Honor Code:
I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code.

Signature ________________________________
**Problem 1 (20 Points):** Potpourri.

a) For the circuit below, find the range of $V_D$ that will simultaneously bias the NMOS and PMOS in the Saturation region.

$\begin{align*}
V_{tn} &= 1V \\
V_{tp} &= -1V
\end{align*}$

\[
\text{NMOS Sat} \\
V_{DS} > V_{GS} - V_{tn} \quad V_{GS} = 2.5V \quad V_{DS} = V_D \\
V_D > 1.5V \quad \circ
\]

\[
\text{PMOS Sat} \\
V_{DS} < V_{GS} - V_{tp} \quad V_{GS} = 2.5 - 5 = -2.5V \quad V_{DS} = V_D - 5 \\
V_D - 5 < -2.5 + 1 \quad V_D < 3.5V \quad \Theta
\]

\[1.5 < V_D < 3.5\]
b) Determine the values of the DC and ac components of the waveforms shown below, assuming the signal are sinusoidal with frequency $\omega = \omega_0$. Additionally, evaluate the ac gain $a_v = v_{out}/v_{in}$ of the circuit.

\[
\text{Total} = \text{DC Component} + \text{ac Component}
\]

\[
v_{IN} = \frac{1}{V} + 0.5 \sin \omega_0 t
\]

\[
v_{OUT} = \frac{2.75 V}{V} + -1.25 \sin \omega_0 t
\]

ac Gain $a_v = v_{out}/v_{in} = -2.5 \frac{V}{V}$
c) Assume the BJT is biased in the FAR region, and that $I_{BIAS}$ is a DC bias source. Substitute the small-signal model for the BJT and find an expression for the small-signal gain $\alpha_v = \frac{v_{out}}{v_{in}}$. Assume the BJT has finite $V_A$ (include base-width modulation).
Problem 2 (25 Points): Use the circuit below and the values given in the table for all parts of this problem. The $C_{BIG}$ capacitors should be treated as bypass capacitors.

<table>
<thead>
<tr>
<th>$\beta_f$</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_A$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$V_{BE,ON}$</td>
<td>0.6V</td>
</tr>
<tr>
<td>$V_{BIAS}$</td>
<td>5V</td>
</tr>
<tr>
<td>$R_B$</td>
<td>10kΩ</td>
</tr>
<tr>
<td>$R_C$</td>
<td>1kΩ</td>
</tr>
</tbody>
</table>

a) Draw the DC equivalent circuit. Calculate the value of $I_{BIAS}$ that will bias the BJT in the FAR region, at the boundary between FAR and Saturation.

Large Signal:

$$ V_B = V_C = \frac{5}{\beta_f R_C - R_B} $$

$$ I_B = I_{BIAS} = \left(\beta_f + 1\right) \frac{5}{\beta_f R_C - R_B} $$

$$ I_{BIAS} = 5.61 \text{mA} $$

FAR/Sat Boundary: $V_{BC} = 0$

$$ V_B = V_C = 5 - \beta_f I_B R_B = 10 - \beta_f I_B R_C $$

$$ I_B = \frac{5}{\beta_f R_C - R_B} $$

$$ I_{BIAS} = \left(\beta_f + 1\right) I_B = 5.61 \text{mA} $$
b) For this part only, assume the bias current $I_C = 2.5mA$. Draw the small-signal (ac) circuit and evaluate the values of the elements in the BJT small-signal model assuming “room temperature”. Assume $V_A = \infty$.

\[ g_m = 100 \text{ mV} \]
\[ R_m = 1k\Omega \]
c) For this part, do NOT use the evaluated values of small signal elements found in part b). Derive an expression for the midband small-signal gain \( a_v = \frac{v_{out}}{v_{in}} \). Leave this expression in terms of variable names (\( r_\pi, g_m \), etc.). Assume \( V_A = \infty \).

\[
\begin{align*}
V_{be} &= -V_{in} \\
V_{out} &= -g_m V_{be} R_c \\
A_v &= g_m R_c
\end{align*}
\]
Problem 3 (30 Points): In this problem you will derive the small-signal (ac) model for a non-linear, 2-port element. For each part of this problem, use the device below with the given expressions for large-signal $I$-$V$ relationships. Show all your work.

