University of Michigan
EECS 311: Electronic Circuits
Fall 2008

Quiz 2

11/3/2008

NAME: Solutions

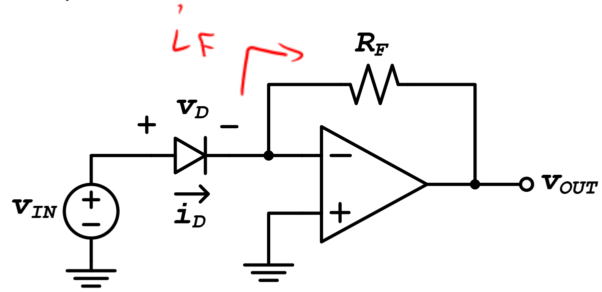
Honor Code:
I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code.

Signature ____

Problem	**Points**	**Score**	**Initials**
1	**26**		
2	**20**		
3	**30**		
4	**24**		
	Total		

Problem 1 (26 Points): Potpourri – this problem has four unrelated parts.

a) Find the expression for gain $A_v = v_{OUT}/v_{IN}$ of the circuit below. Use the exact model for the diode, $i_D = I_S(e^{v_D/V_T} - 1)$. Assume the opamp is ideal.



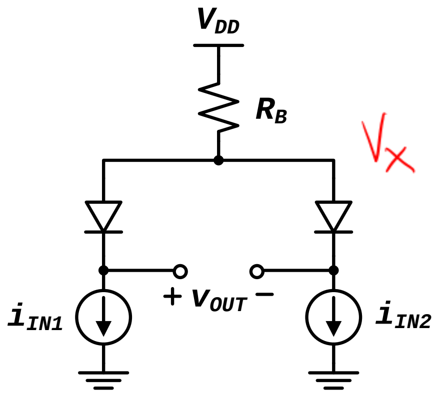
Cannot solve for $\frac{V_{OUT}}{V_{IN}}$!

$$i_F = i_D = I_S\left(e^{V_{IN}/V_T} - 1\right)$$

$$\boxed{V_{OUT} = -R_F I_S\left(e^{V_{IN}/V_T} - 1\right)}$$

Accepted all answers showing the above relationship between V_{OUT} and V_{IN}

b) Find an expression for the voltage v_{OUT} for the circuit below assuming $i_{IN1} > 0$ and $i_{IN2} > 0$. Use the exact model for the diodes, $i_D = I_S\left(e^{v_D/V_T} - 1\right)$, assuming the two diodes are identical (same values for I_S and same temperature).



$$V_X = V_{DD} - R_B\left(i_{IN1} + i_{IN2}\right)$$

$$V_{OUT} = \left(V_X - V_{D1}\right) - \left(V_X - V_{D2}\right)$$

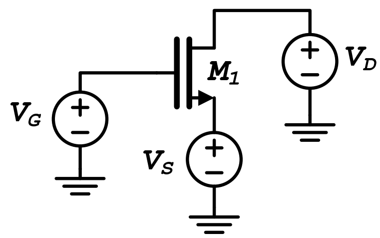
$$= V_{D2} - V_{D1}$$

$$V_{D2} = V_T \ln\left(\frac{i_{IN2}}{I_S} + 1\right)$$

$$V_{D1} = V_T \ln\left(\frac{i_{IN1}}{I_S} + 1\right)$$

$$\boxed{V_{OUT} = V_T \ln\left[\frac{i_{IN2}/I_S + 1}{i_{IN1}/I_S + 1}\right]}$$

c) Identify the region of operation of M_1 in the circuit below when $V_G = 2\,V, V_D = 3\,V$, and $V_S = -1\,V$. Assume $V_{TN} = 1\,V$ and ignore the body-effect on threshold voltage.



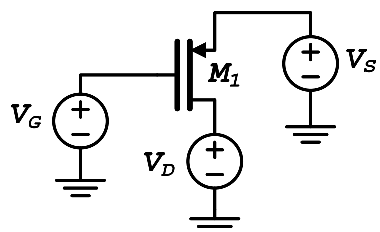
$V_{GS} = 2 - (-1) = 3V$

$V_{GS} > V_{TN} \Rightarrow$ ON

$V_{DS} = 3 - (-1) = 4V$

$V_{DS} > V_{GS} - V_{TN} \Rightarrow$ SAT

d) Identify the region of operation of M_1 in the circuit below when $V_G = 3\,V, V_D = 5\,V$, and $V_S = 5\,V$. Assume $V_{TP} = -1\,V$ and ignore the body-effect on threshold voltage.



