

- 1a. $v = L \frac{di}{dt} \rightarrow i(t) = i(0) + \frac{1}{L} \int_0^t v(\tau) d\tau$ for $0 < t < 0.001$, $0.001 < t < 0.002$, $t > 0.002$:
 $0 < t < 0.001$: $i(t) = 0 + \frac{6mv}{300\mu H} \int_0^t d\tau = (20t)A$ since $\frac{6mv}{300\mu H} = 20 \frac{amp}{sec}$.
 $0.001 < t < .002$: $i(t) = (20 \frac{amp}{sec})(0.001sec) + \frac{6mv}{300\mu H} \int_{0.001}^t (2 - 1000 \frac{msec}{sec} \tau) d\tau = (40t - 10000t^2 - 0.01)A$.
 $t > 0.002$: $i(t) = (40(0.002) - 10000(0.002)^2 - 0.01)A + \frac{6mv}{300\mu H} \int_{0.002}^t 0\tau = 0.03A$.
 CHECK: Continuity of $i(t)$ at $t=0.001$: $(0.02A=0.02A$ OK); $t=0.002$: $(0.03A=0.03A$ OK). Whew!
 1b. See plot below. Note continuity of $i(t)$.

- 2a. $v = L \frac{di}{dt} = (0.02H)[-2500A_1 e^{-2500t} - 7500A_2 e^{-7500t}] = -(50A_1 e^{-2500t} + 150A_2 e^{-7500t})$.
 $i(0) = 0.05 = A_1 + A_2$ and $v(0) = 10 = -50A_1 - 150A_2 \rightarrow A_1 = 0.175$, $A_2 = -0.125$.
 $v(t) = -8.75e^{-2500t} + 18.75e^{-7500t}V$, $t > 0$; $i(t) = 0.175e^{-2500t} - 0.125e^{-7500t}A$, $t \geq 0$.
 2b. $p(t) = i(t)v(t) = 4.375e^{-10000t} - 1.53125e^{-5000t} - 2.34375e^{-15000t}$.
 $p(t) = (4.375x - 1.53125x^2 - 2.34375)/x^3 = 0$ where $x = e^{5000t}$ as suggested.
 Solving quadratic $\rightarrow x = 0.7143 \rightarrow t < 0$; $x = 2.143 \rightarrow t = \frac{\log_e 2.143}{5000} = 152.4\mu sec$.

- 3a. $i = C \frac{dv}{dt} = (0.0000004) \frac{d(-60)}{dt} = 0$, $t < 0$ (this wasn't the nasty derivative).
 3b. $i = C \frac{dv}{dt} = (0.0000004) \frac{d}{dt}[15 - 15e^{-500t}5 \cos(2000t) - 15e^{-500t} \sin(2000t)] = (0.0000004) \times (7500)e^{-500t}[\cos(2000t) + 21 \sin(2000t)] = 3e^{-500t}[\cos(2000t) + 21 \sin(2000t)]mA$.
 3c. $v(0^+) = 15 - 15(e^0)(5 + 0) = -60 = v(0^-)$ so $v(t)$ doesn't jump (it better not!).
 3d. $i(0^+) = 3(e^0)(1 + 0) = 3mA \neq 0 = i(0^-)$ so $i(t)$ does jump (cap. current CAN jump).
 3e. Stored energy $= \frac{1}{2}Cv(\infty)^2 = \frac{1}{2}(0.0000004)(15)^2 = 45\mu Joules$.

4. $(14||6) + 15.8 = 20$. $(20||60) + 5 = 20$. $(20||80) + 24 = 40$. $(40||10) + 12 = 20H$.

