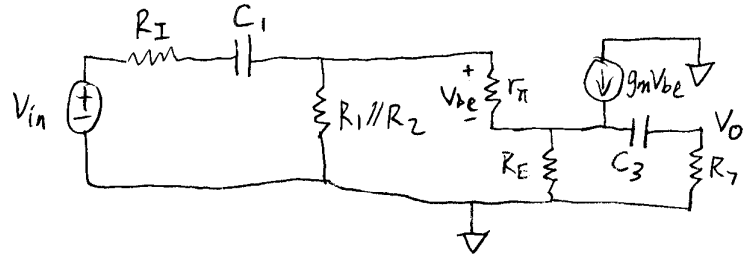
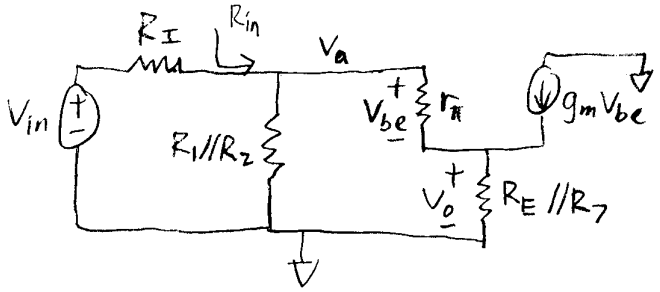
EECS 311 – HW#7 solutions (fall 2008)

1] a) low freq:



midband:



$C_1 = 4.7\,\mu F$
$C_3 = 10\,\mu F$

$R_1 = 100\,k\Omega$
$R_2 = 300\,k\Omega$
$R_E = 13\,k\Omega$
$R_I = 2\,k\Omega$
$R_7 = 100\,k\Omega$

$g_m = \frac{I_C}{V_T}$

$\beta_o = g_m r_\pi = 100$

ignore r_o (V_A not given): $r_o \approx \frac{V_A}{I_C}$

$I_C = 0.25\,mA$
$V_{CC} = 12\,V$

b) SCTC:

C_1: $R_{1,eq} = R_I + \overbrace{[R_1 // R_2 // (r_\pi + (1+g_m r_\pi) R_E)]}^{R_{in}}$

C_3: $R_{3,eq} \approx R_7 + (R_E // \frac{1}{g_m})$

MATLAB attached

$$f_L = \left(\frac{1}{R_{1eq} C_1} + \frac{1}{R_{3,eq} C_3}\right) \frac{1}{2\pi} \approx \boxed{0.623\ Hz}$$

$$A_v = \frac{R_{in}}{R_I + R_{in}} \frac{(1+\beta_o)(R_E // R_7)}{r_\pi + (1+\beta_o)(R_E // R_7)} \approx \boxed{0.964\ V/V}$$

$V_a = V_{be} + V_o$

$= i_b r_\pi + (1+\beta_o) i_b (R_E // R_7) \quad \Rightarrow i_b = \frac{V_a}{r_\pi + (1+\beta_o)(R_E // R_7)}$

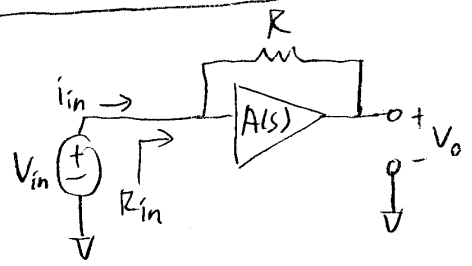
$$V_a \left(1 - \frac{r_\pi}{r_\pi + (1+\beta_o)(R_E // R_7)}\right) = V_o$$

2] $\omega_L = \frac{1}{R_{1eq} C_1} + \frac{1}{R_{3eq} C_3} = 2\pi f_L = 20\pi$

$$C_3 = \frac{\left(20\pi - \frac{1}{R_{1eq} C_1}\right)^{-1}}{R_{3eq}} \approx \boxed{0.167\ \mu F}$$

MATLAB attached

3]

