University of Michigan
EECS 311: Electronic Circuits
Fall 2009

Quiz 2

11/18/2009

NAME: Solutions

Honor Code:
I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code.

Signature ____

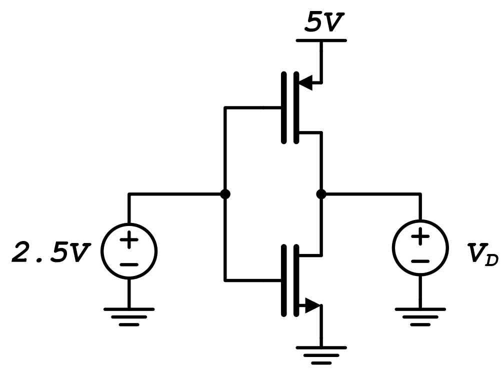
Problem	Points	Score	Initials
1	20		
2	25		
3	30		
4	25		
	Total		

Problem 1 (20 Points): Potpourri.

a) For the circuit below, find the range of V_D that will simultaneously bias the NMOS and PMOS in the Saturation region.

$V_{tn} = 1V$

$V_{tp} = -1V$



NMOS Sat

$V_{DS} > V_{GS} - V_{tn}$

$V_{GS} = 2.5V$

$V_{DS} = V_D$

$V_D > 1.5V$ (1)

PMOS Sat

$V_{DS} < V_{GS} - V_{tp}$

$V_{GS} = 2.5 - 5 = -2.5V$

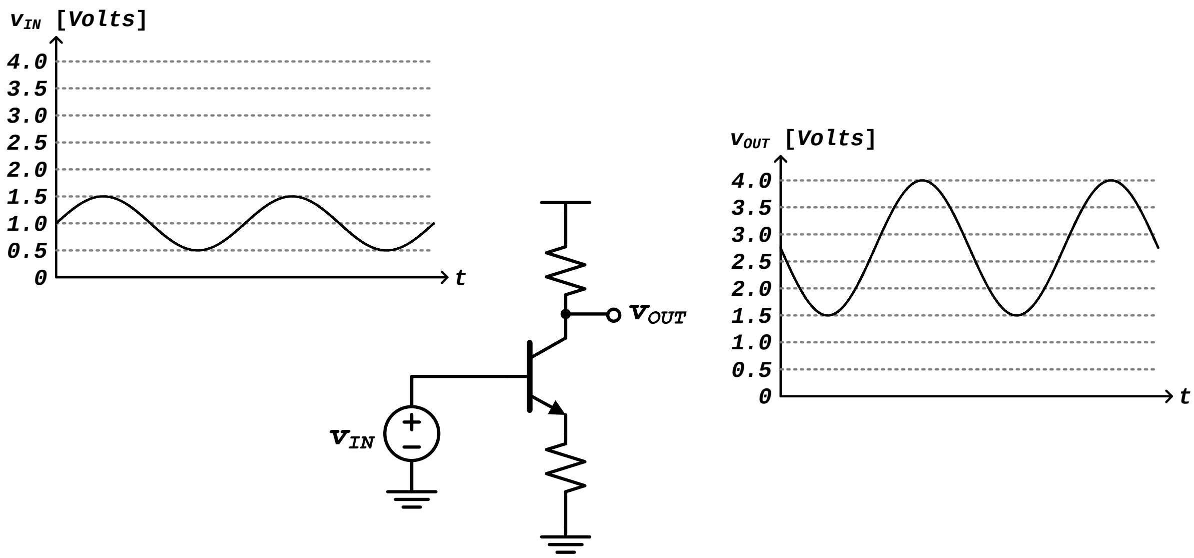
$V_{DS} = V_D - 5$

$V_D - 5 < -2.5 + 1$

$V_D < 3.5V$ (2)

$1.5 < V_D < 3.5$

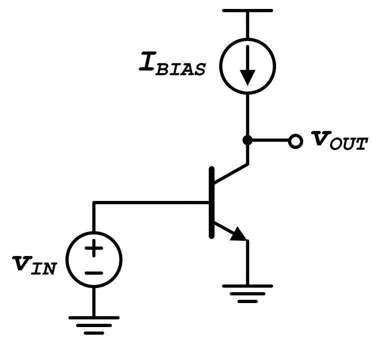
b) Determine the values of the DC and ac components of the waveforms shown below, assuming the signal are sinusoidal with frequency $\omega = \omega_0$. Additionally, evaluate the ac gain $a_v = v_{out}/v_{in}$ of the circuit.



