EECS 311 PS2 Solution

P2.1

a) $$\boxed{\frac{hs}{s^2+ds+1}}$$ #

b) $\boxed{\text{Band pass filter}}$ #

c) $$\boxed{h=\frac{-k}{T_2}}$$ #

d) $$\boxed{T_1T_2=1+k}$$ #

e)

Formula group $\boxed{\text{VII}}$ #

formula for $d(x,K,T)$

$$\Rightarrow \boxed{\frac{1}{T_1}+\frac{T_1(1+x)}{(1+k)}}$$ #

$\boxed{x \text{ is } \gamma \text{ in this equation}}$ #

f) formula for $K(x,d,T)$

$$\boxed{\frac{T_1^2(1+x)+1-dT_1}{dT_1-1}}$$ #

$x=\gamma$

P2.2 Specify γ & T_1 Formula Group IV

$$d = \frac{1}{T_1} + T_1(1+\gamma-K)$$

a) $$H(s) = \frac{1}{\left(1+\frac{s}{10^2}\right)\left(1+\frac{s}{10^4}\right)} = \frac{1}{1+s\left(\frac{1}{10^2}+\frac{1}{10^4}\right)+s^2\frac{1}{10^6}}$$

$$\omega_n = 10^3 \qquad d = 10^3\left(\frac{1}{10^2}+\frac{1}{10^4}\right) = \underline{10.1}$$

$\gamma = 1 \quad K = 1$ (from numerator of $H(s)$)

$$d = \frac{1}{T_1} + T_1 = 10.1 \qquad T_1 = \{10, 0.1\}$$

choose $T_1 = 10 \qquad T_2 = \frac{1}{T_1} = 0.1$

$$\boxed{\begin{aligned} C_1' &= C_2' = 10\,nF \\ R_1' &= 1\,M\Omega \\ R_2' &= 10\,k\Omega \end{aligned}}$$

$$T_1 = R_1' C_1' \omega_n$$

$$R_1' = \frac{T_1}{C_1' \omega_n} = \frac{10}{10^{-8}\, 10^3}$$

b) $H(s) = \dfrac{10}{1 + \frac{s}{10^6} + \frac{s^2}{10^{10}}}$ $\omega_n = 10^5$

$K = 10$

$\gamma = 1$

$d = \dfrac{10^5}{10^6} = 0.1$

$0.1 = \dfrac{1}{T_1} + T_1(1 + 1 - 10)$ $T_1 = 0.35$

$T_2 = \dfrac{1}{T_1} = 2.9$

$$\boxed{\begin{aligned} C_1' &= C_2' = 10\,nF \\ R_1' &= 350\,\Omega \\ R_2' &= 2.9\,k\Omega \end{aligned}}$$

$R_1' = \dfrac{T_1}{C_1' \omega_n} = \dfrac{0.35}{10^{-8} \cdot 10^5}$

c) $H(s) = \dfrac{1}{\left(1 + \frac{s}{100+j100}\right)\left(1 + \frac{s}{100-j100}\right)} = \dfrac{1}{1 + s\frac{1}{10^2} + \frac{s}{2 \cdot 100^2}}$

$\omega_n = 100\sqrt{2}$

$d = 100\sqrt{2} \cdot \dfrac{1}{100} = \sqrt{2}$

$k = 1, \quad \gamma = \frac{1}{2}$

$d = \dfrac{1}{T_1} + T_1\left(1 + \frac{1}{2} - 1\right)$

$T_1 = \sqrt{2} = 1.4$

$T_2 = \dfrac{1}{\sqrt{2}} = 0.7$

$$\boxed{\begin{aligned} &C_1' = 20\,nF \quad C_2' = 10\,nF \\ &R_1' = 500\,k\Omega \\ &R_2' = 500\,k\Omega \end{aligned}}$$

$R_1' = \dfrac{T_1}{C_1' \omega_n} = \dfrac{\cancel{\sqrt{2}}}{10^{-8}\, 100\, \cancel{\sqrt{2}}} = 10^6$

$R_2' = \dfrac{T_2}{C_2' \omega_n} = \dfrac{1}{\sqrt{2} \cdot 2 \cdot 10^{-8} \sqrt{2} \cdot 100} = \frac{1}{4} \cdot 10^6$

d) $H(s) = \dfrac{10^3}{10^4 + 200s + s^2} = \dfrac{0.1}{1 + \frac{2}{10^2}s + \frac{s}{10^4}}$