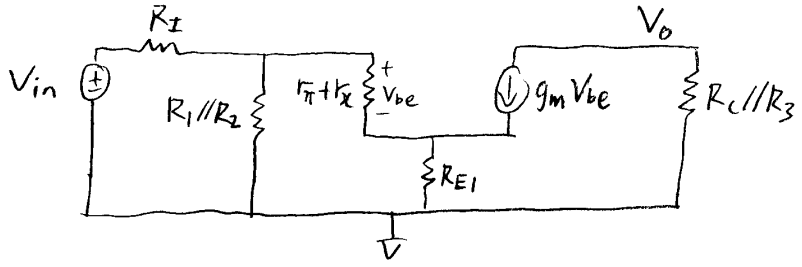


$$R_{in} = \frac{V_{in}}{i_{in}} = \boxed{\frac{R}{1 - A(s)}}$$

$$i_{in} = \frac{V_{in} - V_{out}}{R} = \frac{V_{in}(1 - A(s))}{R}$$

(DC bias in MATLAB attached)

4] midband: ignore r_o



$R_I = 250$ $\quad C_1 = 5\mu$

$R_1 = 10K$ $\quad C_2 = 1\mu$

$R_2 = 30k$ $\quad C_3 = 4.7\mu$

$r_x = 350$ $\quad \beta_0 = 100$

$R_{E1} = 200$ $\quad f_T = 200\,MHz$

$R_C = 4.3\,K$ $\quad C_\mu = 1pF$

$R_3 = 47\,K$

a) $$A_{mid} \approx \frac{R_1 // R_2 // (r_\pi + r_x + (1+\beta_0) R_{E1})}{R_I + R_1 // R_2 // (r_\pi + r_x + (1+\beta_0) R_{E1})} \left[- \frac{g_m (R_C // R_3)}{1 + g_m R_{E1}} \right] \approx \boxed{-17.6\ V/V}$$

b) $R_{1,eq} = R_I + [R_1 // R_2 // (r_\pi + r_x + (1+\beta_0) R_{E1})]$

MATLAB attached

$R_{2,eq} = R_3 + [R_C]$

$R_{3,eq} = R_{E1} + \frac{1}{g_m}$

$$f_L = \frac{1}{2\pi}\left(\frac{1}{R_{1,eq} C_1} + \frac{1}{R_{2,eq} C_2} + \frac{1}{R_{3,eq} C_3}\right) \approx \boxed{166\ Hz}$$

4] c) $f_T = \frac{g_m}{C_\pi + C_\mu} \Rightarrow C_\pi = \frac{g_m}{f_T} - C_\mu$

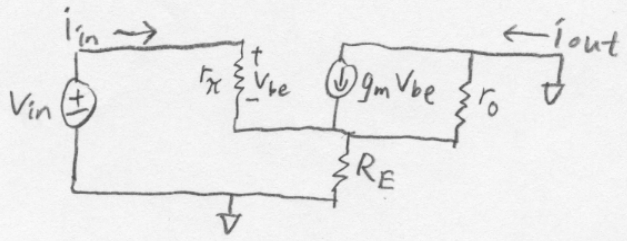
$$R_{\pi,eq} = \left[\frac{1}{r_\pi} + \frac{1+g_m R_{E1}}{R_S + R_{E1}}\right]^{-1}$$

$$R' = R_I /\!/ R_1 /\!/ R_2 /\!/ (r_\pi + r_x + (1+\beta_0)R_{E1})$$
$$R'' = R_C /\!/ R_3$$
$$G_m = \frac{g_m}{1+g_m R_{E1}}$$
$$\Big\} R_{M,eq} = R' + R''(1+G_m R')$$

$$f_H = \frac{1}{2\pi}\left(\frac{1}{R_{\pi,eq} C_\pi + R_{M,eq} C_\mu}\right) \approx \boxed{8.55\ MHz}$$

5]

a)



KCL: $i_{in} + g_m V_{be} = \frac{V_{in} - V_{be}}{R_E /\!/ r_o}$ $\qquad V_{be} = i_{in} r_\pi$

