A Practical Method of Designing RC Active Filters*

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Introduction

IN THE FREQUENCY range below about 30 cps, the dissipation factors of available inductors are generally too large to permit the practical design of inductance-capacitance (LC) or resistance-inductance-capacitance (RLC) filter networks. The circuits described in the following pages were developed and collected to provide an alternative method of realizing sharp cut-off filters at very low frequencies. In many cases the active elements can be simple cathode-follower circuits that have stable gain, low output impedance and a large dynamic range.

General Method of Achieving Arbitrary Transfer Characteristics

A passive two-terminal pair network consisting of resistive and capacitive elements has an open-circuit transfer ratio of the form

$$G(s) = \frac{N(s)}{D(s)} = \frac{a_m s^m + a_{m-1}s^{m-1} + \cdots + a_1 s + a_0}{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}, \quad (1)$$

where s is the complex frequency variable $(\sigma + j\omega)$, the a_i and b_i are real positive constants, and the b_i are nonzero. All the poles of $G(s)$ lie on the negative real axis of the s-plane, a property that severely limits the application of passive RC circuits to sharp cut-off filters.

The unbalanced $(n+1)$-terminal pair RC network shown in Fig. 1 can be characterized by the relation

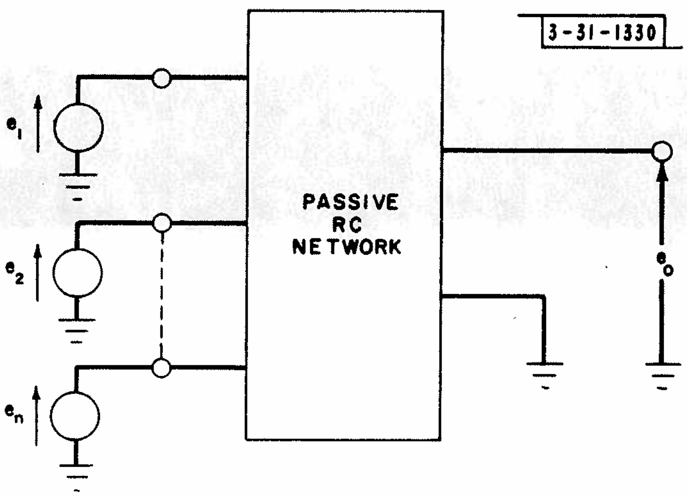

Fig. 1—The $(n+1)$-terminal pair network.

$$e_0(s) = \frac{e_1 N_1(s) + e_2 N_2(s) + \cdots + e_n N_n(s)}{D(s)}, \quad (2)$$

where the individual transfer ratios, $[N_i(s)]/[D(s)]$, have the same properties ascribed to (1).

If active elements are added to the multiterminal network in the manner shown in Fig. 2, the over-all transfer ratio, $[e_0(s)]/[e_1(s)]$, is given by

$$G(s) = \frac{K_{11}N_1(s) + K_{12}N_2(s) + \cdots + K_{1n}N_n(s)}{D(s) - [K_{01}N_1(s) + K_{02}N_2(s) + \cdots + K_{0n}N_n(s)]}. \quad (3)$$

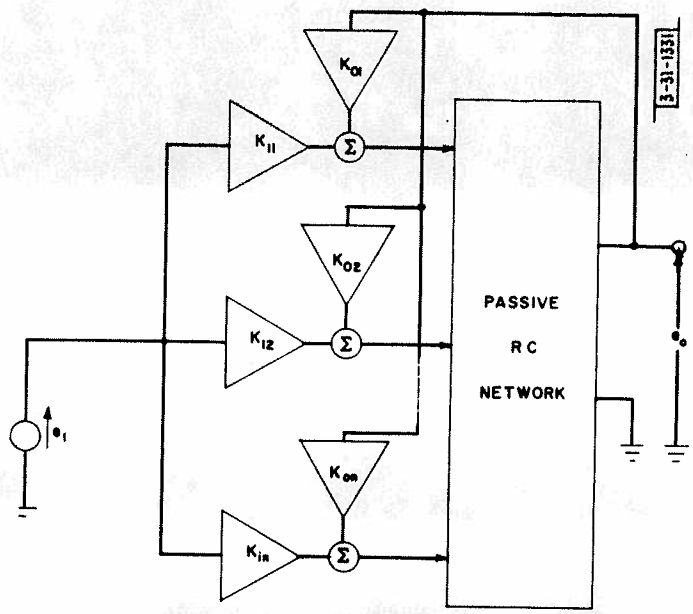

Fig. 2—Multi-terminal active network.

Generally speaking, it is possible to select an appropriate network and a series of constants K_{1i}, K_{0i} so that the poles and zeros of $G(s)$ can be placed anywhere in the complex plane. (Complex critical frequencies will occur, of course, in conjugate pairs.) Under certain conditions, all transfer functions of a given degree can be achieved with one fixed network by selection of appropriate K's.

Actually, a maximum of four active elements are required for the circuit of Fig. 2; two amplifiers for each of the two sets of K's, one with positive gain, one with negative gain. The remaining values of K_{1i} and K_{0i} can be obtained by means of passive attenuators.

Any transfer voltage ratio ordinarily realizable by means of passive RLC networks can be achieved with the circuit of Fig. 2. In addition, a variety of oscillators can also be characterized by the transfer function $G(s)$ in (3).

* This paper is an abridged version of Tech. Rep. No. 50, published by Lincoln Lab., M.I.T., Lexington, Mass., on May 6, 1954, under same title. Reference may be made to that report for useful design charts of Butterworth and Tchebycheff filters. The research in this document was supported jointly by the Army, Navy, and the Air Force under contract with M.I.T.

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In most cases it is desirable to limit the application of the general circuit of Fig. 2 to transfer ratios with only two conjugate poles. Any given transfer ratio can be achieved by a cascade of simpler circuits of this kind and one or more passive RC networks.

The second-order transfer function,

$$G(s) = \frac{a_2s^2 + a_1s + a_0}{b_2s^2 + b_1s + b_0}, \tag{4}$$

can be realized by means of the circuits of Fig. 3, which are special cases of that of Fig. 2. The arrangement of Fig. 3(b) includes two active elements that may be separate amplifiers or one amplifier with two input points. The RC passive networks generally have two capacitors and two resistors each, and a circuit-design procedure is available that affords one considerable control over the orders of magnitude of the components.