$K = 0.1 \qquad d = 2 \qquad \rho = 0.1 \qquad \omega_n = 10^2$

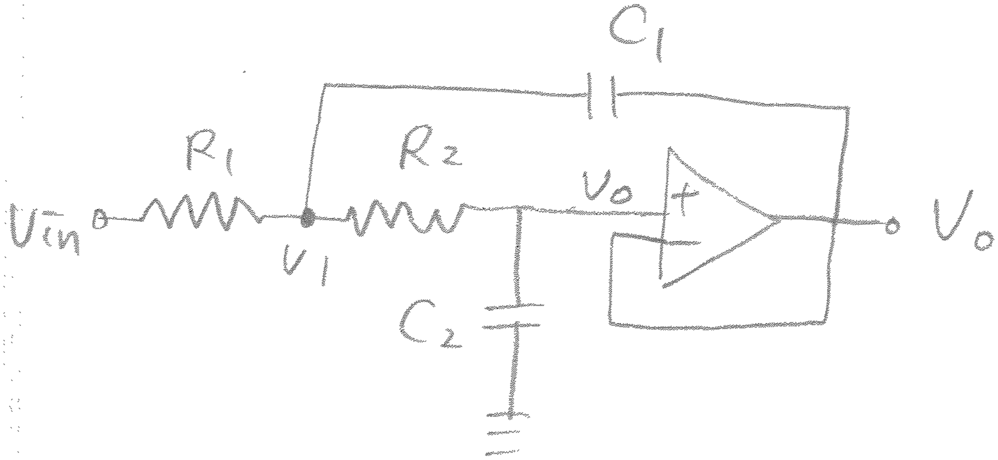
$2 = \dfrac{1}{T_1} + T_1(1 + 0.1 - 0.1) \qquad T_1 = T_2 = 1$

$$\boxed{\begin{array}{ll} C_1' = 100\text{nF} & C_2' = 10\text{nF} \\ R_1' = 100\text{k}\Omega & R_2' = 1\text{M}\Omega \end{array}}$$

$R_1' = \dfrac{T_1}{C_1' \omega_n} = \dfrac{1}{10^{-8} \cdot 10^2}$

$R_2' = \dfrac{T_2}{C_2' \omega_n} = \dfrac{1}{10^{-7}\, 10^2}$

P2.3



a) @ V_1:

$$\frac{V_{in}-V_1}{R_1}+SC(V_o-V_1)+\frac{V_o-V_1}{R_2}=0 \quad —①$$

@ V^+:

$$\frac{V_1-V_o}{R_2}=SC_2V_o \Rightarrow V_1=V_o(SR_2C_2+1) \quad —②$$

plug ② into ①:

$$\frac{V_{in}-V_o(1+SR_2C_2)}{R_1}+SC_1V_o(-SR_2C_2)=V_oSC_2$$

$$V_{in}-V_o(1+SR_2C_2)=V_o(SR_1C_2+S^2R_1R_2C_1C_2)$$

$$V_{in}=V_o(SC_2(R_1+R_2)+S^2R_1R_2C_1C_2+1)$$

$$\boxed{\frac{V_o}{V_{in}}=\frac{1}{R_1R_2C_1C_2S^2+(R_1+R_2)C_2S+1}}$$ #

b) $\boxed{\text{Low-Pass filter}}$ #

c) $R_1 = R_2$, $C_1 = 100C_2$

$$H(s) = \frac{1}{R_1R_2C_1C_2S^2 + C_2(R_1+R_2)S + 1} = \frac{1}{1 + S\frac{1}{\omega_n Q} + S^2\frac{1}{\omega_n^2}}$$

$$\Rightarrow \frac{1}{\omega_n^2} = R_1R_2C_1C_2 , \quad C_2(R_1+R_2) = \frac{1}{\omega_n Q}$$

$$\Rightarrow \omega_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$\boxed{Q = \frac{\sqrt{R_1R_2C_1C_2}}{C_2(R_1+R_2)}} \#$$