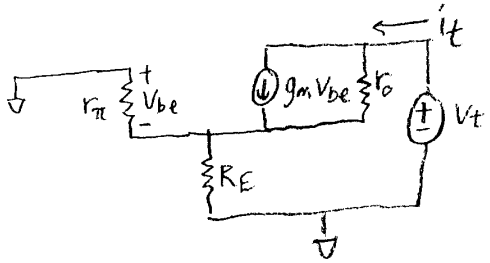
$$i_{in}\left(1 + g_m r_\pi + \frac{r_\pi}{R_E /\!/ r_o}\right) = \frac{V_{in}}{R_E /\!/ r_o}$$

$$R_{in} = \frac{V_{in}}{i_{in}} = \boxed{r_\pi + (1+\underbrace{g_m r_\pi}_{\beta_0})(R_E /\!/ r_o)}$$

$$i_{out} = g_m V_{be} - \frac{(V_{in} - V_{be})}{r_o}$$
$$V_{be} = i_{in} r_\pi = \frac{r_\pi V_{in}}{r_\pi + (1+\beta_0)(R_E /\!/ r_o)} = \frac{r_\pi V_{in}}{R_{in}}$$
$$i_{out} = V_{in}\left(\frac{\beta_0}{R_{in}} - \frac{1}{r_o} + \frac{r_\pi}{R_{in} r_o}\right)$$

$$G_m = \frac{i_{out}}{V_{in}} = \frac{\beta_0 r_o - R_{in} + r_\pi}{r_o R_{in}} = \boxed{\frac{\beta_0 r_o - (1+\beta_0)(R_E /\!/ r_o)}{r_o(r_\pi + (1+\beta_0)(R_E /\!/ r_o))}} \approx \frac{g_m}{1+g_m R_E}$$

↑ assumes $r_\pi \gg R_E$, $r_o \gg r_\pi$, $\beta_0 \gg 1$



$$i_t = \frac{-V_{be}}{r_\pi /\!/ R_E} = g_m V_{be} + \frac{V_t + V_{be}}{r_o}$$

$$\Downarrow \qquad \Downarrow$$

$$V_{be} = -i_t (r_\pi /\!/ R_E) \Rightarrow i_t = -g_m (r_\pi /\!/ R_E) i_t + \frac{V_t - i_t (r_\pi /\!/ R_E)}{r_o}$$

$$i_t \left(1 + g_m (r_\pi /\!/ R_E) + \frac{r_\pi /\!/ R_E}{r_o}\right) = \frac{V_t}{r_o}$$

$$R_{out} = \frac{V_t}{i_t} = \boxed{r_o \left[1 + g_m (r_\pi /\!/ R_E)\right] + (r_\pi /\!/ R_E)} \approx r_o (1 + g_m R_E)$$

assumes $r_o \gg r_\pi \gg R_E$

b) $$R_{in} = r_\pi + (1 + g_m r_\pi)(R_E /\!/ r_o)$$

$$\approx r_\pi + (1 + g_m r_\pi) R_E \qquad (r_o \gg R_E)$$

$$\approx r_\pi + g_m r_\pi R_E \qquad (\beta_o = g_m r_\pi \gg 1)$$

$$\approx r_\pi (1 + g_m R_E)$$

$$G_m = \frac{\beta_o r_o - (1 + \beta_o)(R_E /\!/ r_o)}{r_o (r_\pi + (1 + \beta_o)(R_E /\!/ r_o))}$$

$$\approx \frac{\beta_o r_o - \beta_o R_E}{r_o (r_\pi + \beta_o R_E)} \qquad (r_o \gg R_E,\ \beta_o \gg 1)$$

$$\approx \frac{\beta_o (r_o - R_E)}{r_o (r_\pi + \beta_o R_E)} \approx \frac{\beta_o r_o}{r_o (r_\pi + \beta_o R_E)} \qquad (r_o \gg R_E)$$

$$\approx \frac{g_m r_\pi}{r_\pi + g_m r_\pi R_E} \approx \frac{g_m}{1 + g_m R_E}$$