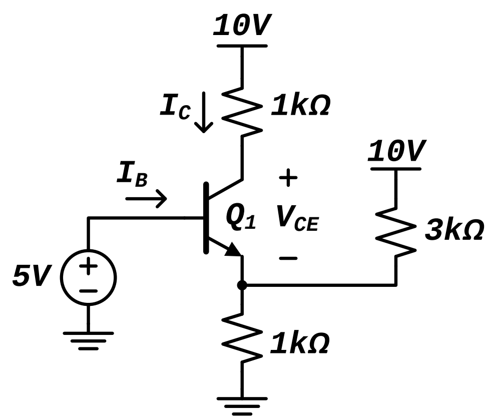
$V_{GS} = 3 - 5 = -2V$

$|V_{GS}| > |V_{TP}| \Rightarrow$ ON

$V_{DS} = 5 - 5 = 0$

$|V_{DS}| < |V_{GS}| - |V_{TP}| \Rightarrow$ LINEAR

Problem 2 (20 Points): Use the following circuit and ignore base-width modulation for all parts.

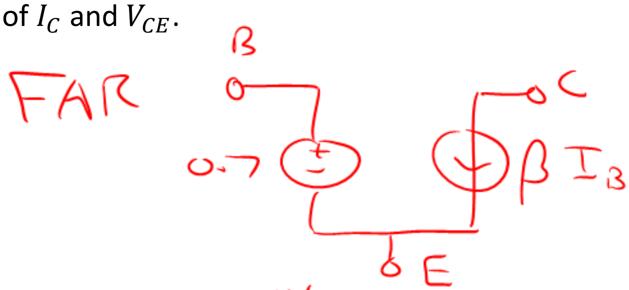


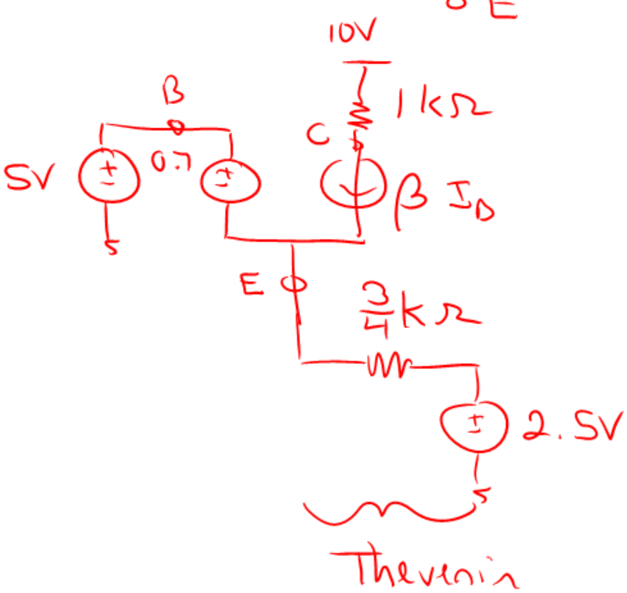
$V_{be(on)} = 0.7\ V$

$V_{ce(sat)} = 0.5\ V$

$\beta_F = 100$

a) Substitute the simplified large-signal model for the BJT in the forward-active region using the constant-voltage source model for the base-emitter junction diode. Solve for the values of I_C and V_{CE}.





$V_E = 4.3V$

$I_E = \frac{4.3 - 2.5}{4/3\ k\Omega} = 2.4mA$

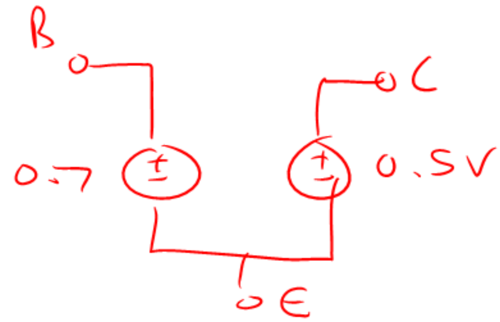
$I_C = \frac{\beta}{\beta+1} I_E = 2.38mA$