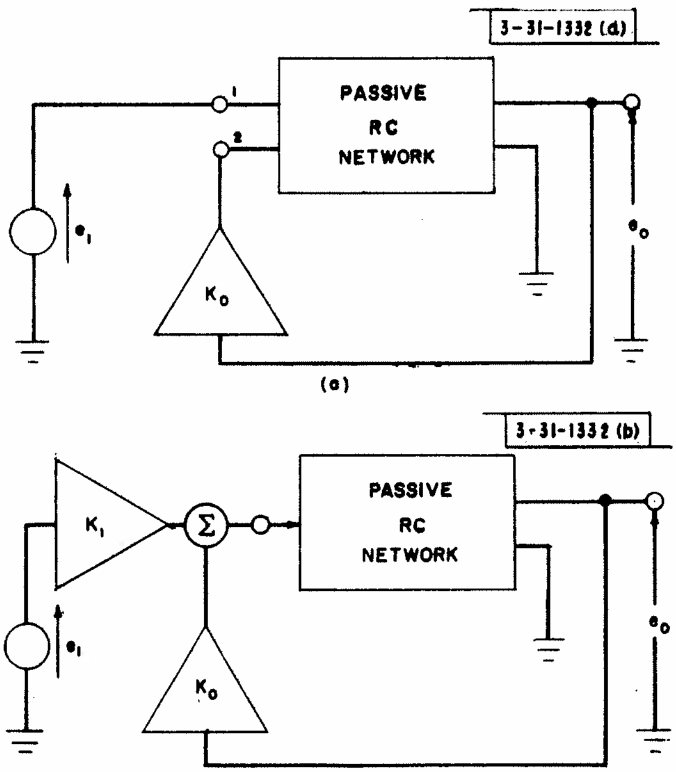


Fig. 3—Second-order active networks.

The design of circuits of the form illustrated in Fig. 3 can be facilitated by means of the catalog of possible circuit arrangements that has been compiled and is given in a later section. There are probably other useful circuits of this form that could be added to the catalog, but for most applications the present list of eighteen networks will be found adequate.

General Approach to Network Design

The basic objective of a design procedure based on the network catalog is the control over the locations of the poles of the transfer voltage ratio (4). These poles are the zeros of the denominator polynomial which, for convenience, can be normalized in the following manner. We have

$$D_0(s) = [D(s) - K_0N(s)] = b_2s^2 + b_1s + b_0 \text{ (Figs. 3 and 4)}$$

$$= b_0\left[\left(\frac{s}{\sqrt{\frac{b_0}{b_2}}}\right)^2 + \frac{s}{\sqrt{\frac{b_0}{b_2}}}\cdot\frac{b_1}{\sqrt{b_0b_2}} + 1\right]$$

$$= b_0\left[\left(\frac{s}{\omega_0}\right)^2 + \left(\frac{s}{\omega_0}\right)d + 1\right], \tag{5}$$

where

$$\omega_0 = \sqrt{\frac{b_0}{b_2}} \quad \text{and} \quad d = \frac{b_1}{\sqrt{b_0b_2}}.$$

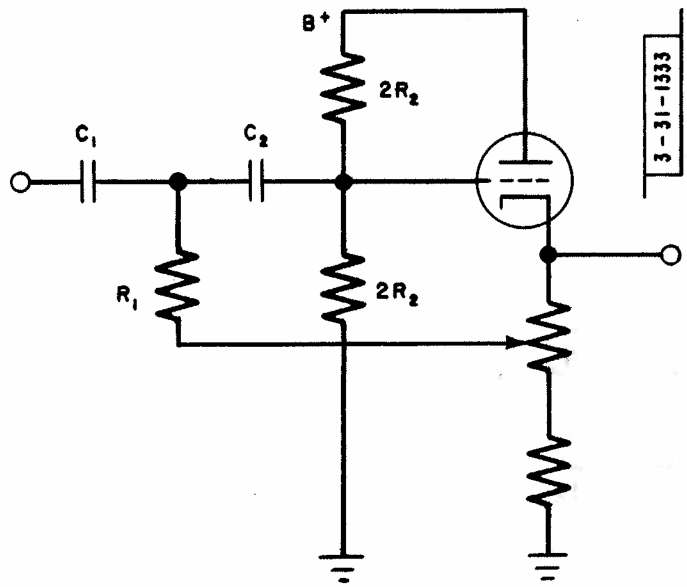


Fig. 4—High-pass filter circuit.

In the s-plane, the zeros of $D_0(s)$ lie on a circle of radius ω_0 and have a real part equal to $-d\omega_0/2$.[1] The shape of the frequency characteristics of $D_0(s)$ are dependent only on the value of the parameter d; the constant ω_0 determines their positions in the frequency domain, and b_0 determines the relative amplitude. The parameter ω_0 can be given the physical interpretations "resonant frequency," "cut-off frequency," etc., depending upon the nature of the numerator of $G(s)$.

It is convenient, in designing a circuit for a given $D_0(s)$, for one to set $\omega_0 = 1$ radian per second temporarily, and to establish the required value of d. The network response can then be shifted in frequency to ω_0 by dividing the resistive elements or the capacitive elements of the circuit by the desired value of ω_0.

In most of the networks in the catalog, there are five basic design variables: two resistances, two capacitors,

[1] This is true only for $d \leq 2$ when $d > 2$, the zeros lie on the negative real axis, a case that is not of present interest.

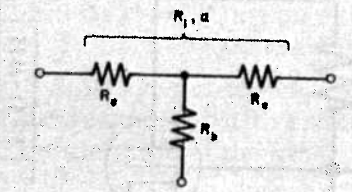
the gain K. The relationships between the variables that are independent of d are given with each network. Several additional parameters that have been found useful for designing a network for a given d include two products of a resistance and capacitance (designated T_1 and T_2), the ratio of the resistances (ρ), and the ratio of the capacitors (γ). The establishment of a specified value of d is accomplished by means of two of these parameters and the gain K. With each network in the catalog is a short table that specifies, for a given choice of parameters, the appropriate group of design relations for d given at the end of the catalog.

The form of the numerator of $G(s)$ is determined by the particular network chosen for the function. In some cases the numerator constants can easily be established at the desired values; in others an attempt to do this may severely limit the parameters affecting the value of d in the denominator and lead to an unsatisfactory circuit design. In this case, the numerator polynomial can be realized by means of additional passive or active networks. A method of network design is discussed later.