conditions used: $r_o \gg R_E$ & $\beta_o = g_m r_\pi \gg 1$

c) $R_{out} = r_o[1+g_m(r_\pi /\!/ R_E)] + (r_\pi /\!/ R_E)$

$\approx r_o[1+g_m R_E] + R_E \qquad (r_\pi >> R_E)$ ☆

$\approx r_o(1+g_m R_E) \qquad (r_o >> R_E)$

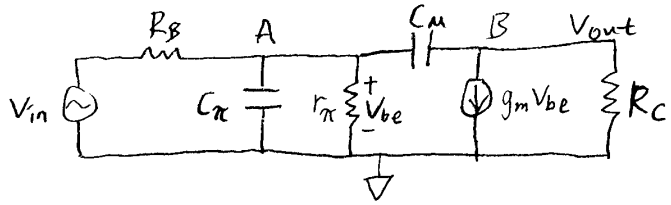
6] a) $A_V = \frac{V_A}{V_{in}} \cdot \frac{V_{out}}{V_A} = \frac{R_{in}}{R_B + R_{in}} \cdot (-G_m R_{out})$

$\approx \boxed{\frac{-r_\pi(1+g_m R_E)}{R_B + r_\pi(1+g_m R_E)} \cdot \frac{g_m R_C}{1+g_m R_E}} = \boxed{-g_m R_C \frac{r_\pi}{R_B + r_\pi}}$ $R_E = 0$

↑ use this

r_o ignored for all of #6

b)



$C_A = C_\pi + C_\mu(1-A_V)$

$C_B = C_\mu(1-A_V^{-1})$

} A_V from part a

c) $R_{A,eq} = R_B /\!/ r_\pi$

$R_{B,eq} = R_C$

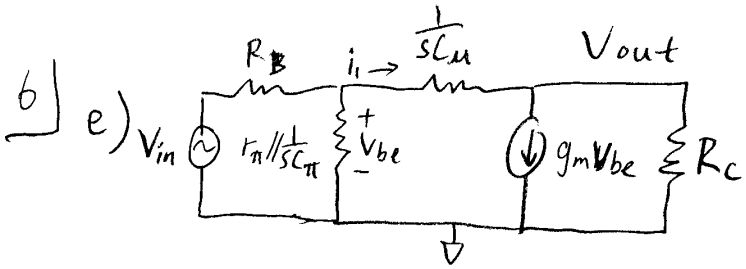
$f_H = \frac{1}{2\pi}\left(\frac{1}{R_{A,eq} C_A + R_{B,eq} C_B}\right)$

d) $A_v \approx -1,389\ V/V$

$f_H \approx 75.0\ kHz$

(plot & code attached)

6] e)



KCL $\quad i_1 = g_m V_{be} + \frac{V_{out}}{R_C}$

KCL $\quad \frac{V_{in} - V_{be}}{R_B} = i_1 + \frac{V_{be}}{r_\pi // \frac{1}{sC_\pi}}$

$$\frac{V_{in}}{R_B} = g_m V_{be} + \frac{V_{out}}{R_C} + \frac{V_{be}}{r_\pi // \frac{1}{sC_\pi}} + \frac{V_{be}}{R_B}$$

$$= V_{be}\left(g_m + \frac{1}{r_\pi // \frac{1}{sC_\pi}} + \frac{1}{R_B}\right) + \frac{V_{out}}{R_C} \quad (1)$$

Ω's Law $\quad V_{be} - \frac{i_1}{sC_\mu} = V_{out}$

$$V_{be} = V_{out} + \frac{g_m}{sC_\mu} V_{be} + \frac{V_{out}}{sR_C C_\mu}$$

$$V_{be}\left(1 - \frac{g_m}{sC_\mu}\right) = V_{out}\left(1 + \frac{1}{sR_C C_\mu}\right)$$

$$\frac{sC_\mu - g_m}{sC_\mu}$$

$$V_{be} = V_{out}\left(\frac{sR_C C_\mu + 1}{sR_C C_\mu}\right)\frac{sC_\mu}{sC_\mu - g_m}$$

$$= V_{out}\,\frac{(sR_C C_\mu + 1)}{R_C(sC_\mu - g_m)} \quad (2)$$

plug (2) into (1):

$$\frac{V_{in}}{R_B} = V_{out}\left[\frac{sR_C C_\mu + 1}{R_C(sC_\mu - g_m)}\left(g_m + \frac{1}{r_\pi // \frac{1}{sC_\pi}} + \frac{1}{R_B}\right) + \frac{1}{R_C}\right]$$

