SOLUTIONS & MARKING SCHEME

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TOTAL: ________ / 75

ELECTROMAGNETICS I
(EECS 230 - WINTER 2000)
Mid-Term Exam #1

Date: February 7, 2000
Time: 7-9pm
Instructor: Professor S. Rand
Duration: 2 hours
LOCATION: 1200 EECS and 1301 EECS

No books, notes or other study materials may be used for this examination. In addition, programmable electronic calculators and note pads are NOT PERMITTED. Show your work and reasoning, to be eligible for part marks. As much as possible, complete problems symbolically before substituting for parameter values.

ANSWER ALL 3 QUESTIONS

Useful Formulae:

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} = \alpha + j\beta \qquad Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

$$u_p = \frac{\omega}{\beta} = f\lambda$$

$$Z_{in} = Z_0\left[\frac{1+\Gamma e^{-2j\beta\ell}}{1-\Gamma e^{-2j\beta\ell}}\right] \qquad \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_{in} = Z_0^2 / Z_L \qquad S = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Honor Code: I have neither given nor received aid on this exam.

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Name (Print)

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Signature

1. (25 marks total) A transmission line is characterized by a resistance per meter of 200 Ω (i.e. 200 Ω/m), an inductance of 37 nH/m, a capacitance of 30 pF/m and zero shunt conductance. Voltage waves on this line obey the phasor wave equation

$$\frac{d^2\tilde{V}}{dz^2} - \gamma^2\tilde{V}(z) = 0.$$

(a) Write down a general solution for $\tilde{V}(z)$ in terms of γ and z. (Note: For this part, γ and z specific values are not needed.) (4 marks)

$$\tilde{V}(z) = V^{+}e^{-\gamma z} + V^{-}e^{+\gamma z}$$

V^{+}, V^{-} are real amplitudes for right and left-going travelling waves and γ is complex in general.

(b) How far along the line (in meters) would a voltage wave have to travel at 85 MHz before its amplitude dropped by a factor of 10 due to absorption? (10 marks)

$\gamma = \sqrt{(R'+j\omega L')(G'+j\omega C')} \Rightarrow 1.203 + j1.323$ or

1 $\gamma = 1.789\, e^{j47.82°}$ upon substitution of R', L', G', C' & ω.

But we also know that

$\gamma = \alpha + j\beta = \sqrt{\alpha^2+\beta^2}\; e^{j\theta}, \quad \theta \equiv \tan^{-1}\left(\frac{\beta}{\alpha}\right).$

$\frac{\beta}{\alpha} = \tan\theta = \tan(47.82°) = 1.10 \longrightarrow \boxed{\beta = 1.10\,\alpha}$ 1

and

$\sqrt{\alpha^2+\beta^2} = 1.789 \longrightarrow \boxed{\alpha^2+\beta^2 = 3.2}$ 2

Solving 1 & 2 for α, β we find

$\alpha^2 + (1.10)^2\alpha^2 = 3.2 \longrightarrow \boxed{\alpha = 1.203\ \mathrm{m}^{-1}}$ 1

The distance over which V drops by 10 is determined by $V_0 e^{-\alpha z} = V_0(0.1) \rightarrow -\alpha z = \ln(0.1) \rightarrow \boxed{z = 1.91\ \mathrm{m}}$

correct 2 1

(c) From the phase constant (also called the propagation constant) for the 85 MHz wave on this particular transmission line, find the value of the wavelength in meters. (3 marks)

$\beta = 1.10\,\alpha = 1.323\ m^{-1}$ (1)

Because $\beta = \frac{2\pi}{\lambda}$, the wavelength must be

$$\boxed{\lambda = \frac{2\pi}{\beta} = 4.75\ m}$$ (1)

In calculating λ, it may not be assumed that $u_p = c$ (velocity of light in vacuum). The cable is not vacuum.