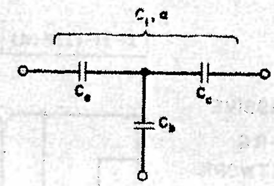
Catalog of Second-Order Active Networks

Definitions of Parameters:

$$T_1 = R_1C_1$$
$$T_2 = R_2C_2$$
$$\rho = \frac{R_1}{R_2}$$
$$\gamma = \frac{C_2}{C_1}$$

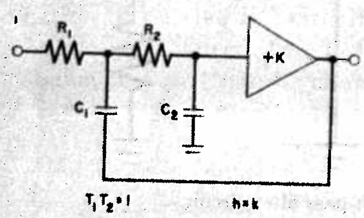


$$R_1 = R_c + \frac{R_aR_b}{R_a + R_b}$$
$$\alpha = \frac{R_a}{R_a + R_b}$$

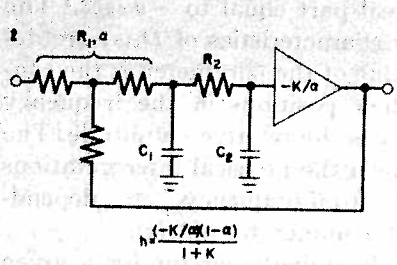


$$C_1 = \frac{C_c(C_a + C_b)}{C_a + C_b + C_c}$$
$$\alpha = \frac{C_b}{C_a + C_b}$$

FUNCTIONS OF FORM $\dfrac{h}{s^2 + ds + 1}$

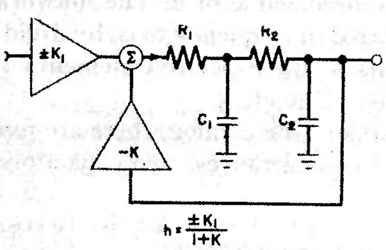


Parameters	d Formulas Group
ρ, T_1	III
ρ, T_2	I
γ, T_1	IV
γ, T_2	II



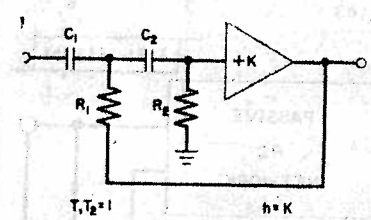
$$\frac{T_1T_2}{1+K} = 1$$

Alternative with 2 active inputs:

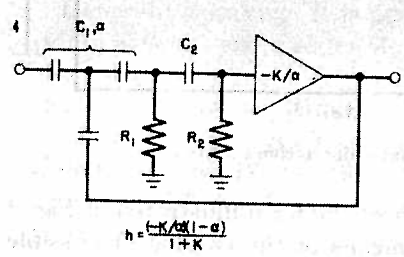


Parameters	d Formulas Group
ρ, T_1	VIII
ρ, T_2	VII
γ, T_1	VII
γ, T_2	VIII

FUNCTIONS OF FORM $\dfrac{hs^2}{s^2 + ds + 1}$

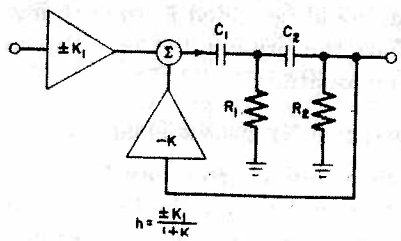


Parameters	d Formulas Group
ρ, T_1	II
ρ, T_2	IV
γ, T_1	I
γ, T_2	III



$$T_1T_2(1 + K) = 1$$

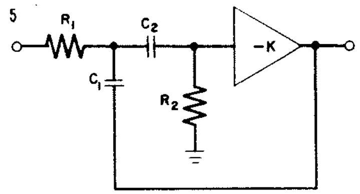
Alternative with 2 active inputs:



Parameters	d Formulas Group
ρ, T_1	X
ρ, T_2	IX
γ, T_1	IX
γ, T_2	X

FUNCTIONS OF FORM $\frac{hs}{s^2 + ds + 1}$

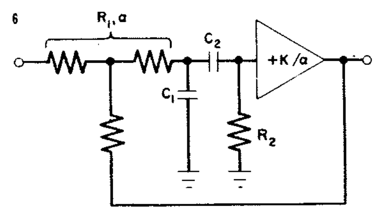
5



$T_1T_2(1+K)=1 \quad h=-KT_2$

Parameters	d Formulas Group
ρ, T_1	X
ρ, T_2	IX
γ, T_1	IX
γ, T_2	X

6

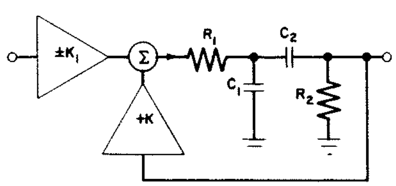


$h=(K/a)(1-a)T_2$

$T_1T_2 = 1$

Parameters	d Formulas Group
ρ, T_1	II
ρ, T_2	IV
γ, T_1	I
γ, T_2	III

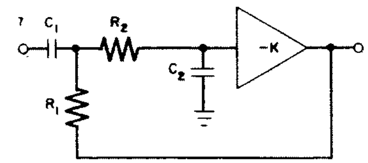
Alternative with 2 active inputs:



$h=\pm K_1T_2$

FUNCTIONS OF FORM $\frac{hs}{s^2 + ds + 1}$

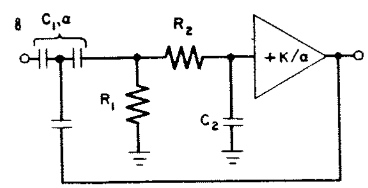
7



$\frac{T_1T_2}{1+K}=1 \quad h=-K/T_2$

Parameters	d Formulas Group
ρ, T_1	VIII
ρ, T_2	VII
γ, T_1	VII
γ, T_2	VIII

8

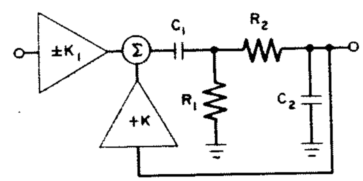


$h=(K/a)(1-a)T_1$

$T_1T_2 = 1$

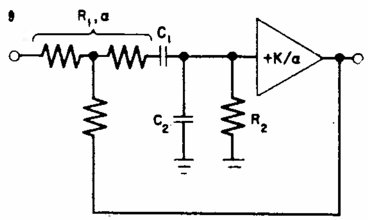
Parameters	d Formulas Group
ρ, T_1	III
ρ, T_2	I
γ, T_1	IV
γ, T_2	II

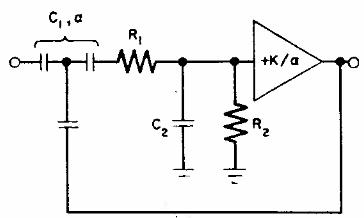
Alternative with 2 active inputs:

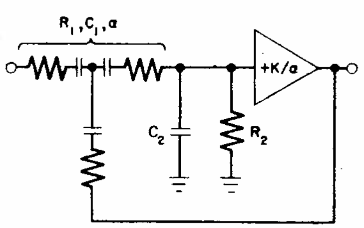


$h=\pm K_1T_1$

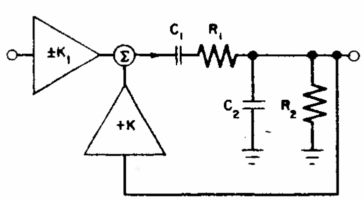
FUNCTIONS OF FORM $\frac{hs}{s^2+ds+1}$







Alternative with 2 active inputs:

