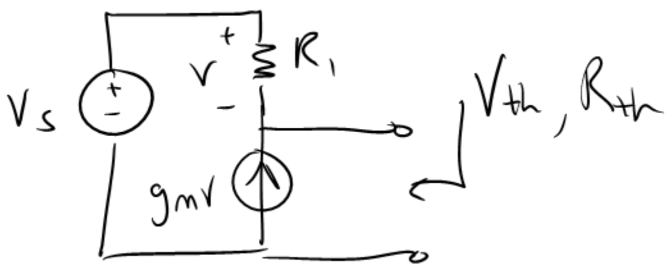
Problem Set 1 Solutions

P1.1 a) J+B 1.24

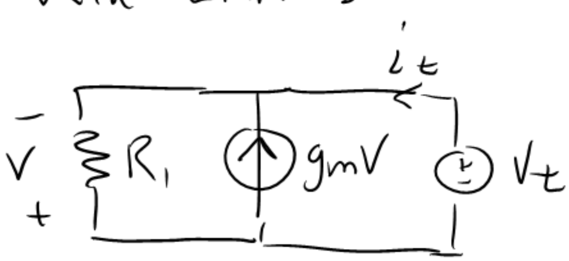


Open Ckt V_{th} KCL @ output

$$g_m V + \frac{V}{R} = 0 \quad \text{only solution } V = 0$$

$$\boxed{V_{th} = V_s}$$

R_{th} with $V_s = 0$

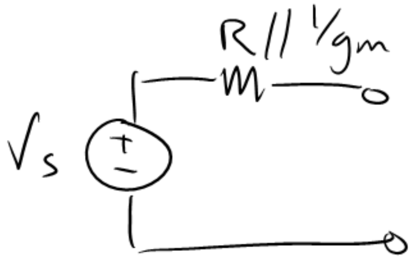


$$V = -V_t$$

$$i_t = \frac{V_t}{R} + g_m V_t$$

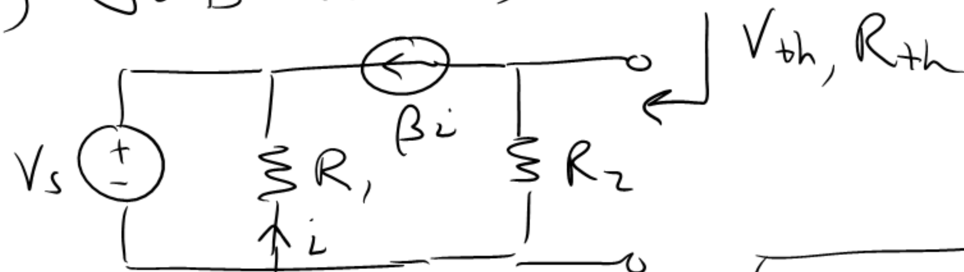
$$\boxed{R_{th} = R \parallel 1/g_m}$$

b) J&B 1.25



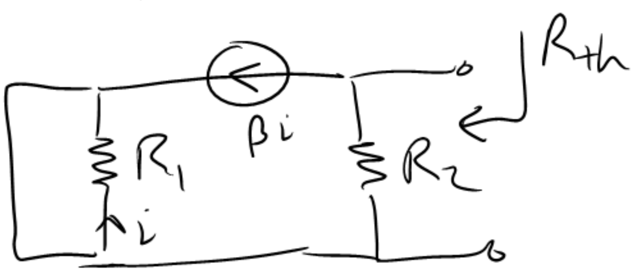
Norton: $\frac{V_s}{R \| 1/g_m}$

c) J&B 1.26a)



$i = \frac{-V_s}{R_1}$ $\quad V_{th} = -\beta i \cdot R_2 = \boxed{V_s \beta \frac{R_2}{R_1}}$

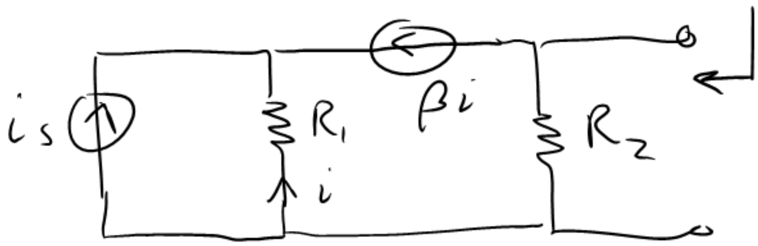
R_{th} w/ $V_s = 0$



$V_{R2} = 0 \Rightarrow i = 0$

$\boxed{R_{th} = R_2}$

J&B 1.26 b)



