Homework #1 Solutions

Problem 2

Solution:

$\sqrt{(A^2 + B^2)} \cos(t - \tan^{-1}(B/A)) \quad = \sqrt{(A^2 + B^2)}\ [\cos(t) \cos[\tan^{-1}(B/A)] + \sin(t) \sin[\tan^{-1}(B/A)]\ ]$ –(**1**)

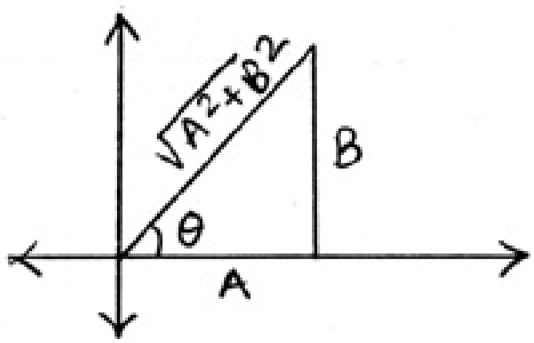
(Since $\cos(a - b) = \cos(a)\cos(b) + \sin(a)\sin(b)$)

Let $\tan^{-1}(B/A) = \theta$

$\tan(\theta) = B/A$

Thus,

$\cos(\theta) = A/\sqrt{(A^2 + B^2)}$ and $\sin(\theta) = B/\sqrt{(A^2 + B^2)}$ (Look at the figure below for explanation)



Replacing the values of $\cos(\theta)$ and $\sin(\theta)$ in equation (**1**), we get

$\sqrt{(A^2 + B^2)} \cos(t - \tan^{-1}(B/A)) = A\cos(t) + B\sin(t)$

Thus, the given expression has been proved.

Problem 3

Solution:

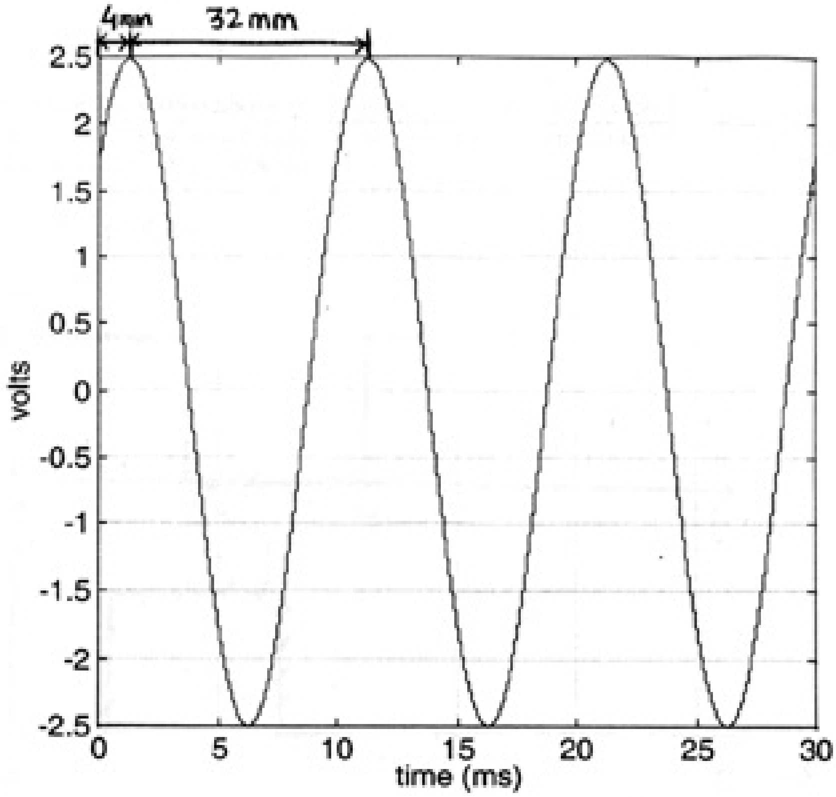
(a) Period = 10 ms

(b) Frequency = 1/10ms = 100 Hz

(c) The period of the sinusoid translates to 32mm on the given plot. Also from the plot, the sinusoid has been shifted towards the right by 4mm. Thus,

phase = (4/32)*2pi = 0.25pi = 45 degrees (Refer to the plot drawn below)

