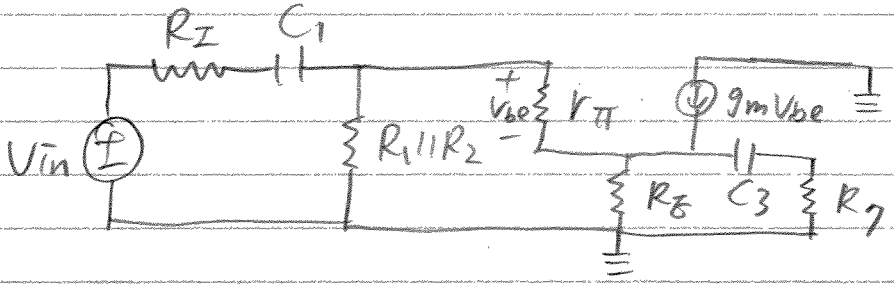
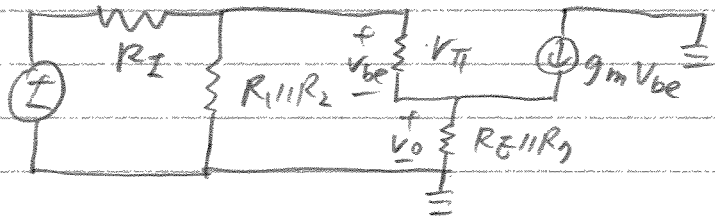
EECS 311 ps9 Solution

p7.1

a) Low frequency



midband:



b) SCTC:

C_1: $R_{1,eq} = R_I + [R_1 \| R_2 \| (r_\pi + (1+g_m r_\pi) R_E]$

C_3: $R_{3,eq} = R_3 + (R_E \| \frac{1}{g_m})$

$C_1 = 4.7\mu F$, $C_3 = 10\mu F$, $R_1 = 100k\Omega$, $R_2 = 300k\Omega$

$R_3 = 13k\Omega$, $R_I = 2k\Omega$, $R_3 = 100k\Omega$

$g_m = \frac{I_C}{V_T}$; $\beta_0 = g_m r_\pi = 100$, $I_C = 0.25mA$, $V_{CC} = 12V$

$$f_L = \left(\frac{1}{R_{1,eq} C_1} + \frac{1}{R_{3,eq} C_3}\right) \frac{1}{2\pi} \approx \boxed{0.623\,Hz} \quad \#$$

$$A_V = \frac{R_{in}}{R_I + R_{in}} \frac{(1+\beta_0)(R_E \| R_3)}{r_\pi + (1+\beta_0)(R_E \| R_3)} \approx \boxed{0.984\ V/V} \quad \#$$

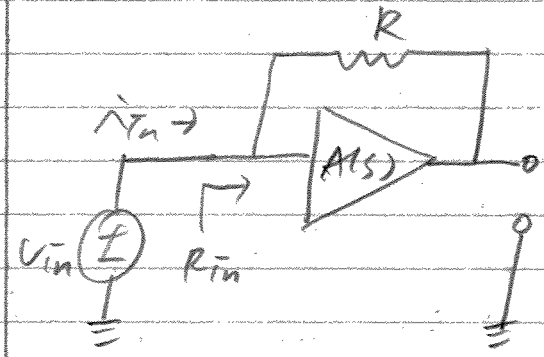
P7.2

$$\omega_L = \frac{1}{R_{1,eq}C_1} + \frac{1}{R_{3,eq}C_3} = 2\pi f_L = 20\pi$$

P.2

$$C_3 = \frac{\left(20\pi - \frac{1}{R_{1,eq}C_1}\right)^{-1}}{R_{3,eq}} \approx \boxed{0.187\,\mu F} \quad \#$$