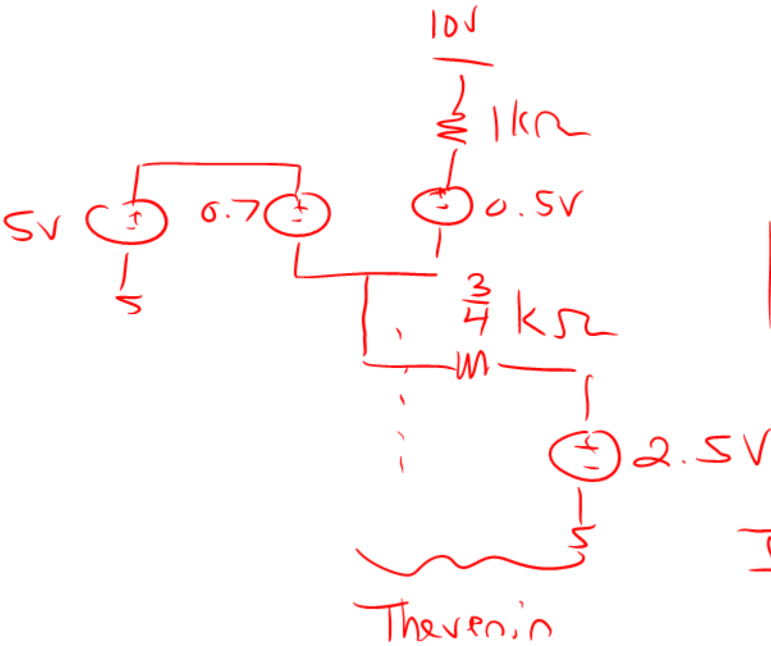
$V_C = 10 - I_C R_C = 7.62V$

$V_{CE} = V_C - V_E = 3.32V$

b) Substitute the simplified large-signal model for the BJT in the saturation region using the constant-voltage source models for the base-emitter and collector-emitter voltages. Solve for the values of I_B and I_C.

SAT





$V_E = 4.3V$

$V_C = 4.8V$

$$I_C = \frac{10 - V_C}{1k\Omega} = 5.2mA$$

$$I_E = \frac{4.3 - 2.5}{\frac{3}{4}k\Omega} = 2.4mA$$

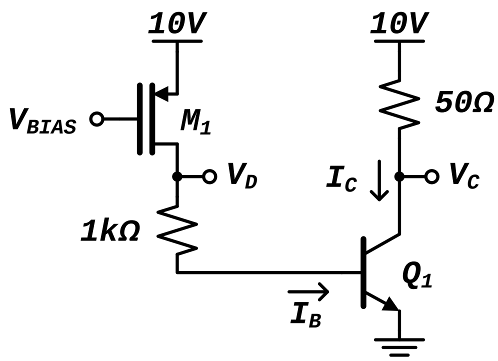
$$I_B = I_E - I_C = -2.8mA$$

c) Given your answers to parts a) and b), is Q_1 in the forward-active or saturation region.

For SAT assumption, $I_B < 0$ which cannot happen for NPN

Forward Active Region

Problem 3 (30 Points): Use the circuit shown below to answer all of the following parts. Use the constant-voltage source model for the base-emitter junction diode of Q_1. Ignore channel-length modulation, body effect, and base-width modulation for all parts.



$\mu_p C_{ox} \frac{W}{L} = 500\ \mu A/V^2$

$V_{tp} = -1.0\ V$

$V_{be(on)} = 0.7\ V$

$V_{ce(sat)} = 0.5\ V$

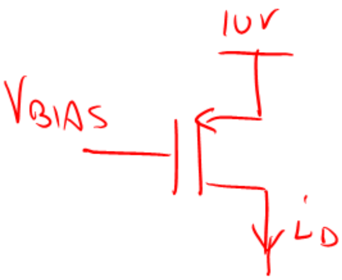
$\beta_F = 100$

a) Assuming Q_1 is kept in the forward-active region, find the numerical value of I_B required to support $I_C = 1\ mA$.