(d) Peak Amplitude = 2.5 V
(e) Peak to Peak Amplitude = 5V
(f) Rms Amplitude = $5/(2\sqrt{2})$ = 1.7677 Vrms
(g) Rms Amplitude relative to 1 Vrms in dBV = 20log(1.7667) = 4.9432 dBV



Problem 4

Solution:

(a)

$x(t) = A\cos(\omega_1 t + \theta)$

Therefore, $x(t-1) = A\cos(\omega_1(t-1) + \theta)(\omega_1 t + \theta)$

$y(t) = x(t) - x(t-1)$

$= A\cos(\omega_1 t + \theta) - A\cos(\omega_1(t-1) + \theta)$

$= A[\cos(\omega_1 t + \theta) - \cos(\omega_1 t - \omega_1 + \theta)]$

$= A[\ -2\sin((2\omega_1 t + 2\theta - \omega_1\ )/2)\ \sin(\omega_1 /2)\ ]$ Since $\cos A - \cos B = -2\sin((A+B)/2)\sin((A-B)/2)$

$= -2A\ \sin(\omega_1 /2)\ \sin(\omega_1 t + (\theta - \omega_1/2))$

$= (-2A\ \sin(\omega_1 /2))\ \cos(\pi /2 - (\omega_1 t + (\theta - \omega_1/2))\ )$ Since $\sin A = \cos(\pi/2 - A)$

$= (-2A\ \sin(\omega_1 /2))\ \cos(\pi - \pi/2 + (\omega_1 t + (\theta - \omega_1/2))\ )$ Since $\cos A = -\cos(\pi - A)$

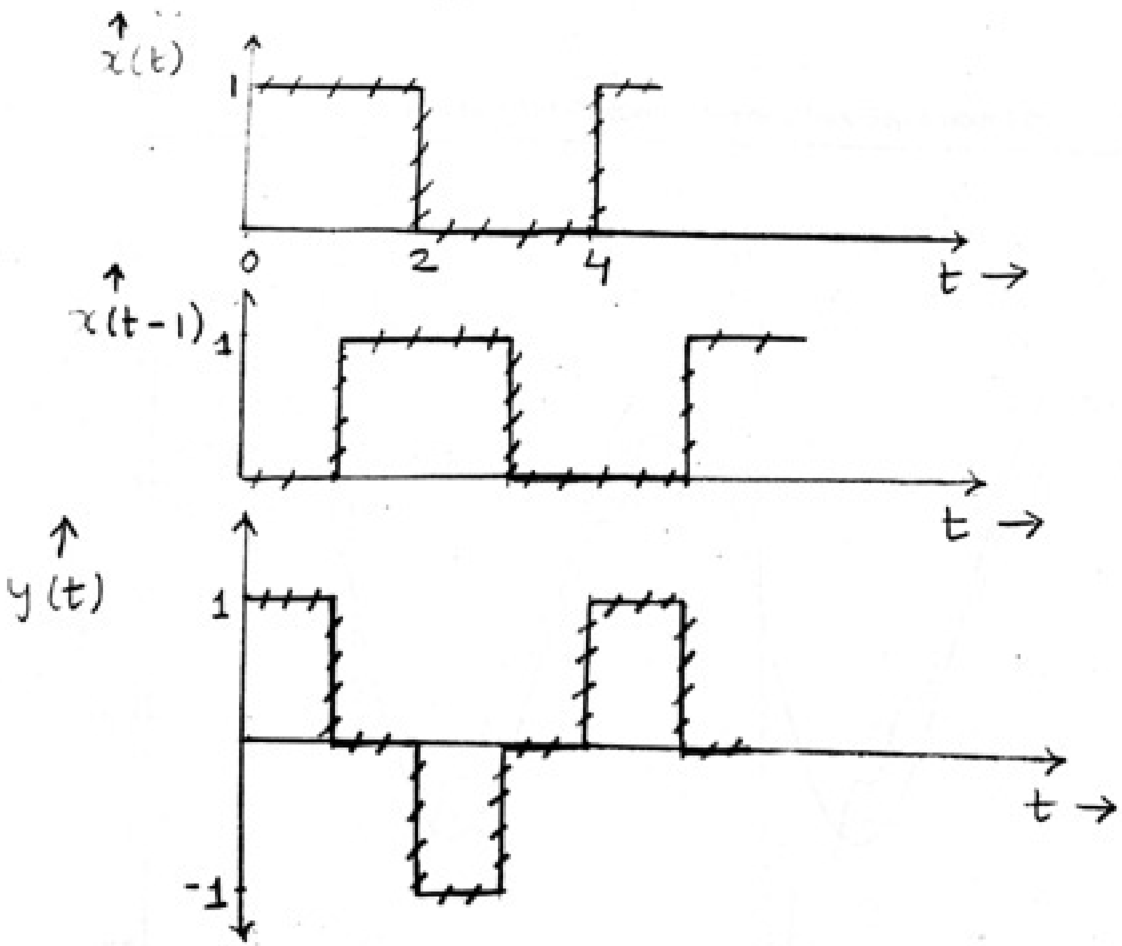
$= (2A\ \sin(\omega_1 /2))\ \cos(\ \omega_1 t\ + (\ \pi/2 + \theta - \omega_1/2)\ )$

