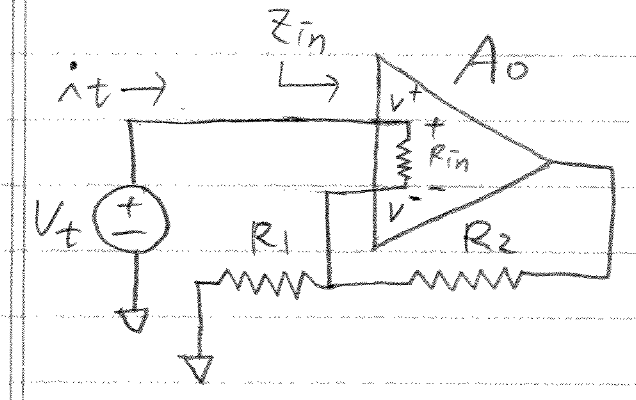
EECS 311 PS3 Solution.

P3.1



$V_o = A_o(V^+ - V^-)$

$V_t = V^+$ — ①

$V^- = V^+ - i_t R_{in}$ — ②

$i_t + \frac{V_o - V^-}{R_2} = \frac{V^-}{R_1}$ — ③

plug ①, ② into ③

$$i_t + \frac{i_t R_{in} A_o - (V_t - i_t R_{in})}{R_2} = \frac{V_t - i_t R_{in}}{R_1}$$

$$i_t \left(1 + \frac{A_o R_{in}}{R_2} + \frac{R_{in}}{R_2} + \frac{R_{in}}{R_1}\right) = V_t \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$i_t \left(\frac{R_1R_2 + A_oR_1R_{in} + R_1R_{in} + R_2R_{in}}{R_1\cancel{R_2}}\right) = V_t \frac{R_1 + R_2}{R_1\cancel{R_2}}$$

$$\Rightarrow R_{in} = \frac{R_1R_2 + R_1R_{in} + R_2R_{in} + A_oR_1R_{in}}{R_1 + R_2} \quad (R_{in} \gg R_1, R_2)$$

$$\Rightarrow R_{in} \approx \frac{R_{in}(R_1 + R_2 + A_oR_1)}{R_1 + R_2}$$

$$\approx \boxed{R_{in}\left(1 + \frac{A_o R_1}{R_1 + R_2}\right)}$$

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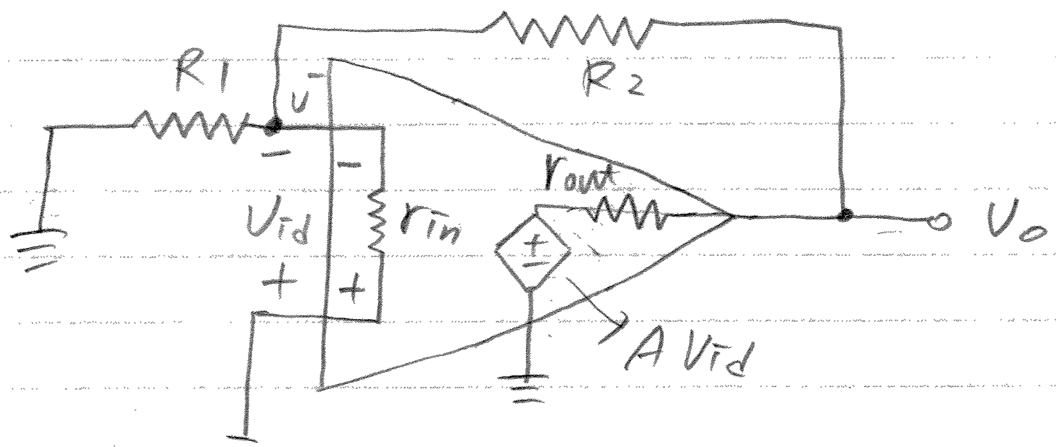
P3.2
a)



$R_1 = 5.6\,k\Omega$, $R_2 = 47\,k\Omega$, $A = 100\,dB = 10^5$

$r_{in} = 400\,k\Omega$, $r_{out} = 200\,\Omega$, $\beta = \frac{R_1}{R_1+R_2} = 0.106$

$$\Rightarrow R_{in} = R_1 + \left(r_{in} \parallel \frac{R_2}{1+A}\right)$$

$$= 5.6k + \left(400k \parallel \frac{47k}{1+10^5}\right)$$

$$\simeq \boxed{5.6\,k\Omega} \ \#$$

$$\frac{V_o - 0}{R_2} = \frac{0 - V_x}{R_1} \Rightarrow \frac{V_o}{R_2} = \frac{-V_x}{R_1}$$

$$\Rightarrow A_v = \frac{V_o}{V_x} = -\frac{R_2}{R_1} = -\frac{47}{5.6} = \boxed{-8.39} \ \#$$

$$\Rightarrow R_{out} = \frac{R_o}{1+A\beta} = \frac{200}{1+10^5 \times 0.106} = \boxed{18.86\,m\Omega} \ \#$$

