NAME: Solutions

Honor Code:
I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code.

Signature ________________________________

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Problem 1 (30 Points): Potpourri – this problem has five unrelated parts.

a) Find the transfer function $H(s) = \frac{i_{out}}{v_{in}}$ of the following circuit.

\[ \frac{I_o}{V_{in}} = \frac{1}{R} \cdot \frac{1}{1 + sBR} \]

\[ I_o = \frac{V_{in}}{R + sL} \]

C does not impact $I_o$

b) Sketch the magnitude of the following transfer function on the following log-log plot. Label the DC magnitude, the high-frequency magnitude, and the frequencies of any poles and zeros.

\[ \frac{v_{out}}{v_{in}} = \frac{0.1s + 1}{s + 1} \]

DC $s = 0 \quad |A_v| = 1$

High $s = \infty \quad |A_v| = 0.1$

Pole $\leftrightarrow \omega_p = 1 \text{ rad/s}$  \quad Zero $\leftrightarrow \omega_z = 10 \text{ rad/s}$
c) Sketch the step response of the following transfer function. Label the initial value \((t = 0^+)\), final value \((t = \infty)\), and time constant on the graph.

\[
\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{0.1s + 1}{s + 1}
\]

Initial value theorem \(s \to \infty\) \(V_{\text{out}}(t=\infty) = 0.1\ V\)

Final value theorem \(s \to 0\) \(V_{\text{out}}(t=\infty) = 1\ V\)

1 pole at \(\omega_p = 1\) \(\Rightarrow \tau = \frac{1}{\omega_p} = 1\ s\)

\[
\text{v}_{\text{out}}\quad 1
\]
\[
0.5
\]
\[
0.1
\]
\[
0\quad 1\quad 2\quad 3\quad t
\]

\[
v_{\text{out}}\to 1\ V
\]

---

d) Solve for the current \(i\) in the following circuit using superposition.

\[
V_2 = 0\quad i_{(1)} = \frac{V_1}{R_1 || R_2}
\]
\[
V_1 = 0\quad i_{(2)} = -\frac{V_2}{R_2}
\]
\[
i = i_{(1)} + i_{(2)} = \frac{V_1}{R_1 || R_2} - \frac{V_2}{R_2}
\]
e) Find the Thevenin voltage and resistance $V_{th}$ and $R_{th}$ as shown in the circuit below.

$V_{th}$: Open-Ckt Voltage

$V_{th} = V_1 - V_2$

$R_{th}$: Zero Sources

$R_{th} = \frac{V_L}{I_t} = 0$
Problem 2 (34 Points): Use the following circuit for this problem. Assume the opamp is ideal unless otherwise stated in the problem.

\[ V_{\text{out}} \text{ is indep of } C \]

\[ \frac{V_{\text{in}}}{V_{\text{in}}} = \frac{O - V_{\text{out}}}{R} \]
\[ \frac{V_{\text{out}}}{V_{\text{in}}} = -1 \]
b) Derive an expression for the input impedance $Z_{\text{in}}(s)$ as defined in the schematic.

\[ Z_{\text{in}} = \frac{V_{t}}{I_{t}} \]

\[ I_{t} = \frac{V_{t}}{R} + \frac{V_{t} - V_{\text{out}}}{1/SC} \]

\[ V_{\text{out}} = -V_{\text{in}} \text{ (from part a)} \]

\[ I_{t} = \left(\frac{1}{R} + s2C\right)V_{t} \]

\[ Z_{\text{in}} = R \cdot \frac{1}{1+s2RC} \]
c) Assume the opamp has a finite offset voltage $V_{OS}$. Derive an expression for the transfer function $K(s) = \frac{v_{out}}{V_{OS}}$.

\[
\text{Zero } V_{in} \quad \hspace{2cm} \text{Non-inverting amp}
\]

\[
V_{out} = 1 + \frac{R}{R} = \frac{2}{V_{os}}
\]

d) Assume the input offset voltage is a DC voltage with amplitude $V_{OS}$. With the input set to zero ($v_{in} = 0$), derive an expression for the magnitude of the output voltage $|v_{out}|$ at time $t = \infty$ resulting from the offset voltage.