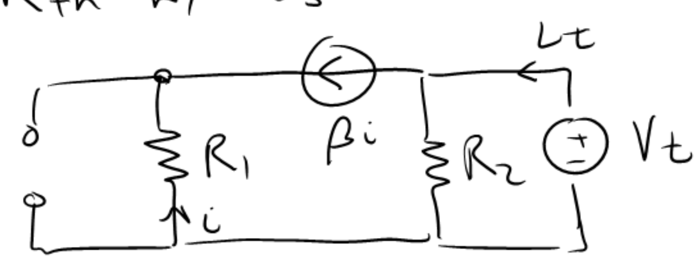
V_{th}

$V_{oc} = -\beta i R_2$

$i = -i_s - \beta i \Rightarrow i = -\dfrac{i_s}{1+\beta}$

$V_{oc} = V_{th} \boxed{= i_s \dfrac{\beta}{1+\beta} R_2}$

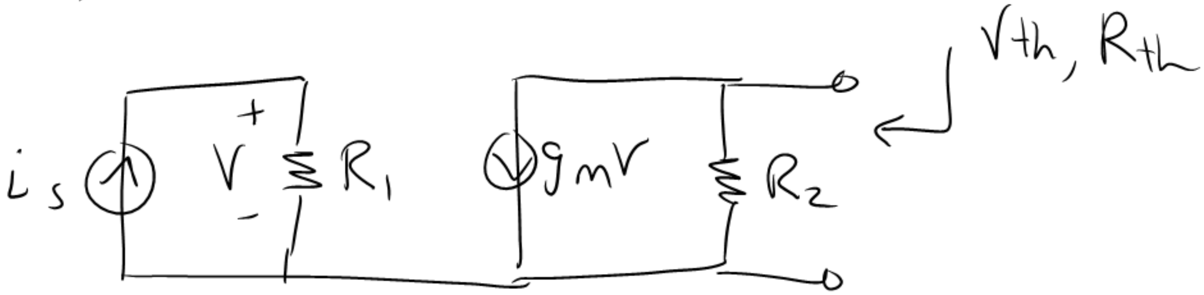
R_{th} w/ $i_s = 0$



KCL: $i + \beta i = 0 \quad i = 0$

$i_t = \dfrac{V_t}{R_2} + \cancel{\beta i}^{\,0} = \dfrac{V_t}{R_2} \qquad \boxed{R_{th} = R_2}$