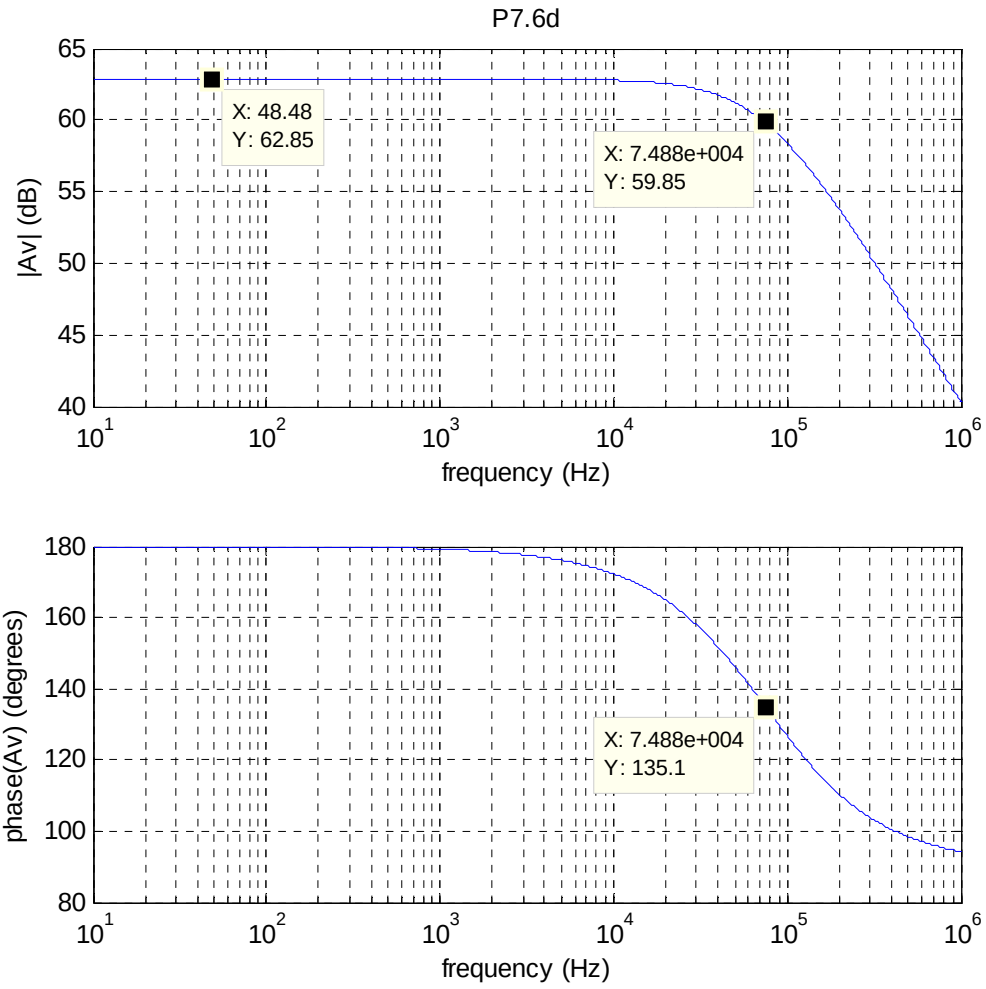
$$\frac{V_{out}}{V_{in}} = \left\{R_B\left[\quad\right]\right\}^{-1}$$

f) plots attached, the gain matches up to ~10kHz,
" phase " " " ~1kHz

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% #1
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

R1 = 100e3;
R2 = 300e3;
Ri = 2e3;
Re = 13e3;
R7 = 100e3;
Bf = 100; % current gain, A/A
Af = Bf/(Bf+1);
Vt = 0.026; % thermal voltage @ room temp.
Ic = 0.25e-3;

gm = Ic/Vt;
rpi = Bf/gm;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Midband gain
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Rin = (1/R1 + 1/R2 + 1/(rpi+(1+Bf)*Re))^-1;
RL = (1/Re + 1/R7)^-1;
Gain = (Rin/(Ri+Rin))*((1+Bf)*RL/(rpi+(1+Bf)*RL))