Assuming FAR $I_C = \beta I_B$

$I_B = 10 \mu A$

b) Assuming M_1 is kept in saturation, find the numerical value of V_{BIAS} required to support I_B from part a).



$$I_B = I_D = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left(V_{gs} - V_{TP}\right)^2$$

$$V_{gs} - V_{TP} = \pm\sqrt{I_D \Big/ \tfrac{1}{2}\mu_p C_{ox}\tfrac{W}{L}} = \pm 0.2V$$

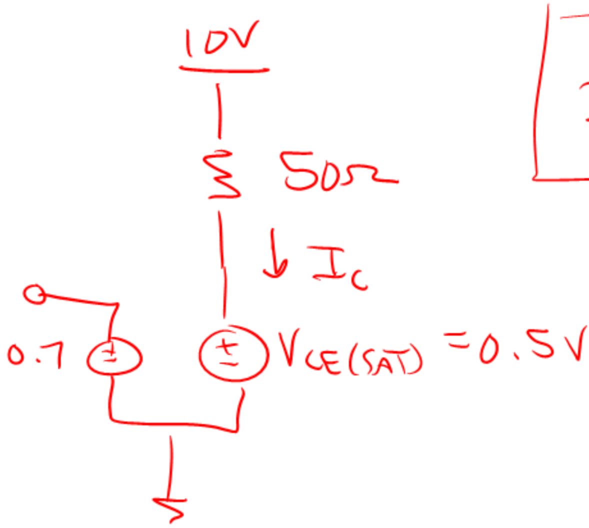
$$V_{GS} = V_{BIAS} - 10V$$

$|V_{GS}| > |V_{TP}|$ for FET ON

$$\boxed{V_{BIAS} = 8.8V}$$

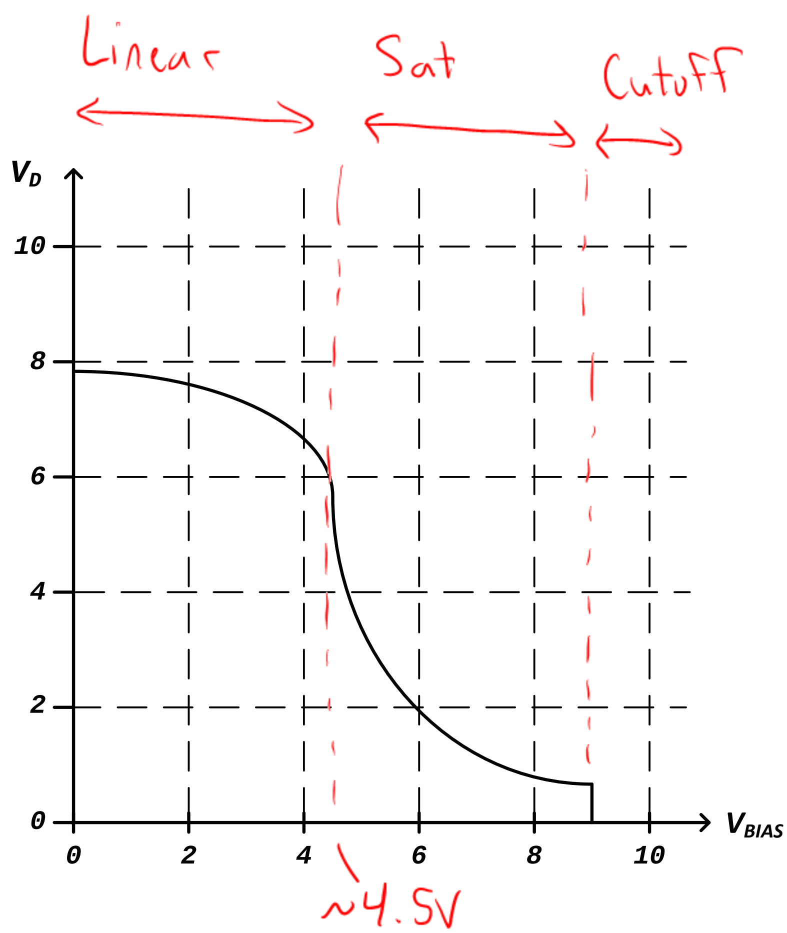
$V_{GS} = -1.2V \Rightarrow |V_{GS}| - |V_{TP}| = 0.2V$
ON

c) Find the maximum value of I_C allowed while keeping Q_1 in the forward-active region.

