

**Note:**  $M\angle\theta = Me^{j\theta}$ ; angles in degrees; impedances in  $\Omega$  throughout.

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1.  $Y = \frac{1}{480} + j(5000)(\frac{5}{9} \times 10^{-6}) = \frac{1}{480} + \frac{j}{360} = \frac{1}{288} \angle 53^\circ$ .  $|I| = |V||Y| = \frac{240}{288} = \frac{5}{6}$ .
  - 1a. Peak inst. power =  $\frac{1}{2}|V||I| \cos(\theta_v - \theta_i) + \frac{1}{2}|V||I| = \frac{1}{2}(240)\frac{5}{6}(\cos(53^\circ) + 1) = 160W$ .
  - 1b. Peak inst. power =  $\frac{1}{2}|V||I| \cos(\theta_v - \theta_i) - \frac{1}{2}|V||I| = \frac{1}{2}(240)\frac{5}{6}(\cos(53^\circ) - 1) = -40W$ .
  - 1c. Average power =  $P = \frac{1}{2}|V||I| \cos(\theta_v - \theta_i) = \frac{1}{2}(240)\frac{5}{6} \cos(53^\circ) = 60W$ .
  - 1d. Reactive power =  $Q = \frac{1}{2}|V||I| \sin(\theta_v - \theta_i) = \frac{1}{2}(240)\frac{5}{6} \sin(-53^\circ) = -80var$ .
  - 1e. Generates vars since  $Q < 0$ . **1f.**  $pf = \cos(53^\circ) = 0.6$ . **1g.**  $rf = \sin(-53^\circ) = -0.8$ .
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2.  $Z = \frac{-j}{0.1} + (5||j5) + 7.5 = 10 - j7.5 = 12.5\angle -37^\circ$ .  $I = \frac{50}{12.5\angle -37^\circ} = 4\angle 37^\circ$ .
  - 2a.  $S = \frac{1}{2}VI^* = \frac{1}{2}(50)(4\angle -37^\circ) = 80 - j60VA$ .  $P = 80W$ ;  $Q = -60var$ ;  $|S| = 100VA$ .
  - 2b.  $I_{5\Omega} = 4\angle 37^\circ \frac{j5}{5+j5} = 2\sqrt{2}\angle 82^\circ$ .  $P_{5\Omega} = \frac{1}{2}|I|^2(5\Omega) = \frac{1}{2}(2\sqrt{2})^2(5) = 20W$ .  
 $P_{7.5\Omega} = \frac{1}{2}|I|^2(7.5\Omega) = \frac{1}{2}4^2(7.5) = 60W$ .  $20 + 60 = 80W$  checks.
  - 2c.  $I_{50\mu H} = 4\angle 37^\circ \frac{5}{5+j5} = 2\sqrt{2}\angle -8^\circ$ .  $Q_{50\mu H} = \frac{1}{2}|I|^2(5) = \frac{1}{2}(2\sqrt{2})^2(5) = 20var$ .  
 $Q_{1\mu F} = \frac{1}{2}|I|^2(-10) = \frac{1}{2}4^2(-10) = -80var$ .  $20 - 80 = -60var$  checks.
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3.  $Z = 2 + j20 + 138 + j460 = 140 + j480 = 500\angle 74^\circ$ .  $I = \frac{V}{Z} = \frac{7200}{140+j480} = 14.4\angle -74^\circ$ .
  - 3a.  $P_{LINE} = |I|^2R = (14.4)^2 2 = 414.72W$ . Note: 7200V is rms.
  - 3b.  $Y_{LOAD} = \frac{1}{138+j460} = \frac{138-j460}{230644}$ . Need  $j\omega C = \frac{j460}{230644} \rightarrow X_C = \frac{1}{\omega C} = \frac{230644}{460} = 501.4\Omega$ .
  - 3c.  $Y_{LOAD} = \frac{138}{230644} \rightarrow Z_{LOAD} = \frac{230644}{138} = 1671.3\Omega$ .
  - 3d.  $I = \frac{7200}{1671+j20} = 4.3\angle -0.68^\circ$ .  $P_{LINE} = |I|^2R = (4.3)^2 2 = 37W$ .
  - 3e. Power dissipated in line reduced from 414W to 37W: a 91% decrease! 8.9%.
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4.  $pf = 0.96 = \cos(\theta_v - \theta_i) \rightarrow \sin(\theta_v - \theta_i) = \sqrt{1 - (0.96)^2} = 0.28$ .  
 $S_L = (25kVA)(0.96 + j0.28) = (24 + j7)kVA$ .  $I^* = \frac{(24+j7)kVA}{125V} = (192 + j56)A$ .
  - 4a.  $V_S = 125 + (192 - j56)(0.006 + j0.048) = 128.84 + j8.88 = 129.1\angle 3.9^\circ \rightarrow |V_S| = 129.1V$ .
  - 4b.  $P_F = |I|^2R = |192 - j56|^2(0.006) = (200)^2(0.006) = 240W$ .
  - 4c.  $(125)^2(\omega C) = 7000 \rightarrow C = \frac{7000}{(125)^2 2\pi(60)} = 1188\mu F$ .
  - 4d.  $V_s = 125 + (192 - j0)(0.006 + j0.048) = 126.15 + j9.22 = 126.5\angle 4.2^\circ \rightarrow |V_S| = 126.5V$ .
  - 4e.  $P_F = |I|^2R = |192|^2(0.006) = 221.2W$ . We have reduced line loss by 19W=7.8%.
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5a. **Hard:**  $Z = (4 + j\omega 0.002) || (4 - \frac{j(10)^6}{\omega 125}) = 4$  after much algebra (don't ask).

5a. **Easy:** Scale  $\omega$  by 2000 (let  $\xi = \omega/2000$ ) and use admittance, not impedance:

$$Y = \frac{1}{4+j4\xi} + \frac{1}{4-j4/\xi} = \frac{1}{4} \left( \frac{1}{1+j\xi} + \frac{j\xi}{1+j\xi} \right) = \frac{1}{4} \rightarrow Z = 4\Omega \text{ independent of } \omega \text{ (and } \xi\text{)!}$$

See how much easier this is if you *think* first before plunging into the algebra?

- 5b. Low frequency currents are steered into the woofer, which reproduces them well.
- 5b. High frequency currents are steered into the tweeter, which reproduces them well.