$R_1 = R_2$, $C_1 = 100C_2$

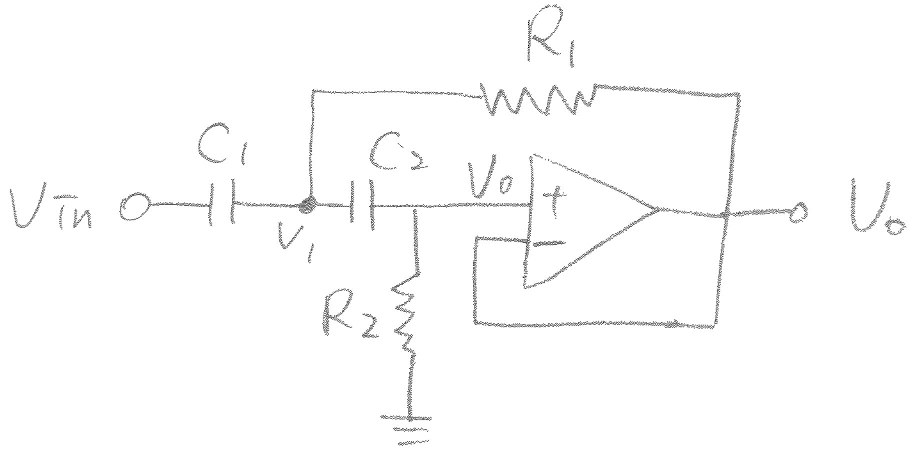
$$Q = \frac{\sqrt{R_1 R_1 100\, C_2 C_2}}{\cancel{C_2}(R_1+R_1)} = \frac{10}{2} = 5$$

d) $$H(s) = \frac{1}{\left(\frac{s}{\omega_n}\right)^2 + \frac{s}{\omega_n}\times\frac{1}{Q} + 1}$$

$$d = \frac{1}{Q} = \frac{C_2(R_1+R_2)}{\sqrt{R_1R_2C_1C_2}} = \frac{C_2 \times 2R_2}{\sqrt{R_2R_2 100C_2C_2}}$$

$$= \frac{2\cancel{R_2}\cancel{C_2}}{10\cancel{R_2}\cancel{C_2}} = \boxed{\frac{1}{5}} \#$$

P2.4 P.8



a) @ V_1:

$$\frac{V_{in}-V_1}{\frac{1}{sC_1}}+\frac{V_o-V_1}{R_1}+\frac{V_o-V_1}{\frac{1}{sC_2}}=0 \quad \text{— ①}$$

@ V^+:

$$\frac{V_1-V_o}{\frac{1}{sC_2}}=\frac{V_o}{R_2} \Rightarrow V_1=\left(1+\frac{1}{sR_2C_2}\right)V_o \quad \text{— ②}$$

plug ② into ①

$$sC_1\left[V_{in}-\left(1+\frac{1}{sR_2C_2}\right)V_o\right]-\frac{V_o}{sR_1R_2C_2}-\frac{V_o}{R_2}=0$$

$$\Rightarrow V_{in}-\left(1+\frac{1}{sR_2C_2}\right)V_o=\frac{1+sR_1C_2}{s^2R_1R_2C_1C_2}V_o$$

$$\Rightarrow V_{in}=\frac{1+sR_1(C_1+C_2+sR_2C_1C_2)}{s^2R_1R_2C_1C_2}V_o$$

$$\Rightarrow \boxed{H(s)=\frac{V_o}{V_{in}}=\frac{s^2R_1R_2C_1C_2}{R_1R_2C_1C_2s^2+R_1(C_1+C_2)s+1}}$$

#

b) $\boxed{\text{High-pass filter}}$ #

P.9

c) $R_1 = R_2$, $C_1 = C_2$, $\rho = \frac{R_1}{R_2} = 1$, $\gamma = \frac{C_2}{C_1} = 1$

Choose parameters ρ, T_1 $\Rightarrow$ group II

$\chi = \gamma = 1$, $T = T_1 = R_1 C_1$

from (b),

$$H(s) = \frac{s^2}{s^2 + \frac{R_1C_1 + R_1C_2}{R_1R_2C_1C_2} s + \frac{1}{R_1R_2C_1C_2}}$$

$\Rightarrow h = 1$, $d = \frac{R_1C_1 + R_1C_2}{R_1R_2C_1C_2}$, $R_1R_2C_1C_2 = 1$