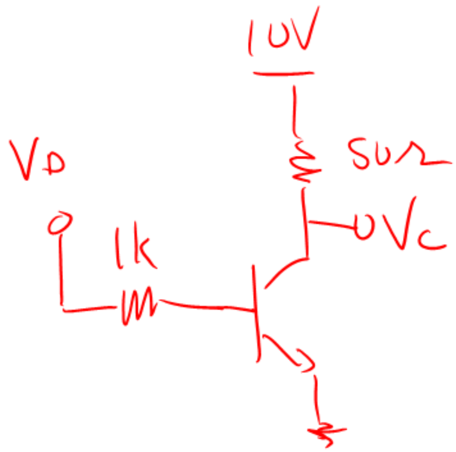


$$\boxed{I_C = \frac{10 - 0.5}{50\Omega} = 190mA}$$

d) As V_{BIAS} is swept from0 to $10\ V$, the drain voltage V_D follows the graph shown below. Label the regions on the graphs where M_1 is in *cutoff*, *linear*, and *saturation*, specifically showing the values of V_{BIAS} at the boundaries between regions.



e) Sketch the voltage at node V_C as V_{BIAS} is swept from0 to $10\ V$. Use the graph from part d), giving you V_D as V_{BIAS} is swept over the same range, to generate your sketch. Label the regions on the graph where Q_1 is in *off*, *forward-active*, and *saturation*.



FAR: $V_C = 10 - \beta \frac{V_D - 0.7}{1k\Omega}$

SAT: $V_C = 0.5V$

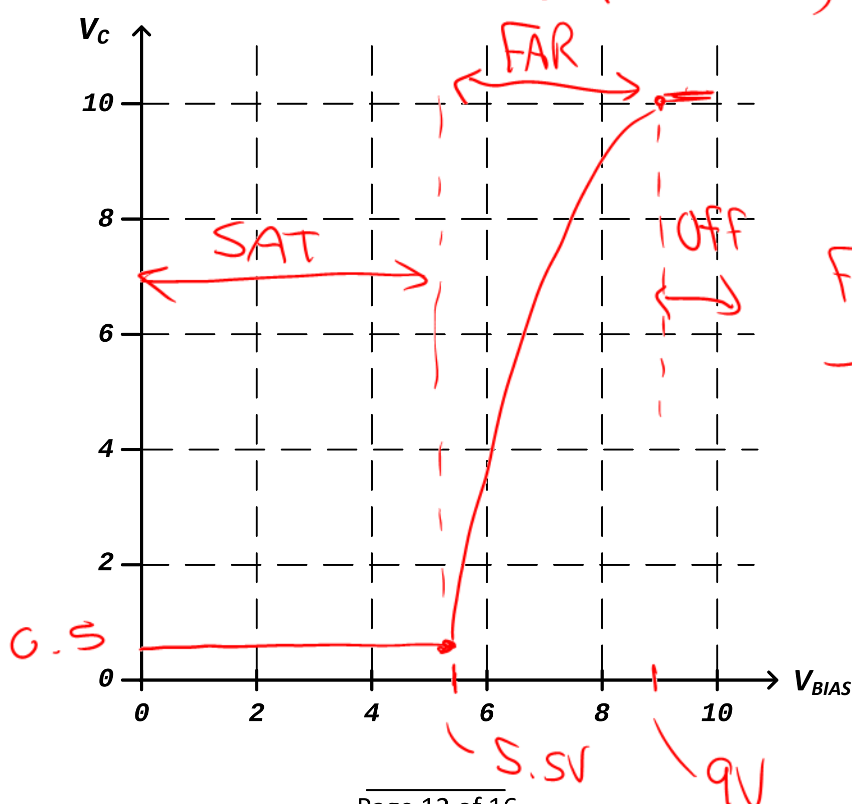
FAR/SAT Boundary:

$I_C = 190mA$ (from part c)

$I_B = 1.9mA$

$V_D = 0.7 + I_B \cdot 1k\Omega = 2.6V$

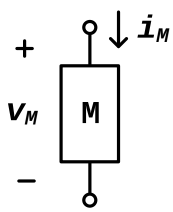
From graph in d), $V_{BIAS} = 5.5V$



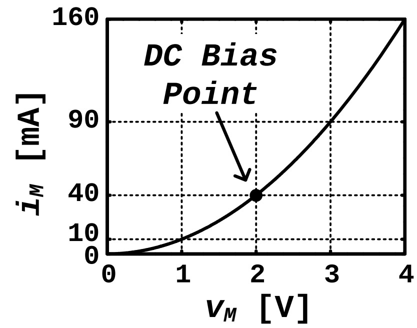
Results in $V_D = 2.6$

FAR/SAT Transition at $V_{BIAS} = 5.5V$

Problem 4 (24 Points): A non-linear two-terminal device M has the following I-V relationship, given in an expression and also plotted. Use this device to answer the following questions about small-signal analysis.