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% SCTC - fL
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

C1 = 4.7e-6;
C3 = 10e-6;
Re_eq1 = Ri+Rin;
Re_eq3 = R7 + ((1/Re + gm)^-1);

FL = (1/(Re_eq1*C1) + 1/(Re_eq3*C3))/(2*pi) %Hz

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% #2
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

C3 = ((20*pi - (1/(Re_eq1*C1)))^-1)/Re_eq3


clc;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% #4
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Vcc = 12;
R1 = 10e3;
R2 = 30e3;
Ri = 250;
rx = 350;
Re1 = 200;
Re2 = 1100;
Rc = 4.3e3;
R3 = 47e3;
Vbeon = 0.7;
```

```
Bf = 100; % current gain, A/A
Af = Bf/(Bf+1);
Vt = 0.026; % thermal voltage @ room temp.

% DC bias

Vb = R1*Vcc/(R1+R2)
Ve = Vb-Vbeon
Ie = Ve/(Re1+Re2);
Ic = Af*Ie
Vc = Vcc - (Ic*Rc) % check for FAR mode

gm = Ic/Vt;
rpi = Bf/gm;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Midband gain
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Rin = (1/R1 + 1/R2 + 1/(rpi+rx+(1+Bf)*Re1))^-1;
Rout = (1/Rc + 1/R3)^-1;
Gain = -(gm/(1+gm*Re1))*(Rout) * (Rin/(Ri+Rin))

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% SCTC - fL
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

C1 = 5e-6;
C2 = 1e-6;
C3 = 4.7e-6;
Re_eq1 = Ri+Rin;
Re_eq2 = R3+Rc;
Re_eq3 = Re1 + (1/gm);

FL = (1/(Re_eq1*C1) + 1/(Re_eq2*C2) + 1/(Re_eq3*C3))/(2*pi) %Hz

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% OCTC - fH
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Cu = 1e-12;
ft = 200e6;
Cpi = (gm/ft) - Cu

GS=1/Ri+1/R1+1/R2;
Rs=1/GS;
Rpi_eq = 1/(1/rpi+(1+gm*Re1)/(Rs+Re1))

Rin = 1/(1/Ri + 1/R1 + 1/R2 + 1/(rpi+rx+(1+Bf)*Re1));
Rout = 1/(1/Rc + 1/R3);
GM = gm/(1+gm*Re1);
Ru_eq = Rin + Rout*(1+GM*Rin);

FH = 1/(2*pi*(Rpi_eq*Cpi + Ru_eq*Cu))
```

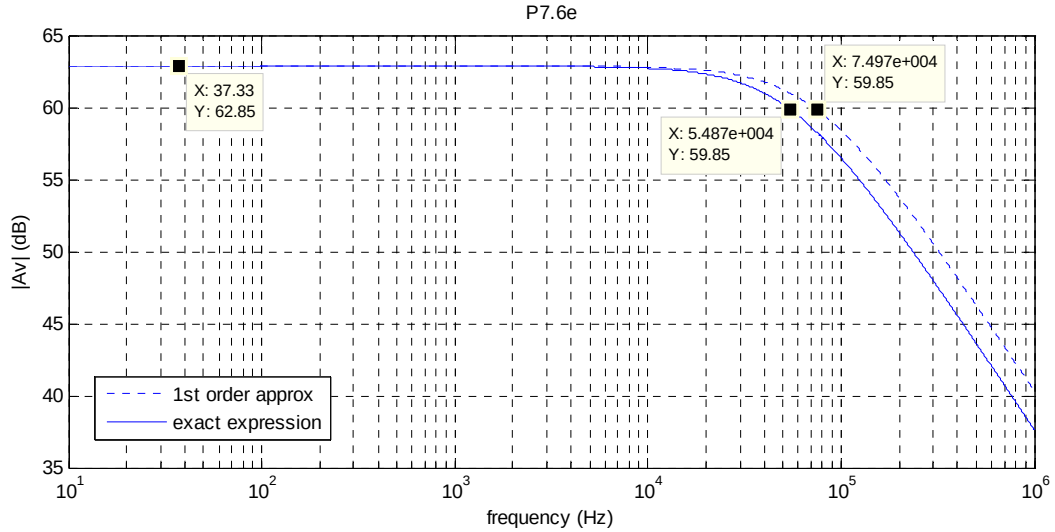
```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% #6
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