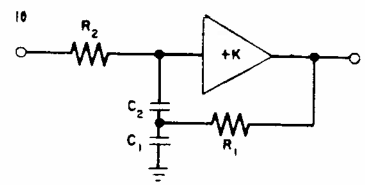


$h = \pm K_1 \rho T_2 = \pm K_1 \gamma T_1$

$$h = (K/\alpha)(1-\alpha)\rho T_2$$
$$= (K/\alpha)(1-\alpha)\gamma T_1$$
$$K > 1$$
$$T_1 T_2 = 1$$

Parameters	d Formulas Group
ρ, T_1	VI
ρ, T_2	V
γ, T_1	V
γ, T_2	VI

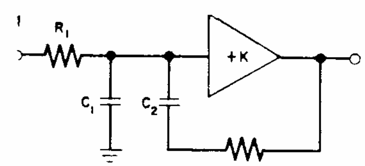
FUNCTIONS OF FORM $\frac{h[s+(1/\tau)]}{s^2+ds+1}$



$T_1 T_2 = 1$ $\quad \tau = [(1/T_2) + \rho T_2] = T_1(1+\gamma)$

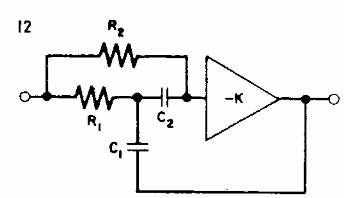
$h = K[(1/T_2) + \rho T_2] = K T_1 (1+\gamma)$

Parameters	d Formulas Group
ρ, T_1	II
ρ, T_2	IV
γ, T_1	I
γ, T_2	III



$T_1 T_2 = 1$ $\quad h = K T_2$ $\quad \tau = T_2$

Parameters	d Formulas Group
ρ, T_1	VI
ρ, T_2	V
γ, T_1	V
γ, T_2	VI

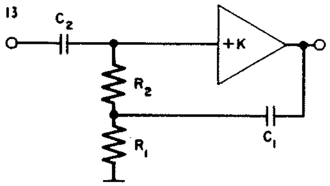


$T_1 T_2 (1+K) = 1$ $\quad h = -Kd$ $\quad \tau = d$

Parameters	d Formulas Group
ρ, T_1	X
ρ, T_2	IX
γ, T_1	IX
γ, T_2	X

FUNCTIONS OF FORM $\frac{hs[s + (1/\tau)]}{s^2 + ds + 1}$

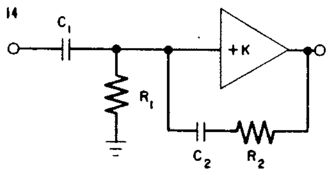
13



$T_1T_2 = 1 \quad h = K \quad \tau = \frac{1}{T_2(1+\rho)} = \frac{T_1}{1+\gamma T_1^2}$

Parameters	d Formulas Group
ρ, T_1	III
ρ, T_2	I
γ, T_1	IV
γ, T_2	II

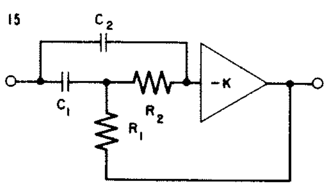
14



$T_1T_2 = 1 \quad h = K \quad \tau = T_2$

Parameters	d Formulas Group
ρ, T_1	II
ρ, T_2	IV
γ, T_1	I
γ, T_2	III

15

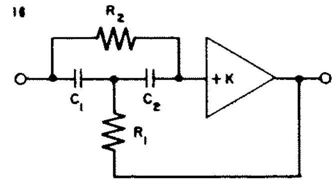


$\frac{T_1T_2}{1+K} = 1 \quad h = -K \quad \tau = 1/d$

Parameters	d Formulas Group
ρ, T_1	VIII
ρ, T_2	VII
γ, T_1	VII
γ, T_2	VIII

FUNCTIONS OF FORM $h\frac{s^2 + bs + 1}{s^2 + ds + 1}$

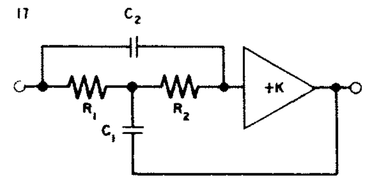
16



$T_1T_2 = 1 \quad h = k \quad b = (1/T_2) + \rho T_2 = T_1(1+\gamma)$

Parameters	d Formulas Group
ρ, T_1	II
ρ, T_2	IV
γ, T_1	I
γ, T_2	III

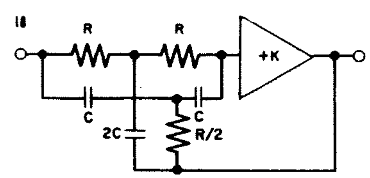
17



$T_1T_2 = 1 \quad h = K \quad b = (1/T_1) + \gamma T_1 = T_2(1+\rho)$

Parameters	d Formulas Group
ρ, T_1	III
ρ, T_2	I
γ, T_1	IV
γ, T_2	II

18



$RC = 1$

$h = K$

$b = 0$

$d = 4(1\text{-}K)$

All exponents are "2" (i.e. d^2, T^2)