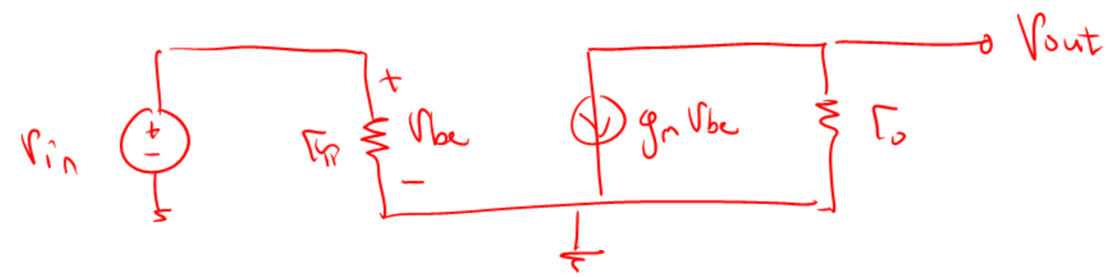
Total	=	DC Component	+	ac Component
v_{IN}	=	1 V	+	0.5 sin $\omega_0 t$
v_{OUT}	=	2.75 V	+	−1.25 sin $\omega_0 t$

ac Gain $a_v = v_{out}/v_{in} =$ −2.5 V/V

c) Assume the BJT is biased in the FAR region, and that I_{BIAS} is a DC bias source. Substitute the small-signal model for the BJT and find an expression for the small-signal gain $a_v = v_{out}/v_{in}$. Assume the BJT has finite V_A (include base-width modulation).



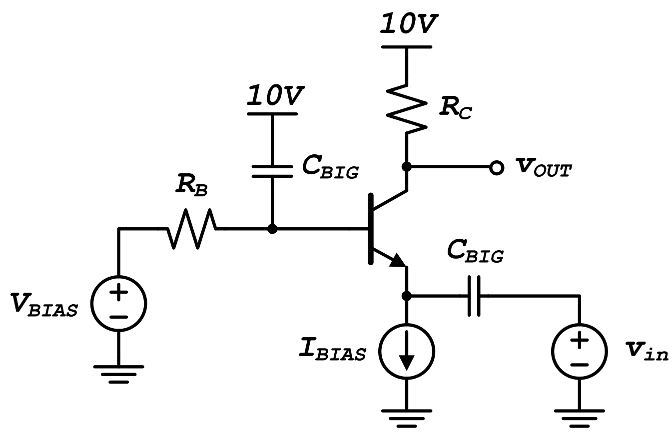
S.S. Ckt:



$$\frac{v_{out}}{v_{in}} = -g_m r_o$$

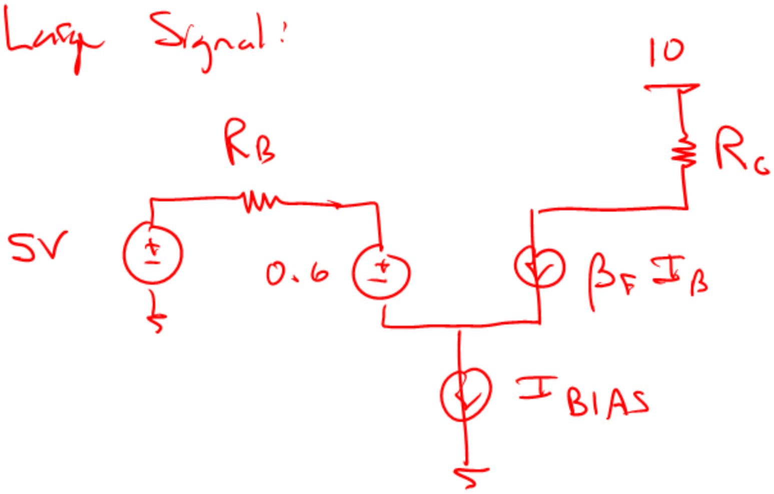
Problem 2 (25 Points): Use the circuit below and the values given in the table for all parts of this problem. The C_{BIG} capacitors should be treated as bypass capacitors.