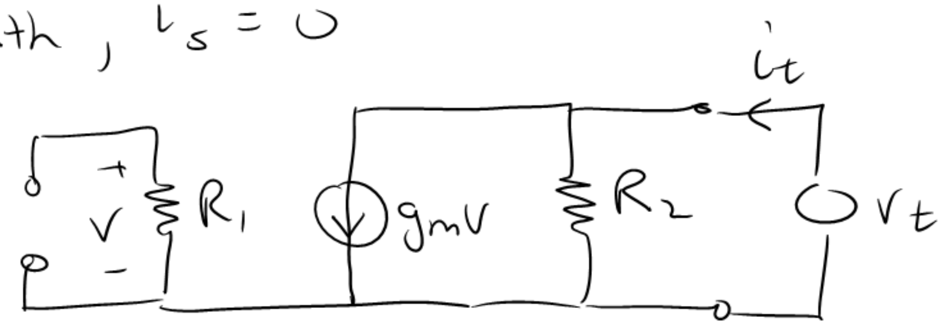
d) J+B: 1.29



$V = i_s R_1 \qquad V_{oc} = -g_m V R_2$

$$\boxed{V_{th} = -g_m i_s R_1 R_2}$$

$R_{th}, \; i_s = 0$



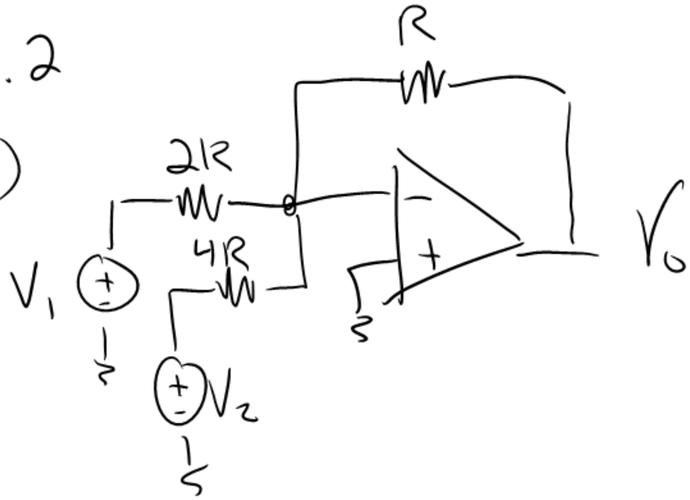
$V = 0, \quad g_m V = 0$

$$i_t = \overset{0}{\cancel{g_m V}} + \frac{V_t}{R_2} = \frac{V_t}{R_2}$$

$$\boxed{R_{th} = R_2}$$

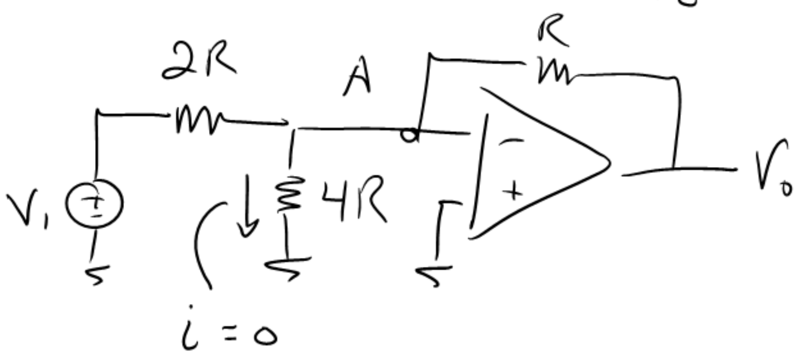
P1.2

a)



Superposition:

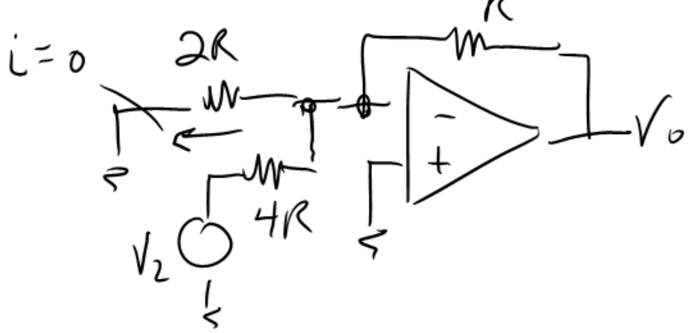
Solve for $V_o = f(V_1)\big|_{V_2=0}$



KCL @ A

$$\frac{V_1 - 0}{2R} + 0 + \frac{V_o - 0}{R} = 0 \qquad \boxed{V_o = -V_1 \frac{R}{2R} = -\frac{1}{2} V_1}$$

Solve for $V_o = f(V_2)\big|_{V_1=0}$



$$\frac{V_2 - 0}{4R} + \frac{V_o - 0}{R} = 0$$

$$\boxed{V_o = -\frac{1}{4} V_2}$$

P1.2 a) (cont)

Putting together:

$$V_o = -\left(\frac{1}{2}V_1 + \frac{1}{4}V_2\right)$$

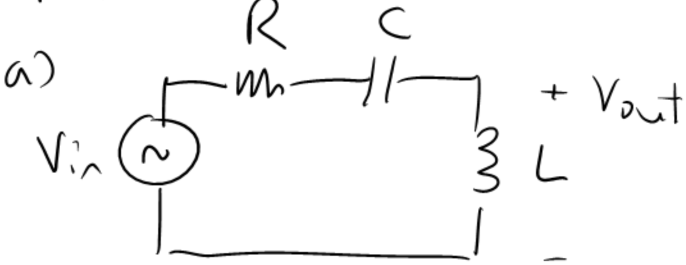
b)