DESIGN FORMULAS* FOR DISSIPATION FACTOR d

Group	(a) $d(x, K, T)$	(b) $T(x, K, d)$	(c) $K(x, d, T)$	(d) K_{min}	(e) x_{min}	(f) $T_{K_{min}}$	(g) K_{max}
I	$\frac{(1-K)}{T}+T(1+x)$	$\frac{d}{2(1+x)}\left[1\pm\sqrt{1-\frac{4(1+x)(1-K)}{d^2}}\right]$	$T^2(1+x)+1-dT$	$\frac{4(1+x)-d^2}{4(1+x)}\ (<1)$	$\frac{d^2-4(1-K)}{4(1-K)}$	$\frac{d}{2(1+x)}$	$T^2(1+x)+1$
II	$\frac{(1+x-K)}{T}+T$	$\frac{d}{2}\left[1\pm\sqrt{1-\frac{4(1+x-K)}{d^2}}\right]$	$T^2+(1+x)-dT$	$\frac{4(1+x)-d^2}{4}$	$\frac{d^2-4(1-K)}{4}$	$\frac{d}{2}$	$T^2+(1+x)$
III	$\frac{(1+x)}{T}+T(1-K)$	$\frac{d}{2(1-K)}\left[1\pm\sqrt{1-\frac{4(1+x)(1-K)}{d^2}}\right]$	$\frac{T^2+(1+x)-dT}{T^2}$	$\frac{4(1+x)-d^2}{4(1+x)}\ (<1)$	$\frac{d^2-4(1-K)}{4(1-K)}$	$\frac{2(1+x)}{d}$	$\frac{T^2+(1+x)}{T^2}$
IV	$\frac{1}{T}+T(1+x-K)$	$\frac{d}{2(1+x-K)}\left[1\pm\sqrt{1-\frac{4(1+x-K)}{d^2}}\right]$	$\frac{T^2(1+x)+1-dT}{T^2}$	$\frac{4(1+x)-d^2}{4}$	$\frac{d^2-4(1-K)}{4}$	$\frac{2}{d}$	$\frac{T^2(1+x)+1}{T^2}$
V	$\frac{1}{T}+T[1+x(1-K)]$	$\frac{d}{2[1+x(1-K)]}\left\{1\pm\sqrt{1-\frac{4[1+x(1-K)]}{d^2}}\right\}$	$\frac{T^2(1+x)+1-dT}{xT^2}$	$\frac{4(1+x)-d^2}{4x}\ (>1)$	$\frac{4-d^2}{4(K-1)}$	$\frac{2}{d}$	$\frac{T^2(1+x)+1}{xT^2}$
VI	$\frac{[1+x(1-K)]}{T}+T$	$\frac{d}{2}\left[1\pm\sqrt{1-\frac{4(1+x)}{d^2(1+K)}}\right]$	$\frac{T^2+(1+x)-dT}{x}$	$\frac{4(1+x)-d^2}{4x}\ (>1)$	$\frac{4-d^2}{4(K-1)}$	$\frac{d}{2}$	$\frac{T^2+(1+x)}{x}$
VII	$\frac{1}{T}+\frac{T(1+x)}{(1+K)}$	$\frac{d(1+K)}{2(1+x)}\left[1\pm\sqrt{1-\frac{4(1+x)}{d^2(1+K)}}\right]$	$\frac{T^2(1+x)+1-dT}{(dT-1)}$	$\frac{4(1+x)-d^2}{d^2}$	$\frac{(1+K)d^2-4}{4}$	$\frac{2}{d}$	∞
VIII	$\frac{(1+x)}{T}+\frac{T}{(1+K)}$	$\frac{d(1+K)}{2}\left[1\pm\sqrt{1-\frac{4(1+x)}{d^2(1+K)}}\right]$	$\frac{T^2+(1+x)-dT}{dT-(1+x)}$	$\frac{4(1+x)-d^2}{d^2}$	$\frac{(1+K)d^2-4}{4}$	$\frac{2(1+x)}{d}$	∞
IX	$\frac{1}{T(1+K)}+T(1+x)$	$\frac{d}{2(1+x)}\left[1\pm\sqrt{1-\frac{4(1+x)}{d^2(1+K)}}\right]$	$\frac{T^2(1+x)+1-dT}{dT-T^2(1+x)}$	$\frac{4(1+x)-d^2}{d^2}$	$\frac{(1+K)d^2-4}{4}$	$\frac{d}{2(1+x)}$	∞
X	$\frac{(1+x)}{T(1+K)}+T$	$\frac{d}{2}\left[1\pm\sqrt{1-\frac{4(1+x)}{d^2(1+K)}}\right]$	$\frac{T^2+(1+x)-dT}{dT-T^2}$	$\frac{4(1+x)-d^2}{d^2}$	$\frac{(1+K)d^2-4}{4}$	$\frac{d}{2}$	∞

* $T=T_1$ or T_2, as appropriate; $x=\rho$ or γ, as appropriate.

Design of Active Networks by Means of the Catalog

While anyone may start with a given transfer function or with the networks in the catalog and work out his own method for selecting parameters, one approach has been found useful and is described here for those wishing to design such circuits most directly. In any event, it is strongly recommended that one work through at least one group of relations in the foregoing section to gain insight into their meaning.

The basis of the general procedure suggested below is the necessity that the practical design of an active filter must be carried out within the limitations imposed by the available components. Restrictions on the size of capacitors, number and complexity of amplifier stages, and requirements for variability are typical factors that impose practical circuit limitations and must be controlled.

Realizing a Specified Value of d

When a particular network has been selected, one chooses a set of two parameters—(ρ, T_1), (ρ, T_2), (γ, T_1) or (γ, T_2)[2]—and locates the appropriate group of design formulas in the foregoing section. If the problem involves restrictions on the size of capacitors, then γ is a useful parameter. On the other hand, if control of the resistance values is more important, one may use ρ. In general, T_1 and T_2 are equally convenient parameters except where one of them determines a factor in the numerator of $G(s)$.

In the same section, each formula group includes:

(a) The expression for d in terms of K and the two parameters (x, T), selected above (x stands for ρ or γ, T for T_1 or T_2);

(b) The solution of the equation in (a) for T;

(c) The solution of the equation in (a) for K;

(d) The minimum value of K satisfying the equation in (a) with arbitrary x, positive T [K_{min} is obtained by solving the equation $\partial K(d, x, T)/\partial T = 0$ for T and substituting the solution, $T_{K_{min}}$ into the expression for $K(d, x, T)$];

(e) The minimum value of x satisfying the equation in (a) with arbitrary K, positive T (x has a minimum in the same sense as K, above);

(f) The value of expression (b) when $K = K_{min}$ ($x = x_{min}$ is the same condition and both values of T are the same in this case);

(g) The value of expression (c) when $d = 0$ (the significance of K_{max} is discussed below).

In establishing the values of x, T and K, one selects any two of them arbitrarily (in an algebraic sense—with more purpose in the practical sense), subject to the algebraic limitations $K \geqq K_{min}$, $x \geqq x_{min}$. The value of the remaining parameter is then determined from formula (b) or (c). For most purposes, a recommended procedure is the assumption of x, K, and the solution for T.

As an example, suppose that a required

$$G(s) = \frac{s^2}{s^2 + 1.414s + 1}$$

is to be realized by means of network No. 3 in the catalog, using one cathode follower as the active element. (The circuit to be used is shown in Fig. 4.) Suppose further that R_2 (parallel combination of the two biasing resistors) shall be 1 megohm and that both capacitors shall have the same value.

First of all, we shall choose (γ, T_2) as our design parameters, so that we may easily control the ratio of the capacitors and the value of R_2. According to the catalog, Formula Group III for d is indicated.

Setting $\gamma = 1$, we have

$$\begin{aligned} K_{min} &= \frac{4(1+\gamma) - d^2}{4(1+\gamma)} \\ &= \frac{4\cdot 2 - (1.414)^2}{4\cdot 2} \\ &= 0.75. \end{aligned}$$

If we set $K = 0.9$, a reasonable value for the amplifier of Fig. 4, then

$$\begin{aligned} T_2 &= \frac{d}{2(1-K)}\left[1 \pm \sqrt{1 - \frac{4(1+x)(1-K)}{d^2}}\right] \\ &- \frac{1.414}{2(1-0.9)}\left[1 \pm \sqrt{1 = \frac{4\cdot 2.(1-0.9)}{(1.414)^2}}\right] \\ &= 1.59, 12.5. \end{aligned}$$

The expression for d in Formula Group III, $d = (1+\gamma)/(T_2) + T_2(1-K)$, suggests that a choice of the smaller value of T_2 above would result in a more stable circuit, in that variations in the active element will have less effect on the value of d.