\[
\begin{align*}
&\text{for } v_A > 0, i_A > 0, i_B > 0 \\
i_A &= K v_A \\
v_B &= \beta i_A^3 (1 + \rho i_B)
\end{align*}
\]

a) Derive an expression to model the small-signal resistance at Port A, $r_a = v_a/i_a$, for the DC operating point: $V_A, I_A, V_B, \text{and } I_B$. Show all your work.

\[
\begin{align*}
r_a &= \left. \frac{\partial v_a}{\partial i_A} \right|_{Q_{dc}} \\
&= \left( \frac{\partial i_A}{\partial v_A} \right)_{Q_{dc}}^{-1} \\
&= \frac{1}{K}
\end{align*}
\]
b) Derive an expression to model the small-signal resistance at Port B, $r_b = v_b/i_b$, for the DC operating point: $V_A$, $I_A$, $V_B$, and $I_B$. Show all your work.

\[ r_b = \left. \frac{\partial v_b}{\partial i_b} \right|_{DC} \]

\[ v_B = \beta i_A^3 (1 + \rho i_B) \]

\[ \left. \frac{\partial v_B}{\partial i_A} \right|_{DC} = \beta I_A^2 \rho_{DC}, \quad \beta I_A^2 \rho = r_b \]
c) Derive an expression to model the small-signal trans-resistance from Port A to Port B, $r_m = \frac{v_b}{i_a}$, for the DC operating point: $V_A, I_A, V_B,$ and $I_B$. Show all your work.

\[
\Gamma_m = \left. \frac{\partial V_B}{\partial i_A} \right|_{DC}
\]

\[
\frac{\partial V_B}{\partial i_A} = 3\beta i_A^2 \left( 1 + \rho i_B \right)
\]

\[
\Gamma_m = 3\beta i_A^2 \left( 1 + \rho i_B \right)
\]
d) Draw an equivalent circuit to model the small-signal response of the 2-port element.

\[
\begin{bmatrix}
V_a \\
V_b
\end{bmatrix} = \begin{bmatrix}
\Gamma_a & 0 \\
\Gamma_m & \Gamma_b
\end{bmatrix} \begin{bmatrix}
I_a \\
I_b
\end{bmatrix}
\]
Problem 4 (25 Points): Use the circuit below and parameters given in the table for each part of this problem.

\[
\begin{align*}
\beta_F &= 50 \\
V_{EB,ON} &= 0.6\text{V} \\
V_{EC,SAT} &= 0.4\text{V} \\
V_A &= \infty
\end{align*}
\]

a) For this part, assume the BJT is in the Saturation region. Use the simplified model for the BJT in saturation, using the constant-voltage-drop model for the E-B junction and \( V_{EC,SAT} \). Solve for the values of \( I_B \), \( I_C \), and \( V_{CB} \) and enter them in the space below.

\[
\begin{align*}
I_C &= 5.2\text{mA} - 0.4\text{mA} = 4.8\text{mA} \\
I_B &= 4.4\text{mA} - 0.4\text{mA} - I_C = -0.8\text{mA} \\
I_C &= \phantom{-}4.8\text{mA} \\
V_{CB} &= 0.2\text{V}
\end{align*}
\]
b) For this part, assume the BJT is in the FAR region. Use the simplified model for the BJT in FAR, using the constant-voltage-drop model for the E-B junction. Solve for the values of $I_B$, $I_C$, and $V_{CB}$ and enter them in the space below.

\[ I_B = \frac{V_E - V_C}{1k} + (\beta_F + 1)I_B \]

\[ \beta_F I_B + \frac{V_E - V_C}{1k} = \frac{V_C}{1k} \]

\[ I_B = 61.5 \text{mA} \]

\[ I_C = \beta_F I_B \]

\[ V_C = 5.6 - 1k(4.4 - (\beta_F + 1)I_B) \]

\[ = 4.34 \text{V} \]

\[ V_{CB} = V_C - 5 \text{V} \]

\[ I_B = \text{_______________________________} \]

\[ I_C = \text{_______________________________} \]

\[ V_{CB} = \text{_______________________________} \]
Based on your answers from parts a) and b), specify the correct region of operation of the BJT. Justify your answer.

**FAR**

1. In Sat model, $I_B < 0$ which is not allowed.
2. In FAR, $I_B, I_C > 0$ and $V_{CB} < 0$
(Space for additional work)