(d) How many oscillations (periods) of the wave occur as it attenuates by the factor of 10 (at 85 MHz)? (3 marks)

Suppose the wave decays in a distance d. The number of oscillations is n, where

$$n = \frac{d}{\lambda} = 1.91m / 4.75m = 0.4 \text{ periods}$$ (1)

(e) What is the value of the characteristic impedance of this transmission line? (3 marks)

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \text{ (1) } = \sqrt{\frac{200 + j2\pi \times 8.5\times10^7 \times 3.7\times10^{-8}}{0 + j2\pi \times 8.5\times10^7 \times 3\times10^{-11}}} \text{ (1)}$$

$$= \sqrt{\frac{200 + j19.8}{j160.2\times10^{-4}}} = 112\,e^{-j\,0.736 \text{ rad}} \text{ OR } 112\,e^{-j42.2^\circ}$$

$$\text{OR } (83.1 - j75.3)\ \Omega$$ (1) Exact

(f) What would the value of the characteristic impedance be if the line were lossless? (2 marks)

Correct substitutions (1)

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \xrightarrow[R'=0]{G'=0} \sqrt{\frac{L'}{C'}} = \sqrt{\frac{3.7\times10^{-8}}{3.0\times10^{-11}}}$$

$$\boxed{Z_0 = 35.1\ \Omega}$$ (1)

2. (25 marks total) Consider a transmission line of characteristic impedance Z_{01} that is terminated with a quarter-wave transformer of characteristic impedance $Z_{02} = \sqrt{Z_{01} Z_L}$ and a load Z_L.

(a) Show explicitly that the input impedance $Z_{in} = Z_0 \left[\frac{1+\Gamma_L e^{-2j\beta\ell}}{1-\Gamma_L e^{-2j\beta\ell}} \right]$ experienced by a continuous sinusoidal voltage at the junction of the two transmission lines is equal to the characteristic impedance of the input line, as intended. (14 marks)

We need to show that $Z_{in} = Z_{01}$. To do this, notice that 2

$$2\beta\ell = 2 \cdot \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \pi \longrightarrow e^{-2j\beta\ell} = e^{-j\pi} = -1.$$ 1

$$\therefore Z_{in} = Z_{02} \left[\frac{1-\Gamma_L}{1+\Gamma_L} \right].$$ 2 1

But $\Gamma_L = \frac{Z_L - Z_{02}}{Z_L + Z_{02}}$, 2 so that we obtain

$$Z_{in} = Z_{02} \left[\frac{1 - \frac{Z_L - Z_{02}}{Z_L + Z_{02}}}{1 + \frac{Z_L - Z_0}{Z_L + Z_0}} \right] = Z_{02} \left[\frac{Z_L + Z_{02} - Z_L + Z_{02}}{Z_L + Z_{02} + Z_L - Z_{02}} \right]$$

1 Substitute

$$= \frac{2 Z_{02}^2}{2 Z_L} = \frac{Z_{01} Z_L}{Z_L} = Z_{01},$$ 2 using $Z_{02} = \sqrt{Z_{01} Z_L}$.

Hence 2

$$\boxed{Z_{in} = Z_{01}},$$ as expected for a $\frac{\lambda}{4}$ transformer.

That is, the transformer achieves impedance matching to the input line characteristic impedance. 1

(b) What is the standing wave ratio with the transformer in place? (1 mark)

As given, $S = \frac{1+|\Gamma|}{1-|\Gamma|}$.

$$\Gamma_{junction} = \text{"}\frac{Z - Z_0}{Z_L + Z_0}\text{"} = \frac{Z_{in} - Z_{01}}{Z_{in} + Z_{01}} = \frac{Z_{01} - Z_{01}}{Z_{01} + Z_{01}} = 0$$

$$\therefore \boxed{S = \frac{1+0}{1-0} = 1}$$ 1

(c) From the formula for Z_{in}, calculate the input impedance if the quarter-wave transformer is replaced by a half-wave line. (10 marks)

$$2\beta l = 2 \cdot \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = 2\pi \;(2) \longrightarrow e^{-2j\beta l} = e^{-j2\pi} = 1 \;(1)$$