Up to this point, we have $R_2 = 10^6$, $T_2 = 1.59$, and $C_1 = C_2$. Then, making use of the relation $T_1 T_2 = 1$, we find $C_1 = C_2 = 1.59\ \mu f$ and $R_1 = 3.93 \times 10^5$.

It can be stated as a general rule of thumb that values of d greater than 0.5 can be realized most easily and with the simplest circuits; as d approaches 0.2, more care becomes necessary in the circuit design. Finally, values of d of the order of 0.1 or less demand active elements that are more complicated and highly stabilized, and passive elements that have been carefully adjusted within close tolerances. The latter values of d are not generally encountered in low-frequency filters.

As stated it has been found most convenient to design second-order networks on the basis of 1 radian per second, and to make a subsequent shift of their characteristics to the appropriate frequency by altering the passive elements. The basic invariants under a frequency transformation of this kind are the parameters

[2] One might also employ (ρ, γ) as design parameters, but this pair appears to be less useful than the others since it provides less control over the actual magnitudes of the components.

ρ and γ; as long as the ratio of resistors and the ratio of capacitors remain constant, the frequency characteristics of the networks will have the same shape. The necessary invariance of ρ and γ indicates the technique for making filters with variable cut-off frequencies.

Imperfections in the Active Elements

It has been assumed, heretofore, that the active elements of the networks in Figs. 2 and 3 and in the catalog possessed the ideal attributes: infinite input impedance, zero output impedance and stable gain. It is therefore important, in the design of an active network of this kind, for one to insure that the imperfections in the amplifiers used do not appreciably deteriorate the desired performance of the circuit.

With regard to finite output impedance, it can be seen that in many cases the active elements drive a portion of the passive network through a resistive element. In this case, the design can be made to incorporate the output impedance in the resistive element and effectively neutralize its effects.

On the other hand, where an amplifier drives a capacitive branch of a network, it is imperative that the output impedance be considerably smaller than any of the resistive elements of the network. This condition is most serious when the value of d is very small and the gain of the active element is close to $+1$. The limitation of the amplifier output impedance to a reasonably small value will generally prevent any significant alteration of the network characteristics in the vicinity of the cut-off frequency(ies). On the other hand, the attenuation achieved in certain networks in regions well beyond cut-off will fall short of the expected value because the output impedance, though small, is still finite. This situation has been observed in low-pass networks at high frequencies and in "notch" circuits at the null frequency. Behavior of this kind can best be investigated by a direct analysis of the particular circuit involved. Since only "very high frequencies" or "null frequencies" are of interest in this case, the analysis can be simplified by the assumption of these extreme frequency conditions. Under these circumstances, the output impedance should be negliglibly small in comparison with $(1-K)$ times the value of resistive elements of the networks. Fortunately, strict requirements of this sort do not occur often in low-frequency filters.

Another interesting departure of the active elements from the ideal is the drift in their gain. With most of the networks in the catalog (those with $T_1T_2=1$), the position of the transfer characteristics in the frequency domain is independent of the active element; with a few others this is not so. In both cases, however, a drift in gain will result in a change in the actual value of d and in the shape of the frequency characteristics. It is often possible, as in the previous example, to reduce this dependence by an appropriate choice of parameters; but in any event the active gain should generally be at least $1/d$ times as stable as the expected value of d.

There is another kind of instability often characteristic of active RC networks of the kind discussed here, namely, their tendency to become oscillators. This tendency is most prevalent when the value of d is small. Even in some circuits where the active gain is ostensibly free from drift, oscillations may be sustained by an amplifier that drives itself into a region of its characteristics where the gain is far greater than expected.

A basic cure for a situation of this kind is the use of a feedback amplifier for the active element, such that the gain K is given by

$$K = \frac{A}{1+\beta A} \tag{6}$$

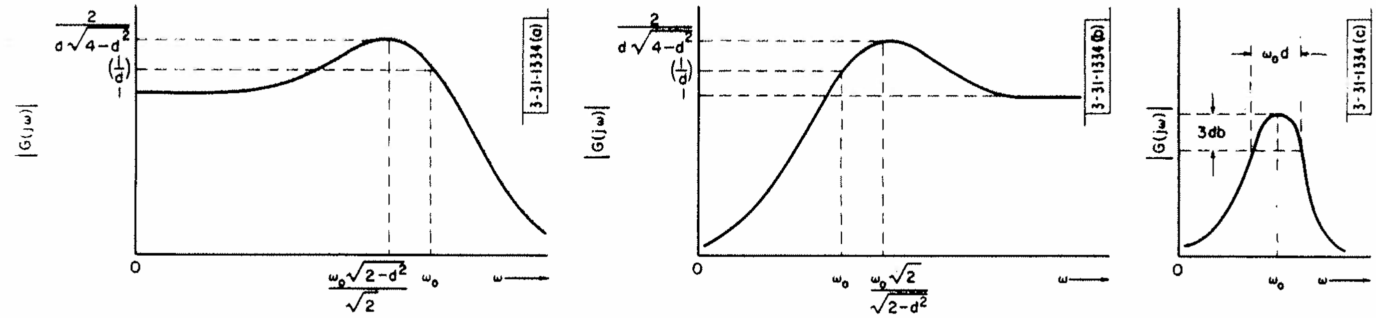
where A is the gain of the amplifier without feedback, β is the feedback ratio derived from passive elements. It is easily seen that the value of K is absolutely limited to $1/\beta$, regardless of the value of A. The simple cathode-follower circuit illustrated in Fig. 4 is an example of this kind of active element. The critical value of K for a given network, K_{max}, is given at the end of the network catalog. A practical circuit design must include means for insuring that the active gain does not approach this value.

Adjustment of Physical Networks

When an active network has been constructed with physical components, minor adjustments in the latter are frequently required to achieve the performance indicated by the design. If the departure from the expected characteristics is not large, the trimming of a single capacitor or resistor may suffice to properly position the network characteristics in the frequency domain. The shapes of the characteristics are most easily altered by adjustment of the gain K.

In the event that the departure from expected characteristics is large and is not accountable to the usual tolerances in components, one may look to the following as possible sources of error: miscalculation of design parameters, excessive amplifier output impedance, poor capacitor "Q". It is unreasonable to ignore the "Q" of large paper capacitors in networks where the resistive elements are of the order of 1 megohm or more.