$$i_M = 0.01 \cdot v_M^2 \; A$$



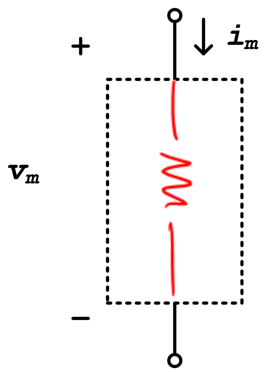
a) Derive an expression for the small-signal conductance i_m/v_m, linearized around a DC operating point defined by variables I_M and V_M.

$$g_m = \left.\frac{\partial i_M}{\partial v_M}\right|_{DC\ op} = 0.02\, v_M \Big|_{V_M}$$

$$\boxed{g_m = 0.02\, V_M}$$

b) Complete the small-signal model by drawing the circuit element in the box that should be used to model the device in small-signal. Evaluate the value of this element at the bias current of $I_M = 40\ mA$ as shown in the graph above.

Small-Signal Model

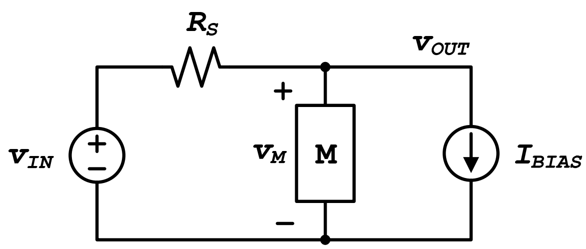


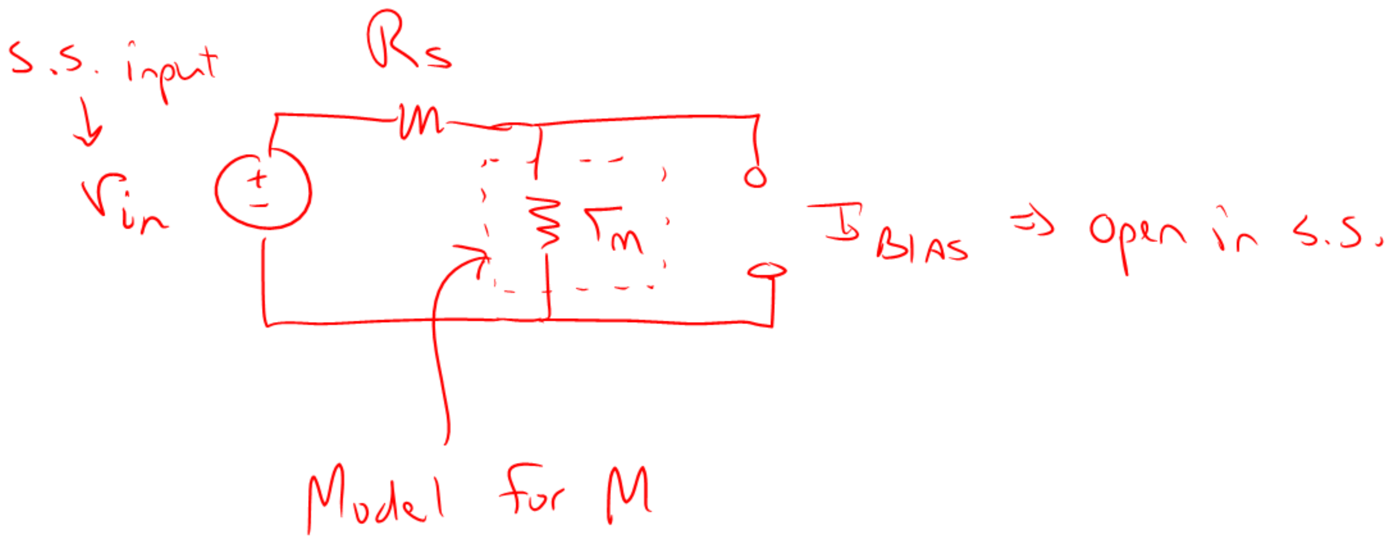
$$r_m = \frac{1}{g_m} = \frac{1}{0.02\,V_M}$$

At $I_M = 40mA$, $V_M = 2V$

$$r_m = \frac{1}{0.02 \cdot 2} = \boxed{25\Omega}$$

c) The device is used in the following circuit. Draw the complete small-signal circuit, substituting the model for the device found in part b). Assume I_{BIAS} is a DC bias current.





(Space for additional work)