$$Z_{in} = Z_{02}\left[\frac{1+\Gamma_L}{1-\Gamma_L}\right] \;(2)$$

In this case, $\Gamma_L = \dfrac{Z_L - Z_{02}}{Z_L + Z_{02}} \cdot \cancel{\dfrac{Z_L - Z_{02}}{Z_L + Z_{02}}}$ (2)

$$\therefore Z_{in} = Z_{02}\left[\frac{1 + \frac{Z_L - Z_{02}}{Z_L + Z_{02}}}{1 + \frac{Z_L - Z_{02}}{Z_L + Z_{02}}}\right] \;(1)\; = Z_{02}\left[\frac{\frac{2Z_L + 0}{Z_L + Z_{02}}}{\frac{2Z_{02} + 0}{Z_L + Z_{02}}}\right]$$

$$\boxed{Z_{in} = Z_L} \;(1)$$

Thus the half-wave line causes the load impedance to appear at the input point (the same junction as in part (a)).

(1)

3. (25 marks total) A lossless 50-Ω transmission line is terminated with a load impedance $Z_L = (100 + j\,75)\ \Omega$. Use the Smith chart provided* to find

(a) the value of the input impedance at 0.75λ from the load (5 marks)

Since an entire rotation on the chart is $0.5\ \lambda$, we consider a point rotated only $0.75 - 0.5 = 0.25\ \lambda$ from the load (WTG), at $0.459\ \lambda$ (WTG).
Here, $Z(0.75\lambda) = (0.32 - j0.24)Z_o = (16 - j12)\ \Omega$.

(b) the value of the input admittance at 0.75λ from the load (3 marks)

$Y(0.75\lambda) = (2 + j1.5)Y_o = (0.04 + j0.03)$ mhos (Ω^{-1}).

(c) the distance of the first voltage minimum (in wavelengths) from the load (5 marks)

Distance to first minimum is given in wavelengths on the WTG scale, starting from the load and rotating to the first intersection with the negative real axis (point M).
Distance $= 0.5\lambda - 0.209\lambda = 0.291\lambda$

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*No marks will be awarded for non-graphical answers. Write values, coordinates, angles, etc. on the chart and on this page.

(d) the length and distance from the load of a shorted stub that achieves impedance matching between the line and the load (12 marks)

Convert load impedance (graphed point A) into admittance of load (B) and rotate away from the load (i.e. in the WTG sense) to $g = 1$ intersection with the SWR circle (C).