$\beta_F = 100$
$V_A = \infty$
$V_{BE,ON} = 0.6V$
$V_{BIAS} = 5V$
$R_B = 10k\Omega$
$R_C = 1k\Omega$



a) Draw the DC equivalent circuit. Calculate the value of I_{BIAS} that will bias the BJT in the FAR region, at the boundary between FAR and Saturation.

Large Signal:



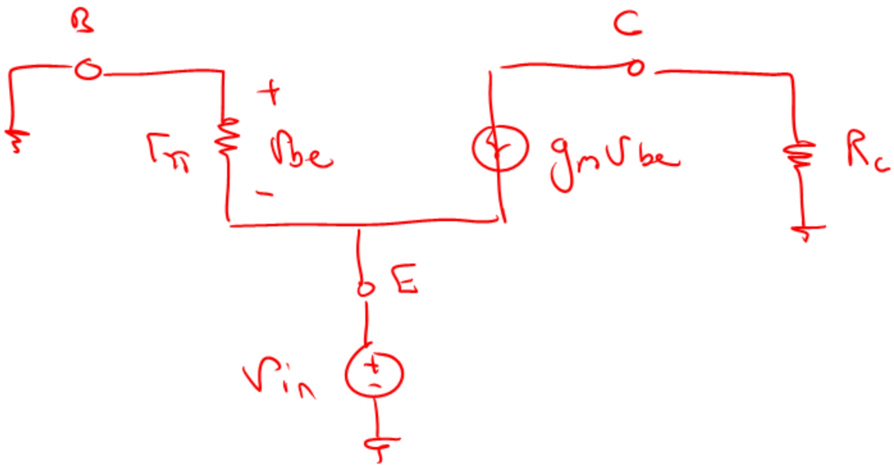
FAR/Sat Boundary : $V_{BC} = 0$

$V_B = V_C \Rightarrow 5 - I_B R_B = 10 - \beta_F I_B R_C$

$I_B = \dfrac{5}{\beta_F R_C - R_B}$ $\quad I_{BIAS} = (\beta_F + 1) I_B = 5.61mA$

b) For this part only, assume the bias current $I_C = 2.5mA$. Draw the small-signal (ac) circuit and evaluate the values of the elements in the BJT small-signal model assuming "room temperature". Assume $V_A = \infty$.

S.S. ckt:



$g_m = 100\ m℧$

$r_\pi = 1k\Omega$

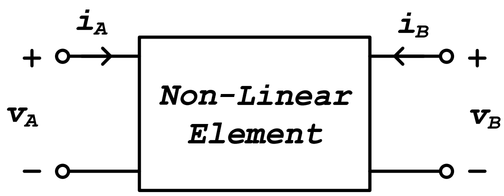
c) For this part, do NOT use the evaluated values of small signal elements found in part b). Derive an expression for the midband small-signal gain $a_v = v_{out}/v_{in}$. Leave this expression in terms of variable names (r_π, g_m, etc.). Assume $V_A = \infty$.