V_1	V_2	V_o
0	0	0
0	V_{ref}	$-\frac{1}{4}V_{ref}$
V_{ref}	0	$-\frac{1}{2}V_{ref}$
V_{ref}	V_{ref}	$-\frac{3}{4}V_{ref}$

This implements a 2-bit digital-to-analog converter (DAC)

P1.3

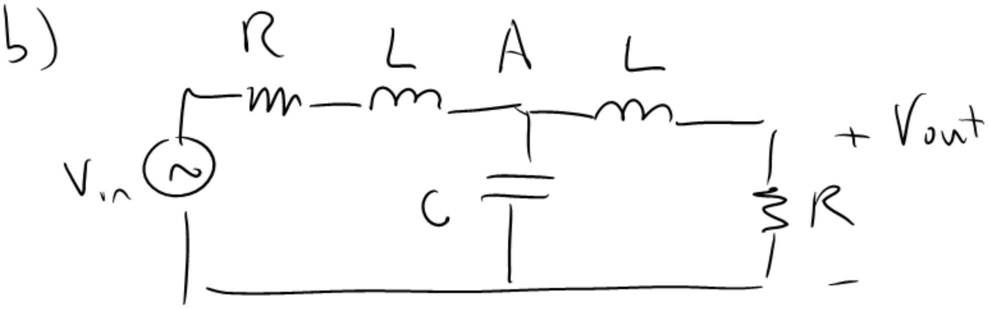
a)



Voltage divider

$$V_{out} = V_{in} \cdot \frac{sL}{R + 1/sC + sL} = \boxed{\frac{s^2LC}{1 + sRC + s^2LC}}$$

b)



① Find V_A

$$V_A = V_{in} \frac{(R+sL) \| 1/sC}{R + sL + \underbrace{(R+sL) \| 1/sC}_{\frac{R+sL}{1+sC(R+sL)}}}$$

$$V_A = V_{in} \frac{\cancel{(R+sL)}}{\cancel{(R+sL)}(1 + sC(R+sL)) + \cancel{(R+sL)}} = V_{in} \frac{1}{2 + sC(R+sL)}$$

② Find $V_{out} = f(V_A)$

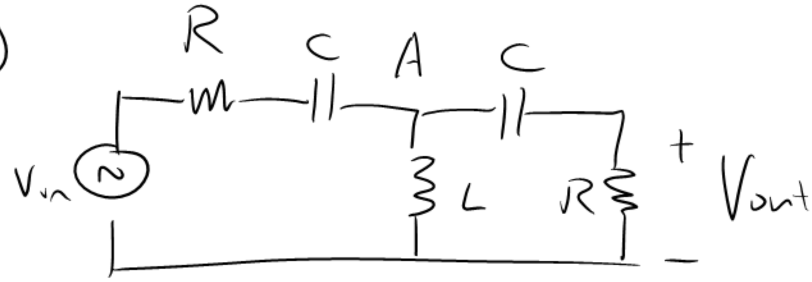
$$V_{out} = V_A \frac{R}{R + sL}$$

③ Substitute V_A

$$V_{out} = V_{in} \frac{1}{2 + sC(R + sL)} \cdot \frac{R}{R + sL}$$

$$A_v = \frac{1}{2} \frac{1}{\left(1 + \frac{sRC}{2} + \frac{s^2LC}{2}\right)\left(1 + sL/R\right)}$$

c)



Find V_A:

$$V_A = V_{in} \frac{sL \,//\, (R + 1/sC)}{(R + 1/sC) + \underbrace{sL \,//\, (R + 1/sC)}_{\frac{sL(R + 1/sC)}{sL + R + 1/sC}}}$$

$$V_A = V_{in} \frac{sL\cancel{\left(R + 1/sC\right)}}{\cancel{\left(R+1/sC\right)}\left(sL + R + 1/sC\right) + sL\cancel{\left(R + 1/sC\right)}}$$

$$= V_{in} \frac{sL}{R + 2sL + 1/sC} = \frac{s^2LC}{1 + sRC + 2s^2LC}$$

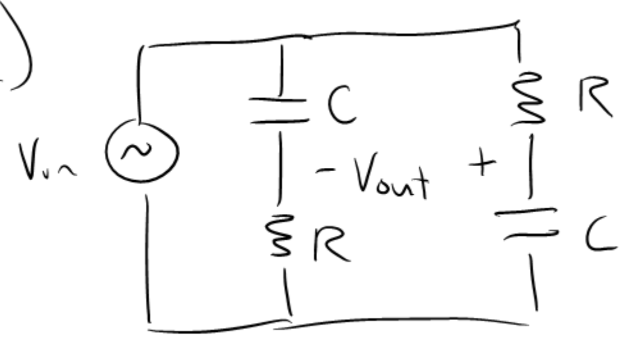
Find V_{out}:

$$V_{out} = V_A \frac{R}{R + 1/sL} = \frac{sRC}{1 + sRC}$$

Substituting:

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{s^3RLC^2}{\left(1 + sRC + 2s^2LC\right)\left(1 + sRC\right)}}$$

d)

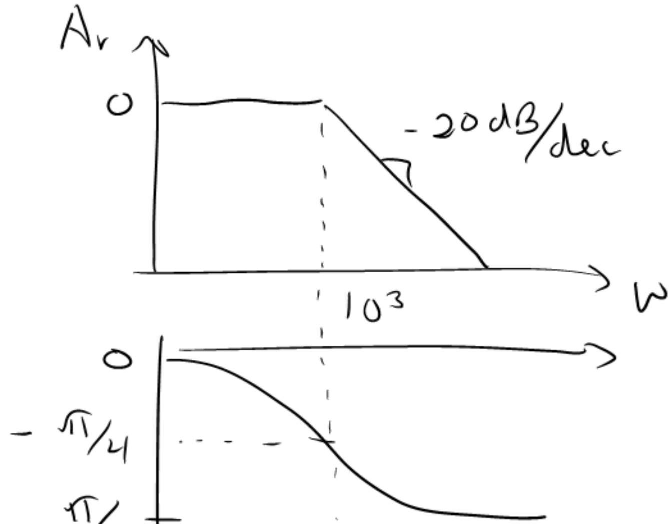


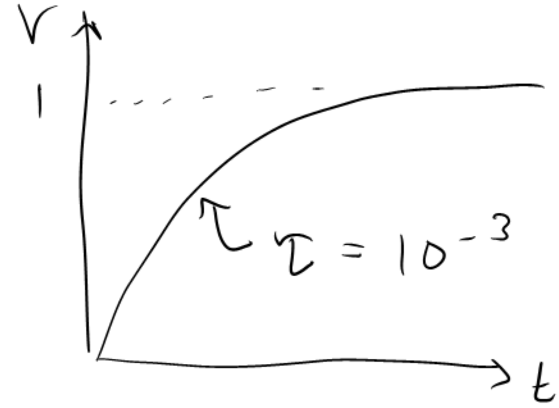
$$V_{out} = V_{in} \frac{1/sC}{R + 1/sC} - V_{in} \frac{R}{R + 1/sC}$$