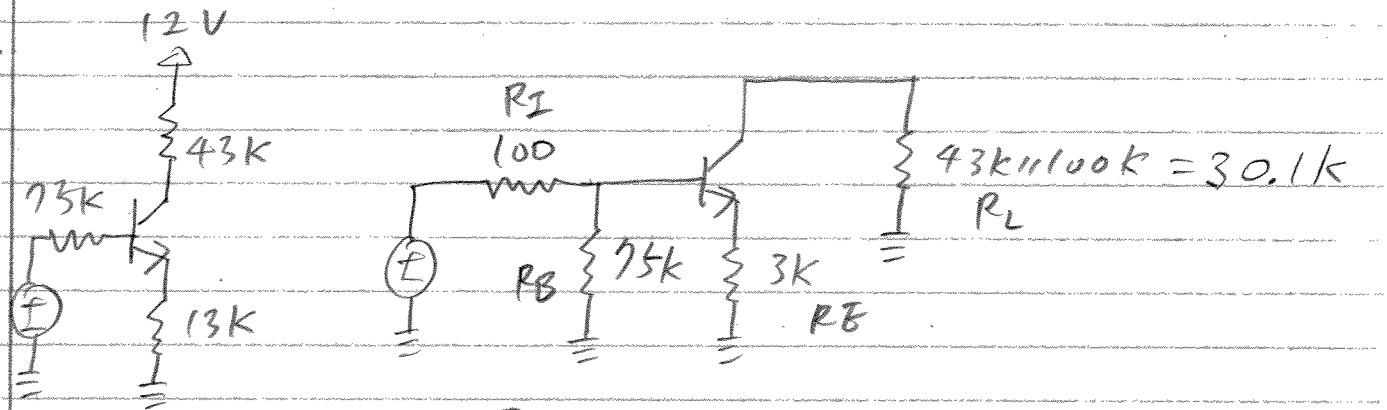
P7.3



$$R_{in} = \frac{V_{in}}{i_{in}} = \boxed{\frac{R}{1-A(s)}} \quad \#$$

$$i_{in} = \frac{V_{in} - V_{out}}{R} = \frac{V_{in}(1-A(s))}{R}$$

P7.4



$$I_C = 100 \times \frac{3 - 0.7}{75K + 101 \times 13K} = 0.166\,mA$$

$$V_{CE} = 12 - 43K \times I_C - 13K\,I_E = 2.68\,V$$

$$g_m = \frac{1}{75m} \times 0.166m = 6.64\,mS$$

$$r_\pi = \frac{100}{6.64m} = 15.1\,k\Omega$$

$$C_\pi = \frac{g_m}{\omega_T} - C_\mu = \frac{6.64m}{2\pi \times 310 \times 10^6} - 0.5 \times 10^{-12} = 3.52\,pf$$

OCTC

From Table 16.2

$r_{\pi O} = r_\pi \,//\, [r_x + (R_I \,//\, R_B)]$, $R_B = R_1 \,//\, R_2$

$= 15.1k \,//\, (300 + (100 \,//\, 75k)) = 390\,\Omega$

$$C_{TB} = \frac{C_\pi}{1+g_m R_E} + C_\mu\left(1 + \frac{g_m R_L}{1+g_m R_E}\right) + (C_u + C_L)\frac{R_L}{r_{\pi O}}$$

$$= \frac{3.52p}{1+6.64m\times 3k} + 0.5p\left[\left(1 + \frac{6.64m\times 30.1k}{1+6.64m\times 3k}\right) + \left(\frac{30.1k\Omega}{390}\right)\right]$$

$\approx 44\,pH$

$$f_H = \frac{1}{2\pi(390)(44p)} = 9.27\,MHz$$

SCTC

$R_{1S} = 100 + 75k \,//\, [300 + 15.1k + (101\times 3k] = 60.8k\Omega$

$R_{xS} = 10k \,//\, \left(3k + \frac{15.1k + 99.9}{101}\right) = 2.4k\Omega$

$R_{3S} = 43k + 100k = 143k\Omega$

$$f_L \approx \frac{1}{2\pi}\left[\frac{1}{(60.8k)(1\mu H)} + \frac{1}{(2.4k)(2.2\mu)} + \frac{1}{(143k)(0.1\mu)}\right]$$

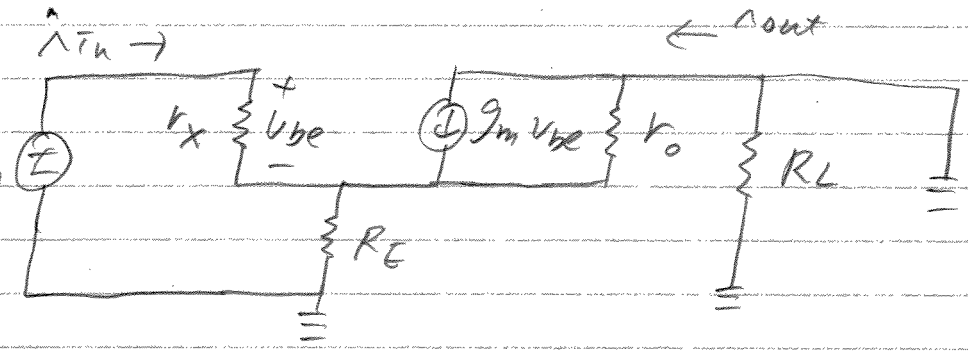
$= 43.9\,Hz$

$$A_{mid} = -\frac{60.7k}{60.8k} \times \frac{(6.64m)(30.1k)}{1+(6.64m)(3k)} = \boxed{-9.5}$$ 4