$v_{be} = -v_{in}$

$v_{out} = -g_m v_{be} R_C$

$$A_v = g_m R_C$$

Problem 3 (30 Points): In this problem you will derive the small-signal (ac) model for a non-linear, 2-port element. For each part of this problem, use the device below with the given expressions for large-signal I-V relationships. Show all your work.



for $v_A > 0$, $i_A > 0$, $i_B > 0$

$$i_A = K v_A$$

$$v_B = \beta i_A^3 (1 + \rho i_B)$$

a) Derive an expression to model the small-signal resistance at Port A, $r_a = v_a / i_a$, for the DC operating point: V_A, I_A, V_B, and I_B. Show all your work.

$$r_a = \left.\frac{\partial v_A}{\partial i_A}\right|_{DC} = \left(\left.\frac{\partial i_A}{\partial v_A}\right|_{DC}\right)^{-1} = \frac{1}{K}$$

b) Derive an expression to model the small-signal resistance at Port B, $r_b = v_b/i_b$, for the DC operating point: V_A, I_A, V_B, and I_B. Show all your work.

$$r_b = \left.\frac{\partial v_B}{\partial i_B}\right|_{DC}$$

$$v_B = \beta i_A^3 (1 + \rho i_B)$$

$$\left.\frac{\partial v_B}{\partial i_B}\right|_{DC} = \left.\beta i_A^3 \rho\right|_{DC} = \boxed{\beta I_A^3 \rho = r_b}$$

c) Derive an expression to model the small-signal trans-resistance from Port A to Port B, $r_m = v_b/i_a$, for the DC operating point: V_A, I_A, V_B, and I_B. Show all your work.

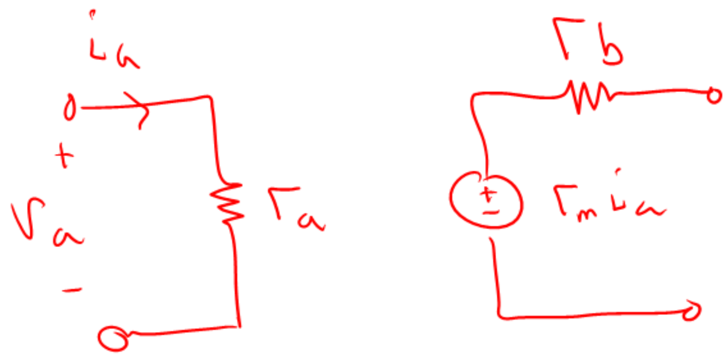
$$r_m = \left.\frac{\partial v_B}{\partial i_a}\right|_{DC}$$

$$\frac{\partial v_B}{\partial i_A} = 3\beta i_A^2 (1 + \rho i_B)$$

$$\boxed{r_m = 3\beta I_A^2 (1 + \rho I_B)}$$

d) Draw an equivalent circuit to model the small-signal response of the 2-port element.

$$\begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} r_a & 0 \\ r_m & r_b \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \end{bmatrix}$$



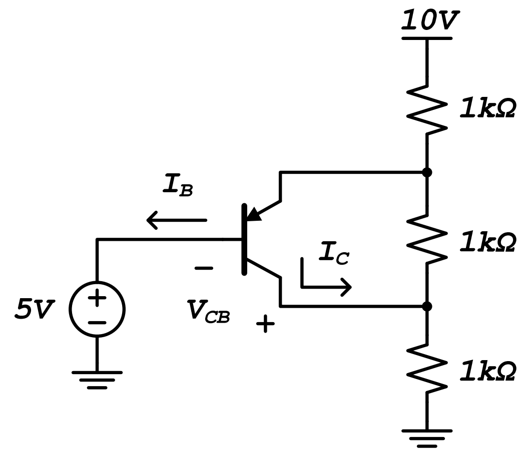
Problem 4 (25 Points): Use the circuit below and parameters given in the table for each part of this problem.