$= B\cos(\omega_2 t + \phi)$

Thus output $y(t)$ has been written in the form $B\cos(\omega_2 t + \phi)$ where $B = 2A\ \sin(\omega_1 /2)$, $\omega_2 = \omega_1$ and $\phi = \pi/2 + \theta - \omega_1/2$

Note: An answer as shown above is adequate. But we can consider a case of $\sin(\omega_1/2)$ being negative when $\omega_1/2 > \pi$ in which case we need to add an additional π in the equation for ϕ and use the absolute value of B i.e. $|2A\sin(\omega_1/2)|$

(b) Sketches of x(t), x(t-1) and y(t)



(c) From the result of part (a) we see that given a sinusoidal input to the system, the output is also sinusoidal. But the notable difference in case of square waves, that we note in part (b), is that a square wave input to the system doesn't give a square wave output.

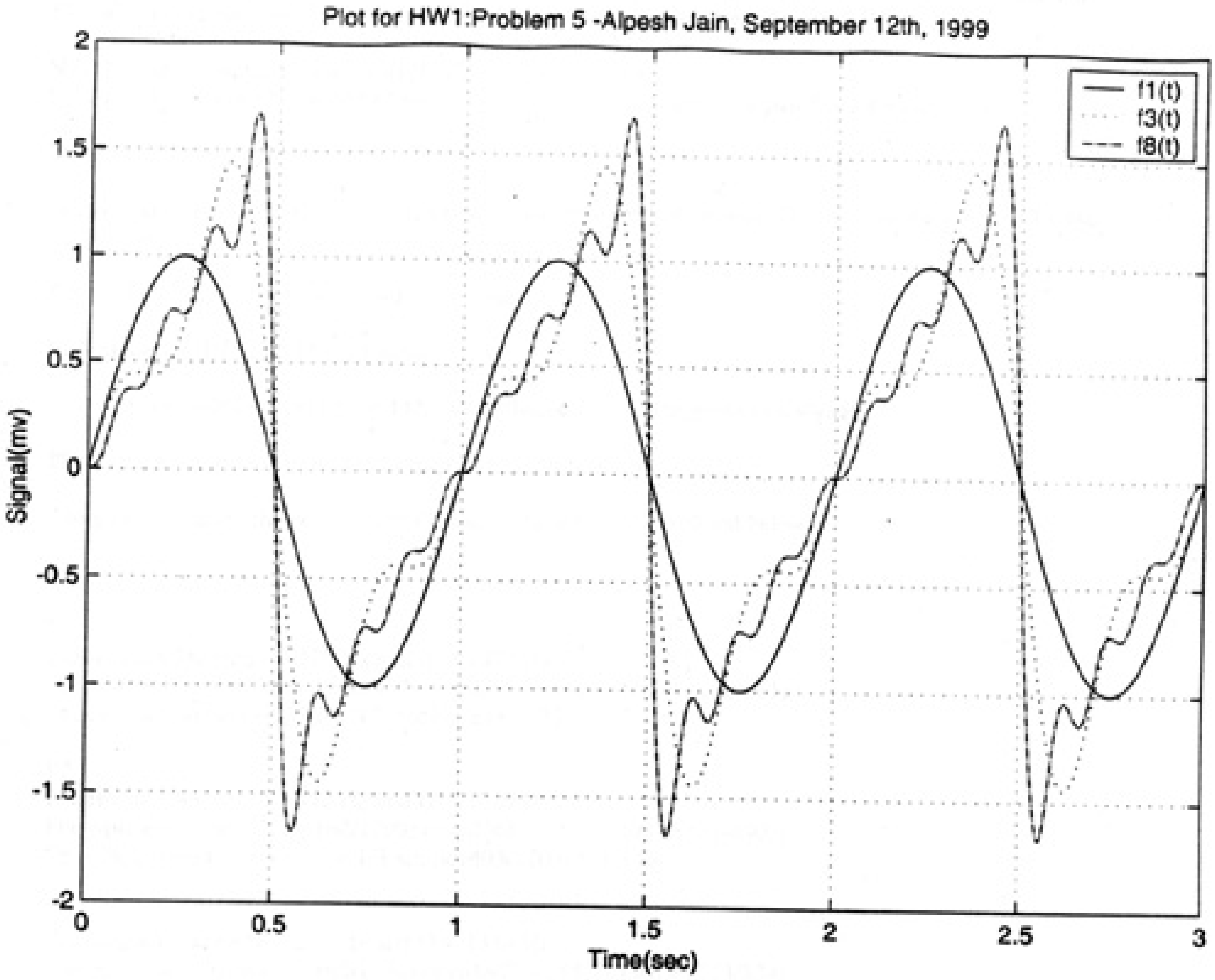
Problem 5

Solution:
(a)
Matlab Code:

```
t = linspace(0,3,10000);
f1 = sin(2*pi*t);
f3 = f1 - (sin(4*pi*t))/2 + (sin(6*pi*t))/3;
f8 = f3 - (sin(8*pi*t))/4 + (sin(10*pi*t))/5 - (sin(12*pi*t))/6 +
(sin(14*pi*t))/7 - (sin(16*pi*t))/8;
plot(t,f1,'-',t,f3,':',t,f8,'-.' );
grid
```

```
xlabel('Time(sec)')
ylabel('Signal(mv)')
title('Plot for HW1:Problem 5 -Alpesh Jain, September 12th, 1999')
legend('f1(t)','f3(t)','f8(t)')
```

Plot:



(b) I think fn(t) would look like a sawtooth wave for a large n.

Problem6

Solution:

In this problem, basically we are calculating the least common multiple (LCM) of the period of the given two sine waves, to find the period of the resultant signal consisting of the given sine waves.

For e.g. we want to find the period of a signal resulting from two signals of periods T1= 2 and T2= 3 respectively.