$GBW = 9.54\times(9.27M - 43.9) \simeq \boxed{88\,MHz}$

4

P9.5
(a)



KCL:

$$\hat{i}_{in} + g_m v_{be} = \frac{V_{in} - V_{be}}{R_E \| r_o}, \quad V_{be} = \hat{i}_{in} r_\pi$$

$$\hat{i}_{in}\left(1 + g_m r_\pi + \frac{r_\pi}{R_E \| r_o}\right) = \frac{V_{in}}{R_E \| r_o}$$

$$R_{in} = \frac{V_{in}}{\hat{i}_{in}} = \boxed{r_\pi + (1 + g_m r_\pi)(R_E \| r_o)} \quad \#$$

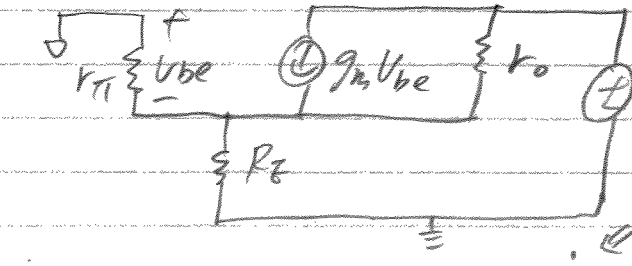
$$\hat{i}_{out} = g_m v_{be} - \frac{(V_{in} - V_{be})}{r_o}$$

$$V_{be} = \hat{i}_{in} r_\pi = \frac{r_\pi V_{in}}{r_\pi + (1+\beta_o)(R_E \| r_o)} = \frac{r_\pi V_{in}}{R_{in}}$$

$$\hat{i}_{out} = V_{in}\left(\frac{\beta_o}{R_{in}} - \frac{1}{r_o} + \frac{r_\pi}{R_{in} r_o}\right)$$

$$G_m = \frac{\hat{i}_{out}}{V_{in}} = \frac{\beta_o r_o - R_{in} + r_\pi}{r_o R_{in}}$$

$$= \boxed{\frac{\beta_o r_o - (1+\beta_o)(R_E \| r_o)}{r_o\left(r_\pi + (1+\beta_o)(R_E \| r_o)\right)}} \quad \#$$



$$\hat{i}_t = \frac{-V_{be}}{r_\pi \| R_E} = g_m v_{be} + \frac{V_t - V_{be}}{r_o}$$

$$V_{be} = -\hat{i}_t (r_\pi \| R_E)$$

$$\hat{i}_t = -g_m (r_\pi \| R_E)\hat{i}_t + \frac{V_t - \hat{i}_t (r_\pi \| R_E)}{r_o}$$

$$R_{out} = \frac{V_t}{\hat{i}_t} = \boxed{r_o[1 + g_m(r_\pi \| R_E)] + (r_\pi \| R_E)} \quad \#$$

b) $R_{in} = r_\pi + (1+g_m r_\pi)(R_E \| r_o)$

$\approx r_\pi + (1+g_m r_\pi) R_E \quad (r_o \gg R_E)$

$\approx r_\pi + g_m r_\pi R_E \quad (\beta_o = g_m r_\pi \gg 1)$

$\approx \boxed{r_\pi (1+g_m R_E)}$ #

$$G_M = \frac{\beta_o r_o - (1+\beta_o)(R_E \| r_o)}{r_o(r_\pi + (1+\beta_o)(R_E \| r_o))}$$

$$\approx \frac{\beta_o r_o - \beta_o R_E}{r_o(r_\pi + \beta_o R_E)} \quad (r_o \gg R_E,\ \beta_o \gg 1)$$

$$\approx \frac{\beta_o (r_o - R_E)}{r_o(r_\pi + \beta_o R_E)} \approx \frac{\beta_o r_o}{r_o(r_\pi + \beta_o R_E)} \quad (r_o \gg R_E)$$

$$\approx \frac{g_m r_\pi}{r_\pi + g_m r_\pi R_E} \approx \boxed{\frac{g_m}{1+g_m R_E}}$$ #