$\because R_1 = R_2$, $C_1 = C_2$

$\therefore R_1^2 C_1^2 = 1 \Rightarrow R_1C_1 = 1$, $R_2C_2 = 1$

$\Rightarrow d = \frac{1+1}{1} = \boxed{2}$ #

P2.5

a)

$$H(s) = \frac{hs}{\left(1+\frac{s}{2\pi 100Hz}\right)\left(1+\frac{s}{2\pi 10kHz}\right)}$$

$$= \frac{hs}{\frac{s^2}{4\pi^2 100\times 10k} + \left(\frac{1}{2\pi 10k}+\frac{1}{2\pi 100}\right)s + 1}$$

$$= \frac{hs}{\left(\frac{s}{\omega_n}\right)^2 + d\left(\frac{s}{\omega_n}\right) + 1}$$

$$\omega_n^2 = 4\pi^2 \times 100 \times 10k \Rightarrow \boxed{\omega_n = 6283.19} \ \#$$

$$\frac{d}{\omega_n} = \frac{1}{2\pi\times 10k} + \frac{1}{2\pi \times 100} \Rightarrow \boxed{d = 10.1} \ \#$$

b) since $c_1 = c_2 \Rightarrow \gamma = \frac{c_2}{c_1} = 1$, $x = \gamma = 1$

We choose $(\gamma, T_2) \Rightarrow$ group VIII

$$K_{min} = \frac{4(1+x)-d^2}{d^2} = \frac{4(1+1)-10.1^2}{10.1^2} < 0$$

If we set $k = 1$, then

$$d = \frac{1+r}{T_2} + \frac{T_2}{1+k}$$

$$T_2 = \frac{d}{2}\left[1 \pm \sqrt{1-\frac{4(1+x)}{d^2(1+k)}}\right]$$

$$= \frac{10.1}{2}\left[1 \pm \sqrt{1-\frac{4(1+1)}{10.1^2(1+1)}}\right]$$

$$= \underset{(\times)}{10} \text{ or } \boxed{0.1} \ \#$$

$$\therefore T_1 T_2 = 1 \Rightarrow T_1 = \boxed{10.} \ \#$$

$$\boxed{T_2 = 0.2}$$

$$\frac{T_1 T_2}{1+k} = 1$$

$$\boxed{T_1 = \frac{2}{0.2} = 10}$$

$$\boxed{\rho = 50}$$

c) $C_1' = C_2' = 10nF$

$R_1' = 159k\Omega$

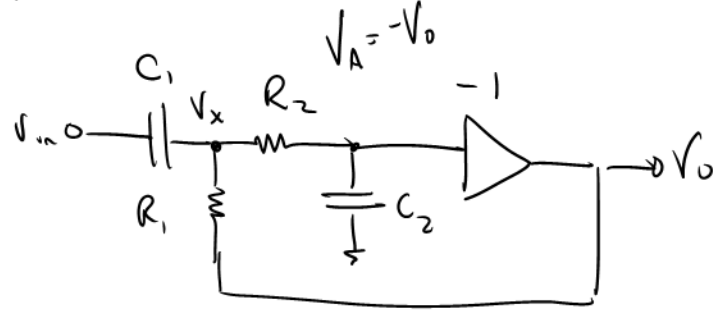
$R_2' = 3.2k\Omega$

$R_1' = \frac{T_1}{C_1' \omega_n} = \frac{10}{10^{-8} 2\pi 1k}$

$R_2' = \frac{T_2}{C_2' \omega_n} = \frac{0.2}{10^{-8} 2\pi 1k}$

$K = 1$

d)



$$\frac{V_{in} - V_x}{1/sC_1} + \frac{V_0 - V_x}{R_1} + \frac{0 - (-V_0)}{1/sC_2} = 0$$