We want to find the period of the resultant signal which will be the LCM of the periods of the given two signals.
LCM of the periods of given signals = 2(n1) = 3(n2), where n1 and n2 are integers. Thus in this case n1 = 3 and n2 = 2 and so LCM = 6, which is the period of the resultant signal.

In the above example, the resultant period was simply the product of the periods of the given two signals. That may not be the case necessarily. Lets take another example where T1 = 8 and T2 = 12

Now Period of resultant signal = n1(T1) = n2(T2) = n1(8) = n2(12).
Now LCM of 8 and 12 is not 8*12 = 96, but is 24. Thus the resultant signal has a period of 24.

To summarize,

The period of T of a signal consisting of two periodic signals of periods T1 and T2 is the LCM of T1 and T2 , that is

T = n1(T1) = n2(T2), where n1 and n2 are integers.

Now since T = 1/f (Period = 1/frequency), we can say

T = (n1/f1) = (n2/f2), where f1 and f2 are the frequencies of the given two signals.

This is the equation given by Dr. Ganago in the lab question.

Using this equation, the periods T6, T7 and T8 have been determined below.

T6
The frequencies are f1 = 770 Hz and f2 = 1477 Hz
The equation is (n1/770) = (n2/1477) or (n1/n2) = (770/1477) = (110/211)
Thus the period is (110/770) = 1/7 sec or (211/1477) = 1/7 sec

T7
The frequencies are f1 = 852 Hz and f2 = 1209 Hz
The equation is (n1/852) = (n2/1209) or (n1/n2) = (852/1209) = (284/403)
Thus the period is (284/852) = 1/3 sec or (403/1209) = 1/3 sec

T8
The frequencies are f1 = 852 Hz and f2 = 1336 Hz
The equation is (n1/852) = (n2/1336) or (n1/n2) = (852/1336) = (213/334)
Thus the period is (213/852) = 1/4 sec or (334/1336) = 1/4 sec