P 3.3

Appendix A3
Effect of resistor tolerance on CMRR of one amplifier differential circuit

In Figure A3.1 the amplifier is assumed ideal, resistors have tolerance 100 × per cent per cent and worst case CMRR is considered. An input common mode signal e_{cm} gives rise to an output signal

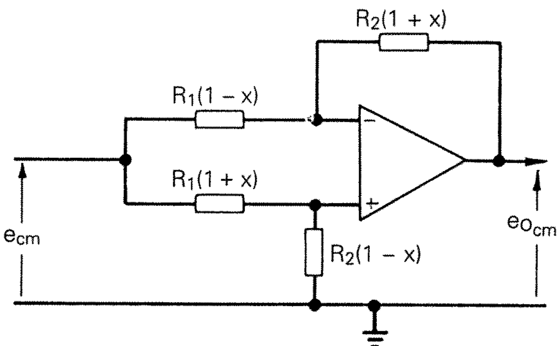


Figure A3.1 *CMRR due to resistor tolerance with worst case distribution*

$$e_{o_{cm}} = e_{cm}\left[\frac{R_2(1-x)}{R_2(1-x) + R_1(1+x)}\,\frac{R_1(1-x) + R_2(1+x)}{R_1(1-x)} - \frac{R_2(1+x)}{R_1(1-x)}\right]$$

$$= e_{cm}\frac{R_2}{R_1}\left[\frac{R_1(1-x) + R_2(1+x)}{R_2(1-x) + R_1(1+x)} - \frac{1+x}{1-x}\right]$$

$$= e_{cm}\frac{R_2}{R_1}\,\frac{R_1\,4x}{R_2(1-x)^2 + R_1(1+x^2)}$$

$$\cong e_{cm}\frac{R_2}{R_1}\,\frac{4x\,R_1}{R_2 + R_1}$$

Thus common mode gain

$$\frac{e_{o_{cm}}}{e_{cm}} = \frac{R_2}{R_1} \frac{4x R_1}{R_2 + R_1}$$

$$\text{and} \quad \text{CMRR} = \frac{\text{differential gain}}{\text{common mode gain}} \cong \frac{\dfrac{R_2}{R_1}}{\dfrac{R_2}{R_1} \dfrac{4x R_1}{R_2 + R_1}}$$

$$\text{CMRR} \cong \frac{1 + \dfrac{R_2}{R_1}}{4x} \qquad \text{(A3.1)}$$

A3.1 CMRR of one amplifier differential circuit due to non-infinite CMRR of operational amplifier

Common mode signal applied to op-amp

$$= e_{cm} \frac{R_2}{R_1 + R_2} \qquad \text{(see Figure A3.2)}$$

Non-infinite CMRR of an op-amp is represented by an equivalent input error signal $e_{\epsilon_{cm}}$ applied directly to the input terminal of the op-amp