$\beta_F = 50$

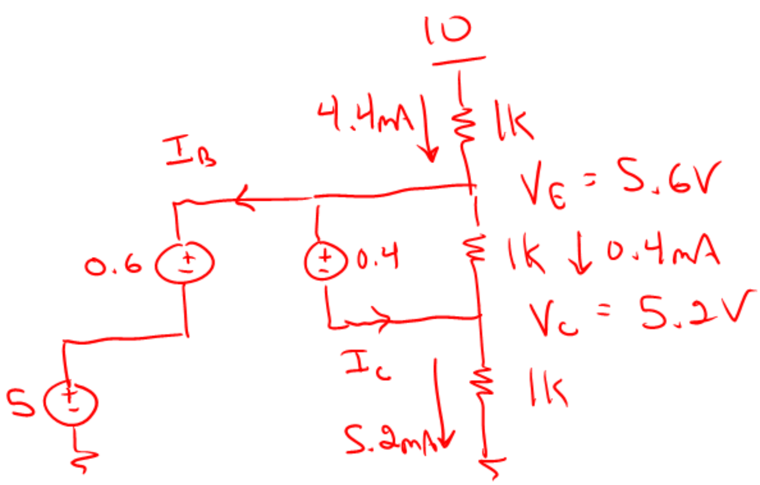
$V_{EB,ON} = 0.6V$

$V_{EC,SAT} = 0.4V$

$V_A = \infty$



a) For this part, assume the BJT is in the Saturation region. Use the simplified model for the BJT in saturation, using the constant-voltage-drop model for the E-B junction and $V_{EC,SAT}$. Solve for the values of I_B, I_C, and V_{CB} and enter them in the space below.



$I_c = 5.2m - 0.4m = 4.8mA$

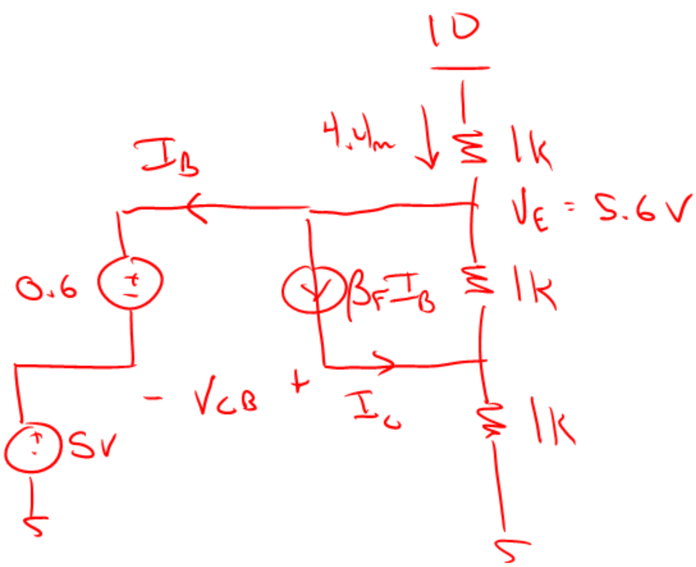
$I_B = 4.4m - 0.4m - I_c = -0.8mA$

$I_B =$ -0.8 mA

$I_C =$ 4.8mA

$V_{CB} =$ 0.2 V

b) For this part, assume the BJT is in the FAR region. Use the simplified model for the BJT in FAR, using the constant-voltage-drop model for the E-B junction. Solve for the values of I_B, I_C, and V_{CB} and enter them in the space below.



$$4.4m = \frac{V_E - V_C}{1k} + (\beta_F + 1) I_B$$

$$\beta_F I_B + \frac{V_E - V_C}{1k} = \frac{V_C}{1k}$$

$$\Rightarrow I_B = 61.5 \mu A$$

$$I_C = \beta_F I_B$$

$$V_C = 5.6 - 1k(4.4 - (\beta_F + 1) I_B)$$

$$\approx 4.34V$$

$$V_{CB} = V_C - 5V$$

$I_B =$ 61.5 μA

$I_C =$ 3.08 mA

$V_{CB} =$ −0.66 V

c) Based on your answers from parts a) and b), specify the correct region of operation of the BJT. Justify your answer.

FAR

① In Sat model, $I_B < 0$ which is not allowed

② In FAR, $I_B, I_C > 0$ and $V_{CB} < 0$

(Space for additional work)