$y_c = (1.0, 1.28)$

The distance to the stub ($B \to C$) is

$\boxed{d_1 = 0.211\ \lambda}$

The length of the stub ($y_{short} \to E$) is

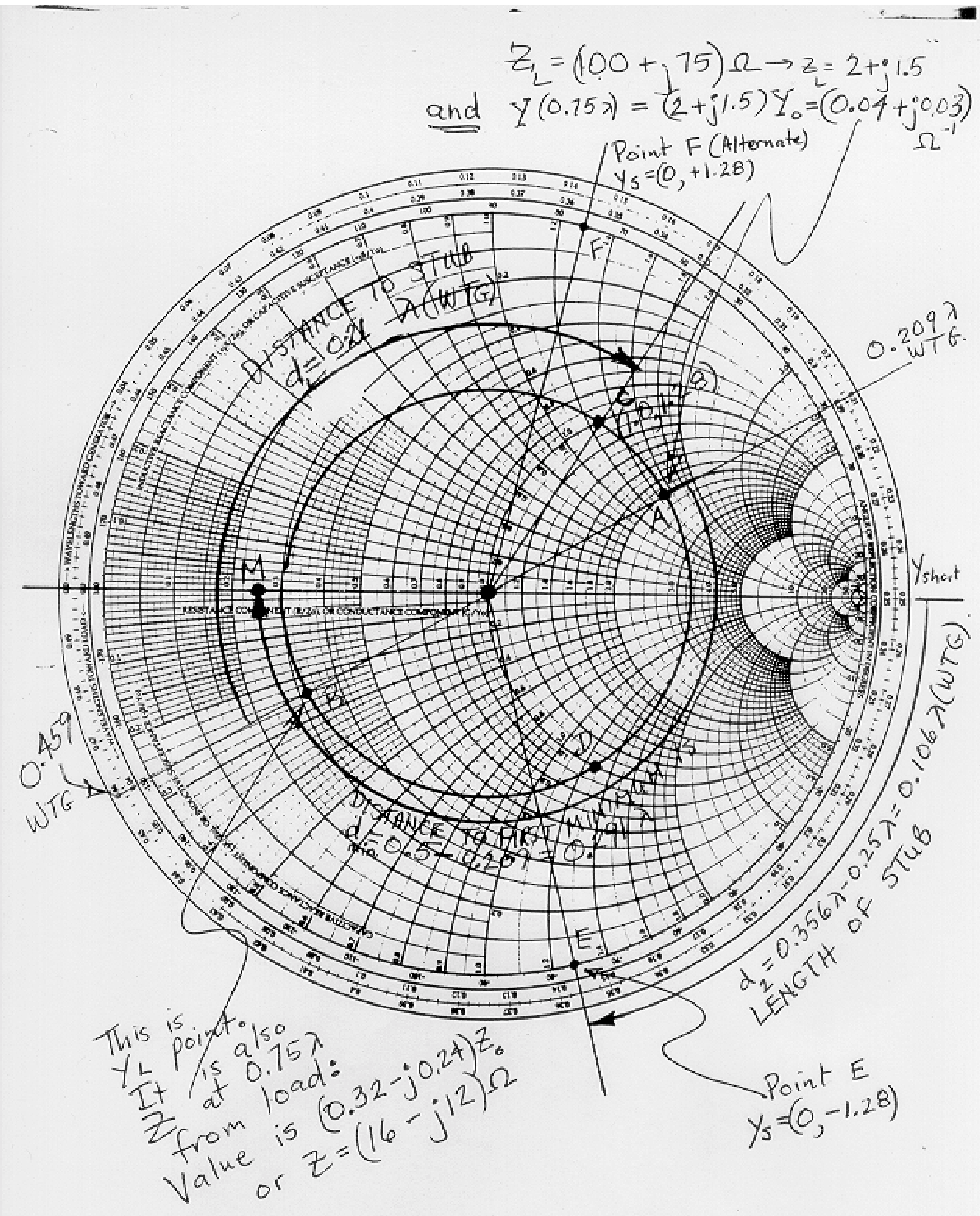
$\boxed{d_2 = 0.106\ \lambda}$

Alternate solution

Using point D, the distance to the stub ($B \to D$) is $d_3 = (0.83 - 0.459)\lambda = 0.371\ \lambda$.

The alternative stub length which must be attached at d_3 is found from the rotation $y_{short} \to F$, which yields

$d_4 = (0.645 - 0.250)\lambda = 0.395\ \lambda$


$Z_L = (100 + j75)\,\Omega \rightarrow z_L = 2 + j1.5$
and $Y(0.75\lambda) = (2 + j1.5)Y_o = (0.04 + j0.03)\,\Omega^{-1}$
Point F (Alternate)
$y_s = (0, +1.28)$
F
DISTANCE TO STUB
$d = 0.21\lambda$ (WTG)
0.209λ WTG
C
A
M
B
D
y_{short}
0.459 WTG
DISTANCE TO FIRST MINIMUM
$d_{min} = 0.5 - 0.209 = 0.291$
E
$d_2 = 0.356\lambda - 0.25\lambda = 0.106\lambda$ (WTG)
LENGTH OF STUB
Point E
$y_s = (0, -1.28)$
This is point y_L
It is also at 0.75λ from load:
Value is $(0.32 - j0.24)Z_o$
or $Z = (16 - j12)\,\Omega$