At $t = \infty$ \quad $V_{out} = \boxed{2V_{os}}$
e) Assume the input $v_{in}$ is a triangle wave with peak-peak amplitude $A_{ppk}$ and frequency $f_0$. Derive an expression for the minimum allowable slew rate specification of the opamp for the circuit to properly amplify and track this input waveform.

From a), $V_{out} = -V_{in}$

Slew Rate > $\text{max } \frac{dV_{out}}{dt}$

$$\frac{dV_{out}}{dt} = \frac{\Delta V}{\Delta t} = \frac{A_{ppk}}{1/(2f_0)} = 2f_0 A_{ppk}$$

S.R. > $2f_0 A_{ppk}$
Problem 3 (24 Points): This problem uses graphical filter design techniques to find the transfer function of a filter. Use the following schematic for all parts of this problem.

![Schematic Diagram]

a) Assume the opamp is ideal (infinite gain and bandwidth). Derive an expression for the transfer function $H(s) = \frac{v_{out}}{v_{in}}$.

\[
\begin{align*}
V_o &= V_{in} \\
10V_{out} - V_{in} &= \frac{V_{in} - 0}{9R} \\
9R &= \frac{R + SL}{R + SL}
\end{align*}
\]

\[
10(R + SL) = 9R V_{in} + (R + SL) V_{in}
\]

\[
\frac{V_{out}}{V_{in}} = \frac{1 + s \frac{L}{10R}}{1 + s \frac{L}{R}}
\]
b) Derive an expression for the feedback factor $\beta(s) = \frac{v_A}{v_{out}}$ as defined in the schematic.

\[ V_A = 10 V_{out} \quad \frac{R + SL}{9R + R + SL} \]

\[
\beta = \frac{1 + s \frac{L}{R}}{1 + s \frac{L}{10R}}
\]

Note $\beta = (\frac{V_{out}}{V_{in}})^{-1}$ for ideal opamp
c) Using graphical filter design techniques, sketch the magnitude of the transfer function $H(s) = \frac{v_{out}}{v_{in}}$ on the graph below. Assume the opamp has finite gain and bandwidth as shown in the graph.

$$\beta = \frac{1 + s \frac{R}{L}}{1 + s \frac{1}{10R}}$$

Plot $\frac{1}{\beta} = \frac{1 + s \frac{1}{10R}}{1 + s \frac{1}{L}}$

Zero @ $\omega_z = \frac{10R}{L}$

Pole @ $\omega_p = \frac{R}{L}$

Lower of $\frac{1}{\beta}$, $A(s)$ is $\frac{V_{out}}{V_{in}}$
Problem 4 (12 Points): This is a multiple choice problem. Use the following page for additional work if needed. For each part, circle only one answer. Refer to the following diode circuit and waveforms for each part. Assume the constant voltage source model for the diode, with an on-voltage of $V_{ON}$.

![Diode circuit and waveforms](image)

a) Keeping everything else constant, the trend indicated on the output waveform will be observed as the value of $C$ is:

- Increased  |  Decreased  |  Doesn’t cause trend

b) Keeping everything else constant, the trend indicated on the output waveform will be observed as the value of $R$ is:

- Increased  |  Decreased  |  Doesn’t cause trend

c) Keeping everything else constant, the trend indicated on the output waveform will be observed as the period of the input sine wave $T'$ is:

- Increased  |  Decreased  |  Doesn’t cause trend

d) Keeping everything else constant, the trend indicated on the output waveform will be observed as the amplitude of the input sine wave $A$ is:

- Increased  |  Decreased  |  Doesn’t cause trend
Initials: __________

(Space for additional work)