When the network design includes an active element whose gain is slightly less than $+1$ (e.g., the circuit of Fig. 4), it is usually difficult to measure or adjust the quantity $(1-K)$ directly with necessary accuracy. If a potentiometer is available for trimming the gain (as in Fig. 4), one may effect the adjustment in a simple manner by observing the over-all network amplitude-frequency response. The expected frequency-response characteristic for three transfer functions is illustrated in Fig. 5. Note that, although the latter are written on the basis of 1 radian per second, the frequency response is indicated at the true "resonant" frequency ω_0.



(a) $G(s) = \dfrac{1}{s^2 + ds + 1}$ (b) $G(s) = \dfrac{s^2}{s^2 + ds + 1}$ (c) $G(s) = \dfrac{s}{s^2 + ds + 1}$

Fig. 5—Amplitude-frequency response of several active networks.

It is sometimes desirable to avoid the necessity for adjusting the elements of a network, particularly the passive ones, by their prior selection within specified tolerances. Although the subject of tolerances has not been studied in detail, the expected variability of the network characteristics will generally be of the same order of magnitude as the tolerances in the passive elements, when the active elements are properly adjusted.

Factoring a High-Order $G(s)$ into Second-Order Transfer Ratios

A third- or higher-order transfer ratio can be written in the form

$$G(s) = \frac{N(s)}{D(s)} = \frac{N_1(s)}{D_1(s)} \times \frac{N_2(s)}{D_2(s)} \times \cdots \times \frac{N_n(s)}{D_n(s)}, \quad (7)$$

in a variety of ways such that the transfer ratios, $G_i(s) = [N_i(s)]/[D_i(s)]$, contain first- or second-order polynomials with real coefficients in the numerator and denominator.

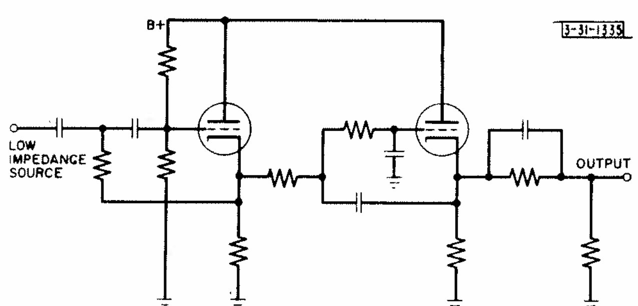
All of the first-order denominators can be achieved with one passive RC network or by means of a cascade of isolated RC networks which, incidentally, can be made to absorb some of the factors of $N(s)$. The remaining second-order denominator polynomials are each identified with an individual active RC network of a form shown in the catalog. Practical considerations determine the pairing of the appropriate numerator factors with the second-order denominators; the relative properties of the various networks are the key to the pairing process.

No general rules can be given for selecting a group of second-order networks, for the choice is dependent on the particular requirements of the over-all circuit. However, an example can be used to indicate the kind of reasoning one may employ in a given situation. Consider the transfer ratio

$$G(s) = \frac{s^2(s+1)}{(s^2 + 0.5s + 1)(s^2 + s + 1)(s + 2)},$$

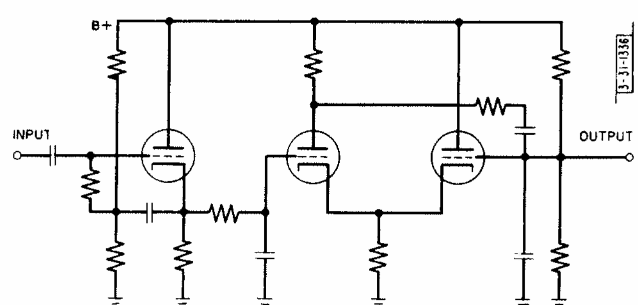
and assume we are interested in realizing $G(s)$ with the simplest and most easily designed circuits possible.

First of all, one passive network is indicated: either $1/(s+2)$, $s/(s+2)$ or $(s+1)/(s+2)$. The possible second-order active networks will then have functions of the form



$$\frac{S^2}{S^2 + 0.5s + 1} \; \frac{1}{S^2 + S + 1} \; \frac{S+1}{S+2}$$

Fig. 6—Circuit with transfer function.



$$\frac{S(S+1)}{S^2 + 0.5s + 1} \; \frac{1}{S+2} \; \frac{S}{S^2 + S + 1}$$

Fig. 7—One alternative to circuit of Fig. 6.

$$\frac{hs^2(s+1)}{(S^2+0.5s+1)(S^2+s+1)(S+2)}$$

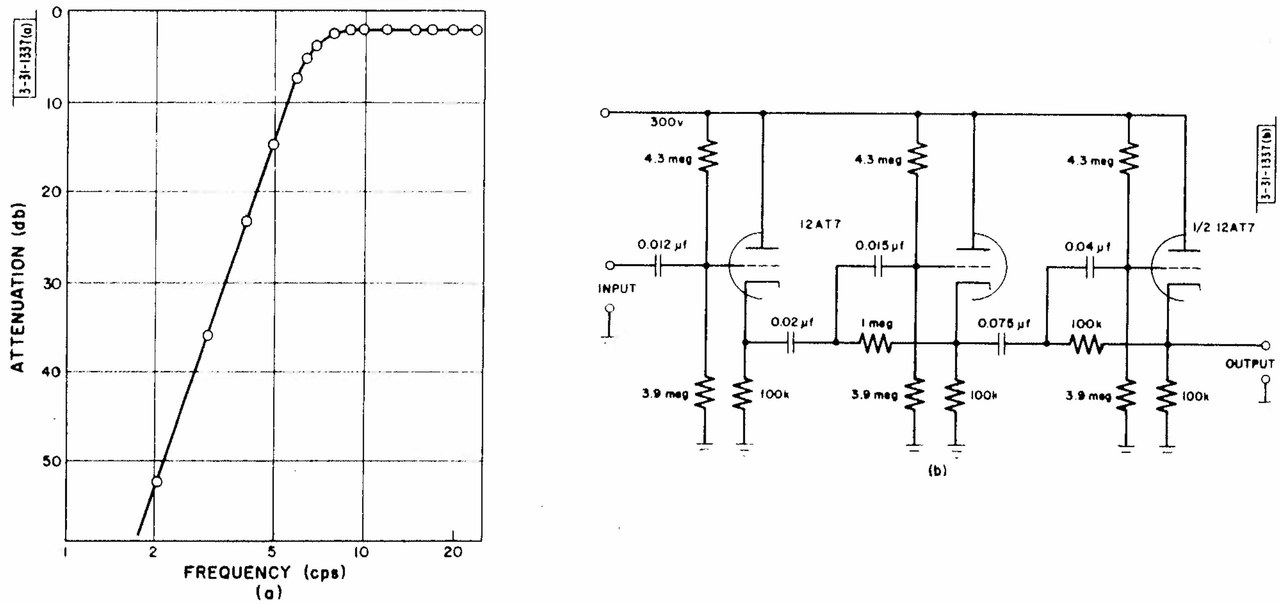


Fig. 8—Active high-pass filter.

