^2 L_2^2 R_2}{R_2^2 + \omega^2 L_2^2} \qquad L_1 = \frac{R_2^2 L_1}{R_2^2 + \omega^2 L_2^2}$$

b) $R_2 = 50\,k\Omega \qquad L_2 = 2.5\,H \qquad \omega = 20\ krad/sec$

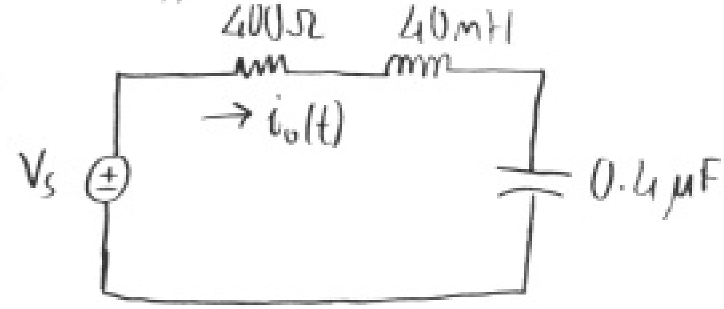
$$\Rightarrow R_1 = \frac{(2.10^4)^2 \cdot (2.5)^2\, 5.10^4}{(5.10^4)^2 + (2.10^4)^2 (2.5)^2} = \frac{4 \cdot 6.25 \cdot 5 \cdot 10^4}{(5.10^4)^2 + (2.10^4)^2 (2.5)^2}$$

$$= \frac{125.10^4}{25+25} = 25\ k\Omega$$

$$L_1 = \frac{(5\cdot 10^4)^2\ 2.5}{(5\cdot 10^4)^2 + (2\cdot 10^4)^2(2.5)^2} = \frac{25\cdot 2.5\cdot 10^8}{25\cdot 10^8 + 4\cdot 6.25\cdot 10^8}$$

$$= \frac{62.5}{50} = 1.25\ H$$ //

(10) Problem 9.14)



$V_s = 750 \cos 5000t$ mV   Equivalent resistance of R,L,C:

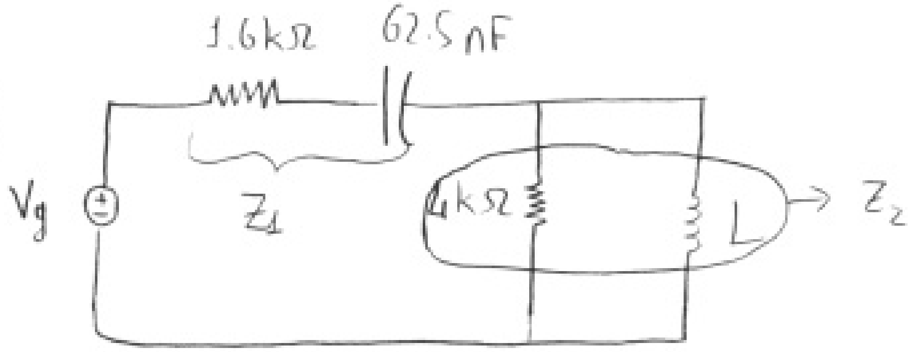
$$Z = 400 + j(5\cdot\cancel{10^3})(4\cdot\cancel{10^{-3}}) + \frac{1}{j(5\cdot 10^3)(0.4\cdot 10^{-6})}$$

$$= 400 + j200 - j\frac{1000}{2} = 400 - j300 = 500\angle{-36.86^\circ}\ \Omega$$

$$I_o = \frac{V_s}{Z} = \frac{0.75\angle 0^\circ}{500\angle{-36.86^\circ}} = 1.5\angle 36.86^\circ \text{ mA}$$

$$\Rightarrow i_o(t) = 1.5\cos(5000t + 36.86^\circ) \text{ mA}$$

11 Problem 9.21)



$V_g = 96 \cos 10000t$ V

a) $Z_1 = 1600 + \dfrac{1}{j(10^4)(62.5\cdot 10^{-9})} = 1600 - j1600\ \Omega$

$$Z_2 = \frac{4000(j10^4 L)}{4000 + j10^4 L} = \frac{j16.10^4 L + 4.10^5 L^2}{16 + 100L^2}$$

$$Z_T = Z_1 + Z_2 = 1600 + \frac{4.10^5 L^2}{16+100L^2} - j1600 + j\frac{16.10^4 L}{16+100L^2}$$

For i_g and V_g to be in phase require Z_T to be purely resistive $\Rightarrow \mathrm{Im}(Z_T) = 0$

$$\Rightarrow 1600 = \frac{16.10^4 L}{16+100L^2} \Rightarrow 16.10^4 L = 25600 + 160\,000L^2$$

$$\Rightarrow 1600L^2 - 1600L + 256 = 0 \Rightarrow 25L^2 - 25L + 4 = 0$$

$$\Delta = 625 - 400 = 225 \Rightarrow L_{1,2} = \frac{25 \pm 15}{50} = 0.8,\ 0.2\ \mathrm{H}$$

b) $L = 0.8H \Rightarrow Z_T = 1600 + \dfrac{4 \cdot 10^5 \cdot 0.64}{16+64} = 4800\,\Omega$

$$I_g = \frac{V_g}{Z} = \frac{96\angle 0^\circ}{4800} = 20\angle 0^\circ \text{ mA}$$

$\Rightarrow i_g = 20 \cos 10000t$ mA

$L = 0.2H \Rightarrow Z_T = 1600 + \dfrac{4 \cdot 10^5 (0.04)}{16+4} = 2400\,\Omega$

$$I_g = \frac{V_g}{Z} = \frac{96\angle 0^\circ}{2400} = 40\angle 0^\circ \text{ mA}$$

$\Rightarrow i_g = 40 \cos 10000t$ mA