$$\boxed{A_v = \frac{1 - sRC}{1 + sRC}}$$

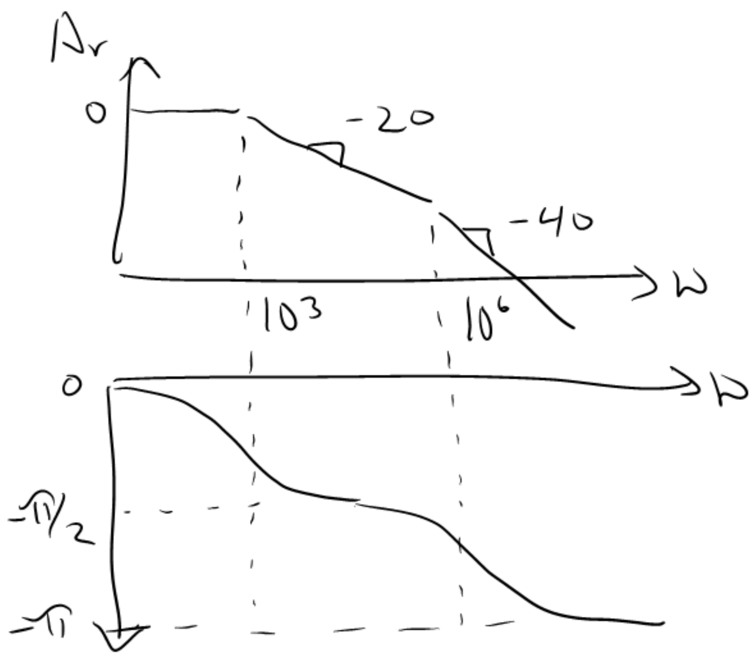
P1.4

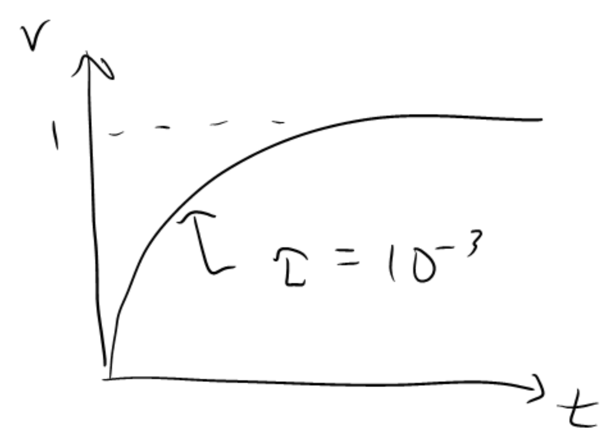
a) $A_v = \dfrac{1}{1 + s/10^3}$





b) $A_v = \dfrac{1}{\left(1 + \frac{s}{10^3}\right)\left(1 + \frac{s}{10^6}\right)}$

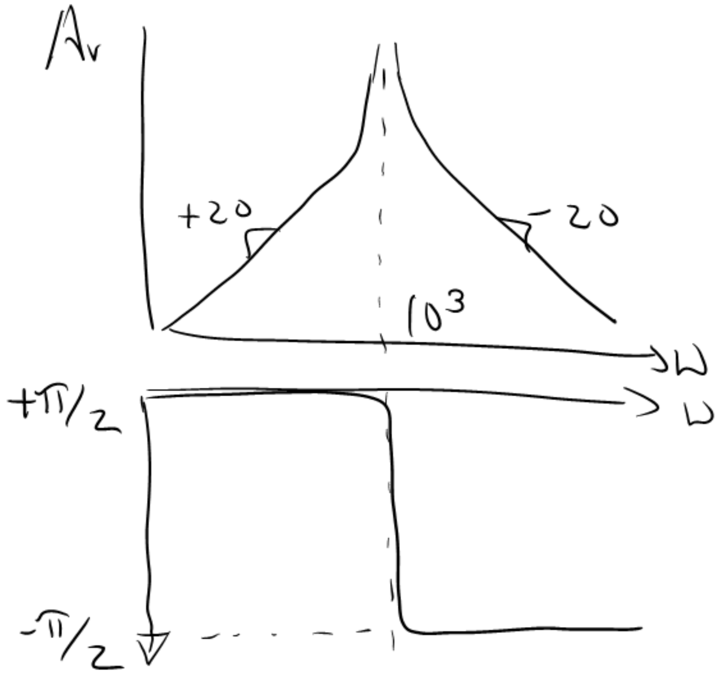


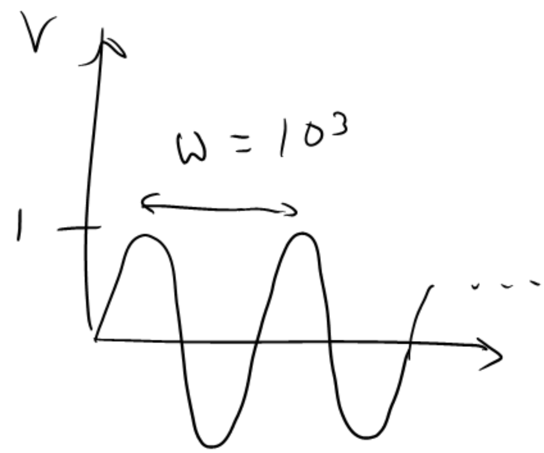


Second time constant is 10^{-6} → much faster than 10^{-3}

Approx as 1st order

c) $A_v = \dfrac{s/10^3}{1+\left(s/10^3\right)^2} \qquad Q = \infty$





d) $A_v = \dfrac{1}{1+\dfrac{s}{10^4}+\left(\dfrac{s}{10^3}\right)^2} \qquad Q = 10$