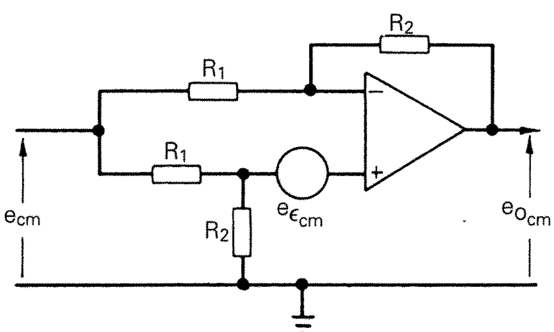


Figure A3.2 *CMRR of circuit due to non-infinite CMRR of amplifier*

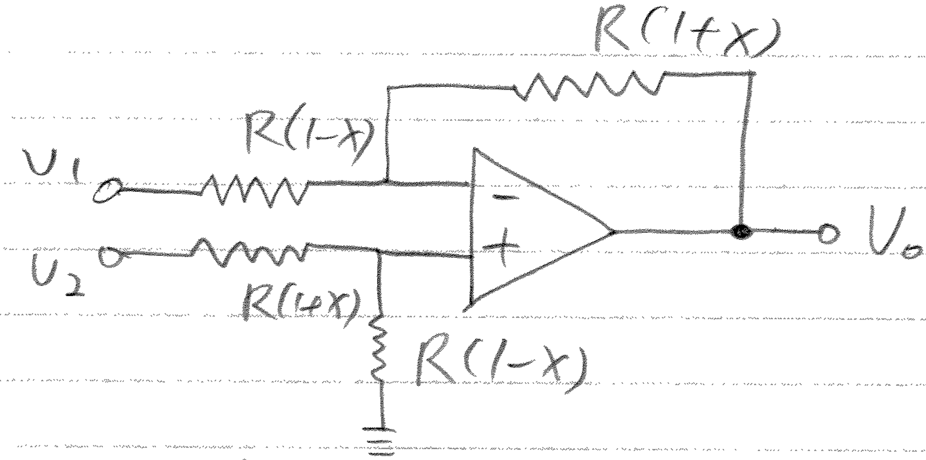
$$e_{\epsilon_{cm}} = \frac{e_{cm} \dfrac{R_2}{R_1 + R_2}}{\text{CMRR}_{(A)}}$$

$e_{\epsilon cm}$ gives an output signal

$$e_{o_{cm}} = e_{\epsilon_{cm}} \left(1 + \frac{R_2}{R_1}\right)$$

P3.3

Worst case consideration



$x = 0.05\%$

CMRR (due to resistor tolerance)

$$= \frac{1 + \frac{R_2}{R_1}}{4x} = \frac{1+1}{4 \times 0.05\%} = 1000 = \boxed{60\,dB}$$

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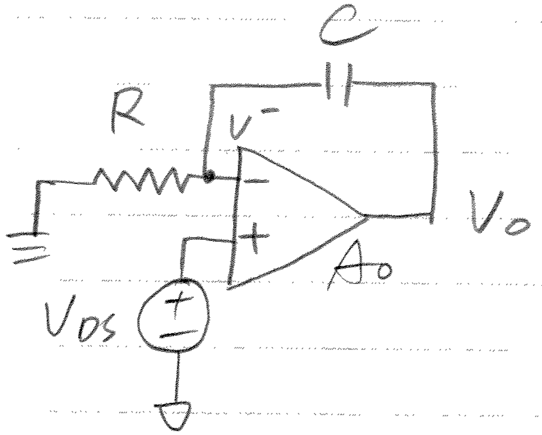
P3.4



a)

$$V_o = A_o(V_{OS} - V^-)$$

$$\Rightarrow V^- = V_{OS} - \frac{V_o}{A_o}$$

$$\frac{V_o - V^-}{\frac{1}{sC}} = \frac{V^-}{R}$$

Substitute V^-

$$\Rightarrow V^-\left(\frac{1}{R} + sC\right) = sC V_o$$

$$\Rightarrow \left(V_{OS} - \frac{V_o}{A_o}\right)\left(\frac{1}{R} + sC\right) = sC V_o$$

$$\Rightarrow V_{OS} - \frac{V_o}{A_o} = \frac{sRC V_o}{1+sRC}$$

$$\Rightarrow V_{OS} = V_o\left(\frac{sRC}{1+sRC} + \frac{1}{A_o}\right) = V_o\left(\frac{sRCA_o + 1 + sRC}{A_o(1+sRC)}\right)$$

$$\Rightarrow \frac{V_o}{V_{OS}} = \frac{A_o(1+sRC)}{1+sRC+sRCA_o} = \boxed{\frac{A_o(1+sRC)}{1+sRC(1+A_o)}}$$ #

b) at DC,

$$\frac{V_o}{V_{OS}} = \frac{A_o}{1} = A_o$$

$$\therefore \boxed{V_o = A_o V_{OS}}$$ #

P3.5 Both inverting and noninverting amplifiers will have the same output resistance.

$$Z_{out} \cong \frac{R_o}{1+A\beta}$$

$$A(s) = \frac{A_o}{1+\frac{s}{\omega_B}}$$ where ω_B = open loop bandwidth of the op amp.

$$Z_{out} = \frac{R_o}{1+\frac{A_o\beta}{1+\frac{s}{\omega_B}}} = \boxed{\frac{R_o(s+\omega_B)}{s+\omega_B(1+A_o\beta)}}$$

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