$$(V_{in} - V_x) sR_1C_1 + (V_0 - V_x) + V_0 sR_1C_2 = 0$$

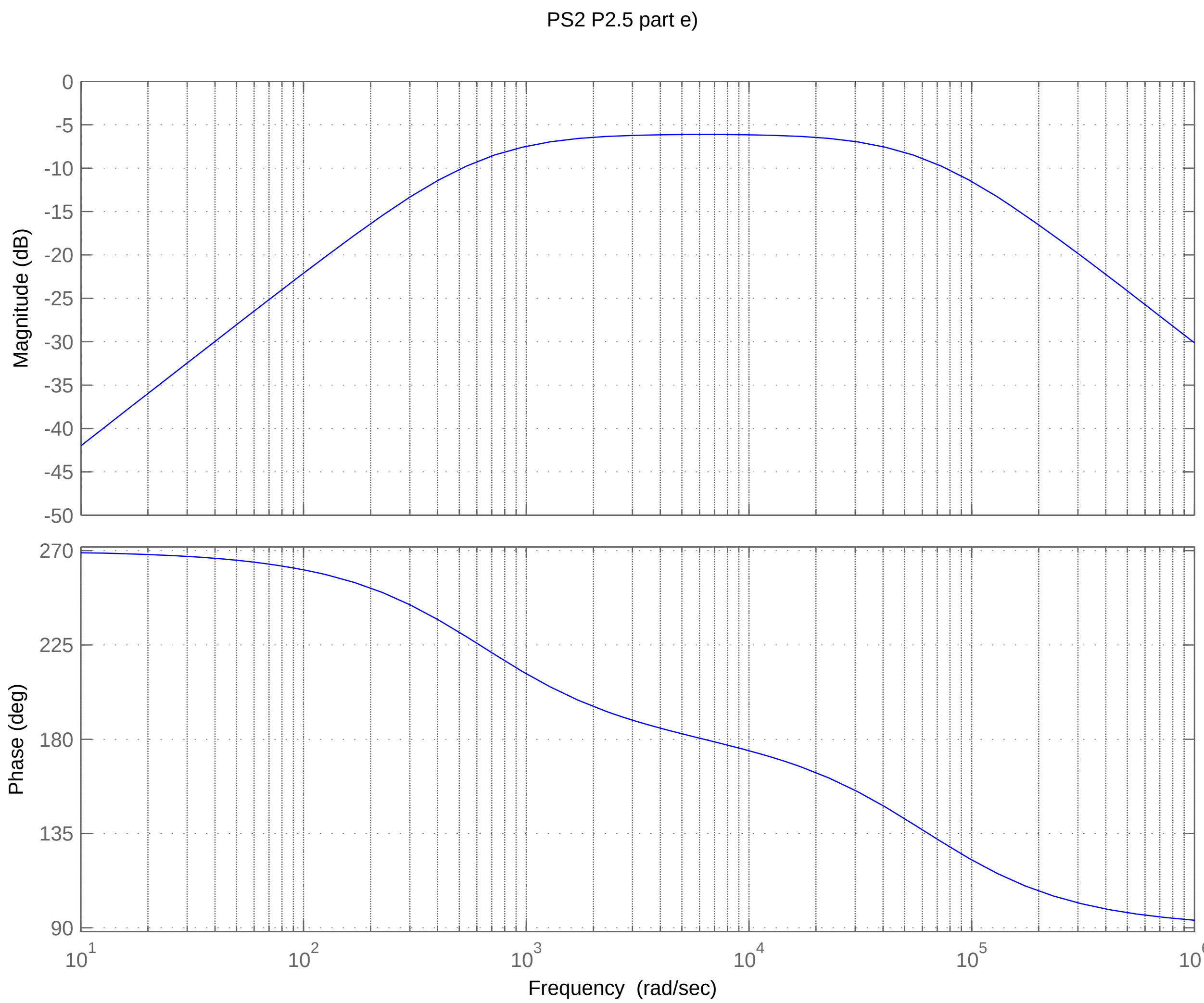
$$\frac{V_x - (-V_0)}{R_2} + \frac{0 - (-V_0)}{1/sC_2} = 0$$

$$V_x + V_0 + V_0 sR_2C_2 = 0$$

$$V_x = -V_0(1 + sR_2C_2)$$

$$V_{in} sR_1C_1 + V_0(1 + sR_2C_2) sR_1C_1 + V_0(2 + sR_2C_2) + V_0 sR_1C_2 = 0$$

$$\frac{V_0}{V_{in}} = \frac{sR_1C_1}{2 + s(R_1C_1 + R_2C_2 + R_1C_2) + s^2 R_1C_1R_2C_2}$$


PS2 P2.5 part e)
Magnitude (dB)
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
Phase (deg)
270
225
180
135
90
10^1
10^2
10^3
10^4
10^5
10^6
Frequency (rad/sec)