- 5a. $i_o(0) = i_1(0) + i_2(0) = 10 - 5 = 5A$. $(32||8) + 3.6 = 10H$ for (b).
 5b. $i_o(t) = i_o(0) - \frac{1}{L} \int_0^t v(\tau) d\tau = 5 - 0.1 \int_0^t 1250e^{-25\tau} d\tau = 5e^{-25t}A$, $t > 0$.
 5c. $i(t) = i(0) + \frac{1}{L} \int_0^t v(\tau) d\tau \rightarrow i_1(t) = C_1 + 4e^{-25t}$, $t > 0$ and $i_2(t) = C_2 + e^{-25t}$, $t > 0$.
 Where C_1 and C_2 are chosen to match the given $i_1(0) = 10$ and $i_2 = -5$.
 $\rightarrow i_1(t) = 4e^{-25t} + 6$ and $i_2(t) = e^{-25t} - 6$.
 5e. Stored energy $= \frac{1}{2}8(10)^2 + \frac{1}{2}32(-5)^2 + \frac{1}{2}3.6(5)^2 = 845Joules$.

6. $(4 + j3)(5 + j12) = (4 \cdot 5 - 3 \cdot 12) + j(3 \cdot 5 + 4 \cdot 12) = -16 + j63$.
 $(4 + j3)(5 + j12)^* = (4 + j3)(5 - j12) = (4 \cdot 5 + 3 \cdot 12) + j(3 \cdot 5 - 4 \cdot 12) = 56 - j33$.
 $\frac{4+j3}{5+j12} = \frac{4+j3}{5+j12} \frac{5-j12}{5-j12} = \frac{1}{169}(4+j3)(5-j12) = \frac{56}{169} - j \frac{33}{169}$ ($5^2 + 12^2 = 169$).
 $1/[(4+j3)(5+j12)] = \frac{1}{-16+j63} = \frac{-16-j63}{-16-j63} = -\frac{16}{4225} - j \frac{63}{4225}$ ($16^2 + 63^2 = 4225$).
 $(4 + j3)^*(5 + j12) = (4 - j3)(5 + j12) = (4 \cdot 5 + 3 \cdot 12) + j(-3 \cdot 5 + 4 \cdot 12) = 56 + j33$.

Now redo these using polar form (answers don't match perfectly due to rounding):

$$4 + j3 = 5e^{j37^\circ}; \quad 5 + j12 = 13e^{j67^\circ}. \quad \text{Now these are } \textit{easy}:$$

$$(4 + j3)(5 + j12) = 5e^{j37^\circ} 13e^{j67^\circ} = 65e^{j104^\circ} = -16 + j63.$$

$$(4 + j3)(5 + j12)^* = 5e^{j37^\circ} 13e^{-j67^\circ} = 65e^{-j30^\circ} = 56 - j33.$$

$$\frac{4+j3}{5+j12} = 5e^{j37^\circ} / 13e^{j67^\circ} = 0.385e^{-j30^\circ} = 0.333 - j0.192.$$

$$1/[(4 + j3)(5 + j12)] = \frac{1}{5}e^{-j37^\circ} \frac{1}{13}e^{-j67^\circ} = 0.0154e^{-j104^\circ} = -0.00372 - j0.0149$$

$$(4 + j3)^*(5 + j12) = 5e^{-j37^\circ} 13e^{j67^\circ} = 65e^{j30^\circ} = 56 + j33.$$

7. Magnitude squared = $[(25)(65)(100)(169)(125)] / [(676)(65)(625)(125)] = 1$.

How did I come up with these? Note the various Pythagorean triangles.

8. HARD WAY: $5 \cos(3t) \cos(30^\circ) - 5 \sin(3t) \sin(30^\circ) + 5 \sin(3t) + 5 \cos(3t) \cos(210^\circ) + 5 \sin(3t) \sin(210^\circ) = 0 \cos(3t) + 0 \sin(3t) = 0$ *identically*. Why?

8. EASY WAY: $5e^{j30^\circ} + 5e^{-j90^\circ} + 5e^{-j210^\circ} = 0$. Note $5 \sin(3t) = 5 \cos(3t - 90^\circ)$.

See below right for a pretty picture of why these sum to zero.

9. UNGRADED: The Norton equivalent box has current flowing through a resistor.

The Thevenin equivalent box has no current flowing. The Norton equivalent is *warmer*.