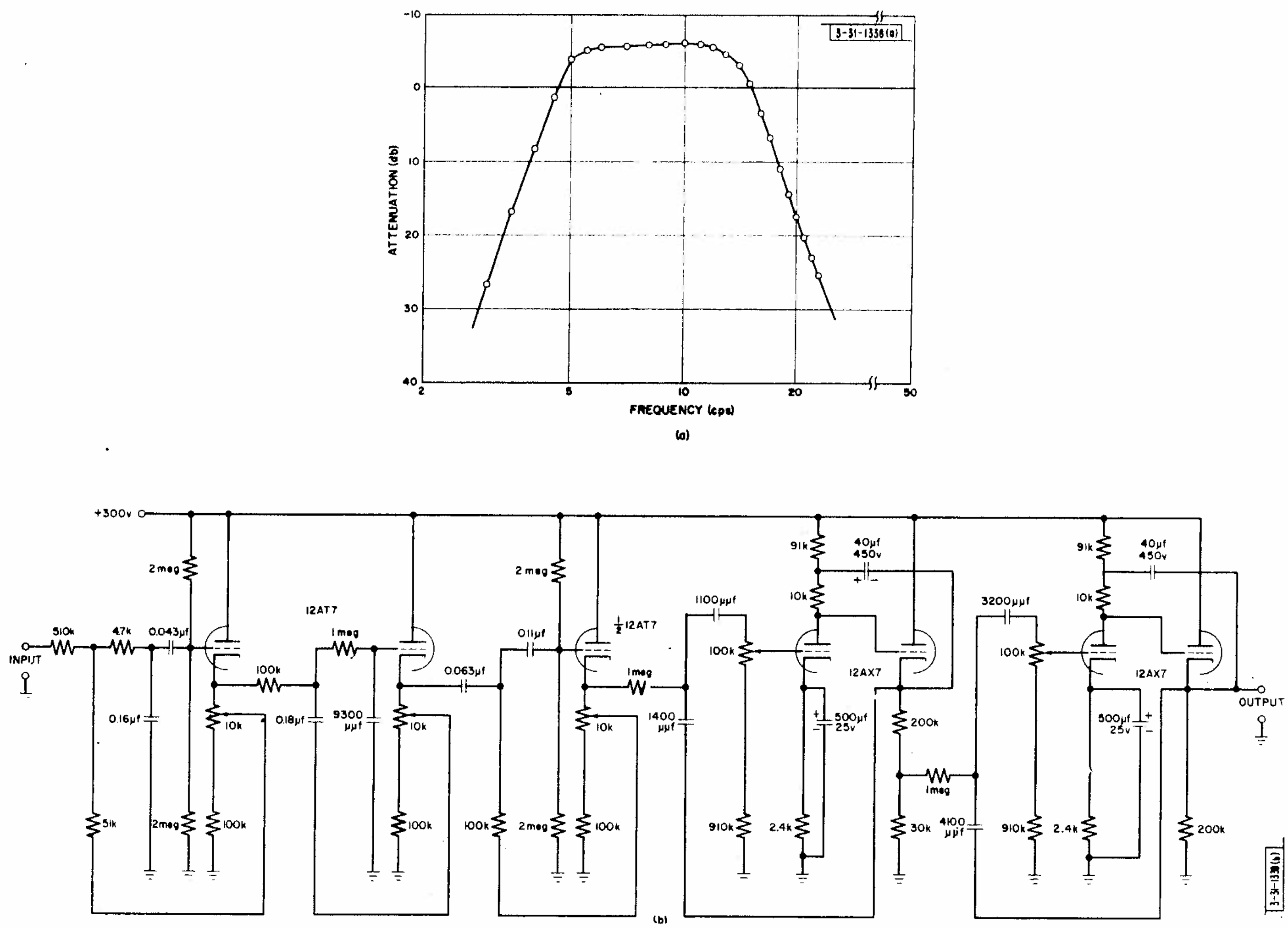


Fig. 9—Active bandpass filter.

$$\frac{1}{(s^2 + ds + 1)}, \quad \frac{s}{(s^2 + ds + 1)}, \quad \frac{s^2}{(s^2 + ds + 1)},$$

$$\frac{s + 1}{(s^2 + ds + 1)} \quad \text{or} \quad \frac{s(s + 1)}{(s^2 + ds + 1)}.$$

Because the last two functions contain zeros at $s = -1$, the establishment of which will tend to inhibit the freedom usually desired in the design for d in the denominator, we may justifiably simplify the problem by incorporating the zero at -1 in the passive network.

Reference to the catalog further indicates that networks for numerators 1 and s^2 are simple and can be achieved with positive gains less than 1. Finally, since the smaller values of d require amplifiers that more closely approximate the ideal active element, we shall associate the s^2 numerator (network No. 3) with the denominator whose $d = 0.5$, so that R_1 can absorb the output impedance of the amplifier. A reasonable set of factors of $G(s)$ is then

$$G(s) = \frac{s^2}{(s^2 + 0.5s + 1)} \times \frac{1}{(s^2 + s + 1)} \times \frac{s + 1}{s + 2}.$$

A possible circuit is shown in Fig. 6 (page 83), to which an alternative is given in Fig. 7 (page 83) where the network is based on a different set of factors of $G(s)$.

In general, there are a great number of possible circuits realizing a given $G(s)$ that can be justified on the basis of particular requirements for simplicity, output impedance, stability and so on.

Several additional points should be mentioned in regard to the cascading of second-order networks. First, a series of networks involving cathode-follower amplifiers as active elements, or a series of cathode-coupled amplifier networks, has a useful property whereby the bias given to the first amplifier can usually be carried over from stage to stage. This is important where resistive elements couple adjacent networks, since it avoids the need for coupling capacitors which, at low frequencies, might become fairly large.

In designing a circuit, it is always desirable for one to employ only capacitors whose impedance-frequency characteristics help to determine the transfer ratio; i.e., capacitors that are part of the basic networks in the catalog. Other capacitors, which might be used solely for bypassing, coupling, etc., will usually have impedances negligible at the frequencies of interest and will be appreciably larger than those in the former category. In this regard, it is often advisable, when one is designing an active RC filter to be followed by amplifier circuits, to incorporate single zeros and poles of the $G(s)$ into the amplifier interstage coupling networks, or even to add additional ones that can be cancelled by the filter network.

Finally, where possible, amplifiers in a chain of active networks should be placed so that those with the smallest dynamic range appear last.

Two examples of low-frequency filters that have been constructed and tested in the laboratory are shown in Figs. 8 and 9, on the previous page.