1. From the log plots, the gain and phase at DC ($\omega = 0$) and $\omega = 100$ are:

$\omega = 0$: Gain = 0 dB = 1; Phase = 0.
$\omega = 100$: Gain = (-20 dB) = 0.1; Phase = -45°.

1a. (i) $0.1 \cos(100t - 45°)$; (ii) $7 + 0.3 \cos(100t - 25°)$.

1b. (i) $10 \cos(100t + 45°)$; (ii) $7 + 30 \cos(100t + 65°)$.

2. Plotted below left. Approaching an exponential function ($e^{3-t}$, $0 < t < 3$, in fact).

3. Gain at $\omega = 2$ is $15/3 = 5$; Gain at $\omega = 8$ is $10/5 = 2$. Solve 2 equations in 2 unknowns:

$B/(2^2 + A^2) = 5 \rightarrow B - 5A^2 = 20$; $B/(8^2 + A^2) = 2 \rightarrow B - 2A^2 = 128$. Solving $A = \pm 6, B = 200$.

4a. $x(t) = \begin{cases} 1 - t, & \text{for } 0 < t < 1; \\ 1 + t, & \text{for } -1 < t < 0. \end{cases}$ Period = T = 2; $\omega_o = \frac{2\pi}{2} = \pi$.

Since $x(t)$ is an even function (symmetric about $t = 0$), we have $b_n = 0$.

$$a_n = \frac{2}{\pi} \int_{-1}^{1} (1 + t) \cos(n\pi t) dt + \frac{2}{\pi} \int_{0}^{1} (1 - t) \cos(n\pi t) dt = 2 \int_{0}^{1} \cos(n\pi t) dt = \frac{2}{\pi} \sin(n\pi t)|_0^{1} - 2 \cos(n\pi t)|_0^{1} - \frac{n^2}{n^2} \sin(n\pi t)|_0^{1} = \frac{2}{\pi^2 n^2} (1 - (-1)^n) = \begin{cases} \frac{4}{\pi^2 n^2}, & \text{for } n \text{ odd}; \\ 0, & \text{for } n \text{ even}. \end{cases}$$

Only middle term above is non-zero.

$$a_o = \frac{1}{\pi} \int_{-1}^{1} (1 + t) dt + \frac{1}{\pi} \int_{0}^{1} (1 - t) dt = \frac{2}{\pi} \int_{0}^{1} (1 - t) dt = 1/2 \text{ using symmetry.}$$

$$x(t) = \frac{1}{2} + \frac{4}{\pi^2} \cos(\pi t) + \frac{4}{9\pi^2} \cos(3\pi t) + \frac{4}{25\pi^2} \cos(5\pi t) + \frac{4}{49\pi^2} \cos(7\pi t) + ...$$

4b. 2 Hertz $\Leftrightarrow \omega = 4\pi \rightarrow$ keep 1st 3 terms of this series: $\frac{1}{2} + \frac{4}{\pi^2} \cos(\pi t) + \frac{4}{9\pi^2} \cos(3\pi t)$.

Plotted below right. Attenuation of high freqs → round corners: can’t change quickly.