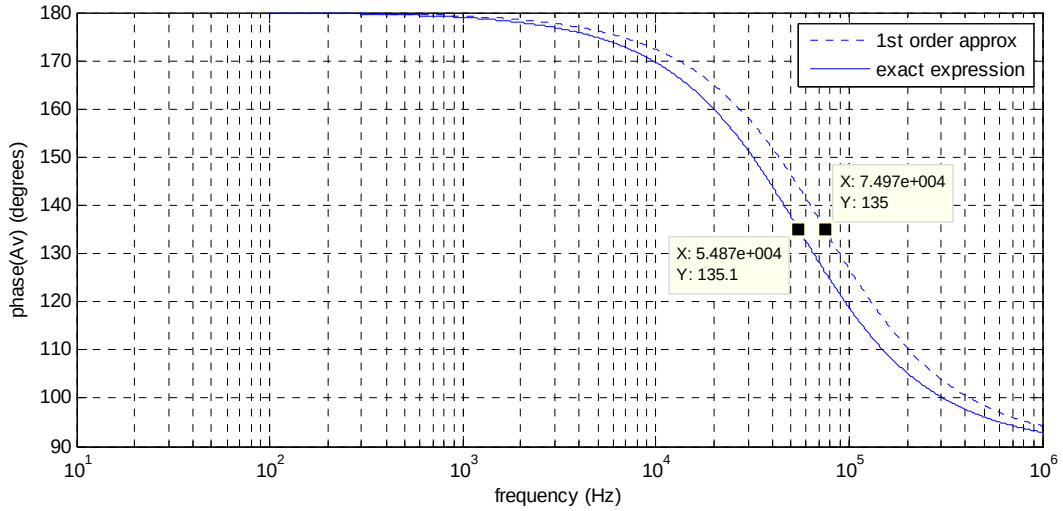



```
Rb = 1e3;
Rc = 50e3;
Bf = 100;
Cu = 2e-12;
Cje = 0; %no depletion cap, just consider diffusion cap for B-E junction
Ic = 1e-3;
Tf = 500e-12;
Vt = 0.026;

gm = Ic/Vt;
rpi = Bf/gm;
Cpi = gm*Tf;

Av = -gm*Rc*(rpi/(Rb+rpi))
CA = Cpi + Cu*(1-Av);
CB = Cu*(1-(1/Av));
RAeq = (1/Rb + 1/rpi)^-1;
RBeq = Rc;

fH = (1/(RAeq*CA + RBeq*CB))/(2*pi)

f = logspace(1,6,10000);
```

```
Gain = Av./(1+(j*f./fH));
subplot(2,1,1);
semilogx(f,20*log10(abs(Gain)));
grid on
xlabel('frequency (Hz)');
ylabel('|Av| (dB)');
title('P7.6d');
subplot(2,1,2);
semilogx(f,angle(Gain)*180/pi);
grid on
xlabel('frequency (Hz)');
ylabel('phase(Av) (degrees)');
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% #6e
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```





```
Gain_e = 1./(Rb.*(((1+j*2*pi.*f.*Rc*Cu)./(Rc.*(j*2*pi.*f.*Cu-
gm))).*(gm+((1/rpi)+j*2*pi.*f.*Cpi)+(1/Rb))+(1/Rc)));
subplot(2,1,1);
semilogx(f,20*log10(abs(Gain)),':');
hold on
semilogx(f,20*log10(abs(Gain_e)));
hold off
grid on
xlabel('frequency (Hz)');
ylabel('|Av| (dB)');
title('P7.6e');
legend('1st order approx','exact expression');
subplot(2,1,2);
semilogx(f,angle(Gain)*180/pi,':');
hold on
semilogx(f,angle(Gain_e)*180/pi);
hold off
grid on
xlabel('frequency (Hz)');
ylabel('phase(Av) (degrees)');
legend('1st order approx','exact expression');
```