conditions used: $r_o \gg R_E$ & $\beta_o = g_m r_\pi \gg 1$

c) $R_{out} = r_o[1+g_m(r_\pi \| R_E)] + (r_\pi \| R_E)$

$\approx r_o[1+g_m R_E] + R_E \quad (r_\pi \gg R_E)$

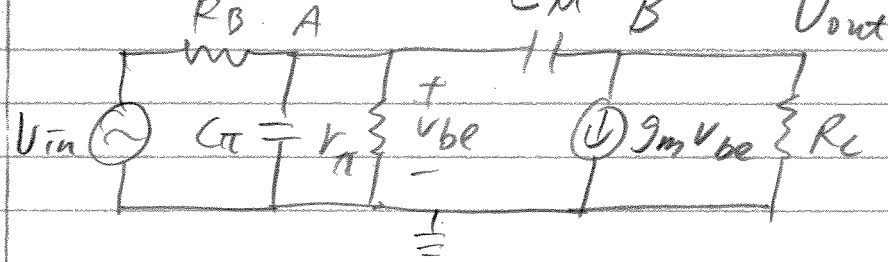
$\approx \boxed{r_o(1+g_m R_E)}$ # $\quad (r_o \gg R_E)$

P7.6 Ignored r_o

a) $$A_V = \frac{V_A}{V_{in}} \times \frac{V_{out}}{V_A} = \frac{R_{in}}{R_B + R_{in}} \times (-G_m R_{out})$$

$$\approx \left[\frac{-r_\pi (1 + g_m R_E)}{R_B + r_\pi (1 + g_m R_E)} \times \frac{g_m R_C}{1 + g_m R_E} \right] \#$$

b)



$$C_A = C_\pi + C_\mu (1 - A_V)$$
$$C_B = C_\mu (1 - A_V^{-1})$$
} A_V from part a)

#

c) $R_{A,eq} = R_B \parallel r_\pi$

$R_{B,eq} = R_C$

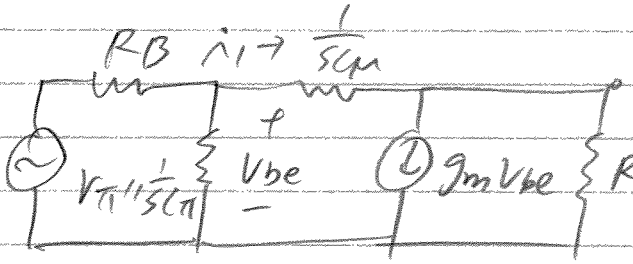
$$f_H = \frac{1}{2\pi} \left(\frac{1}{R_{A,eq} C_A + R_{B,eq} C_B} \right) \#$$

d) $A_V \approx -138 \frac{V}{V}$

$f_H \approx 75\ kHz$

(See attached)

e)



KCL $\hat{i}_1 = g_m V_{be} + \frac{V_{out}}{R_C}$

KCL $\frac{V_{in} - V_{be}}{R_B} = \hat{i}_1 + \frac{V_{be}}{r_\pi \| \frac{1}{sC_\pi}}$

$$\frac{V_{in}}{R_B} = g_m V_{be} + \frac{V_{out}}{R_C} + \frac{V_{be}}{r_\pi \| \frac{1}{sC_\pi}} + \frac{V_{be}}{R_B}$$

$$= V_{be}\left(g_m + \frac{1}{r_\pi \| \frac{1}{sC_\pi}} + \frac{1}{R_B}\right) + \frac{V_{out}}{R_C} \quad ①$$

Ohm's Law $V_{be} - \frac{\hat{i}_1}{sC_\mu} = V_{out}$

$$V_{be} = V_{out} + \frac{g_m}{sC_\mu} V_{be} + \frac{V_{out}}{sR_C C_\mu}$$

$$V_{be}\left(1 - \frac{g_m}{sC_\mu}\right) = V_{out}\left(1 + \frac{1}{sR_C C_\mu}\right)$$

$$\underbrace{1 - \frac{g_m}{sC_\mu}} = \frac{sC_\mu - g_m}{sC_\mu}$$

$$\Rightarrow V_{be} = V_{out}\left(\frac{sR_C C_\mu + 1}{sR_C C_\mu}\right)\frac{sC_\mu}{sC_\mu - g_m}$$

$$= V_{out}\frac{(sR_C C_\mu + 1)}{R_C(sC_\mu - g_m)} \quad ②$$

Plug ② into ①

$$\frac{V_{in}}{R_B} = V_{out}\left[\frac{sR_C C_\mu + 1}{R_C(sC_\mu - g_m)}\left(g_m + \frac{1}{r_\pi \| \frac{1}{sC_\pi}}\right) + \frac{1}{R_B}\right] + \frac{1}{R_C}$$

$$\frac{V_{out}}{V_{in}} = \left\{ R_B \left[\qquad \right] \right\}^{-1}$$

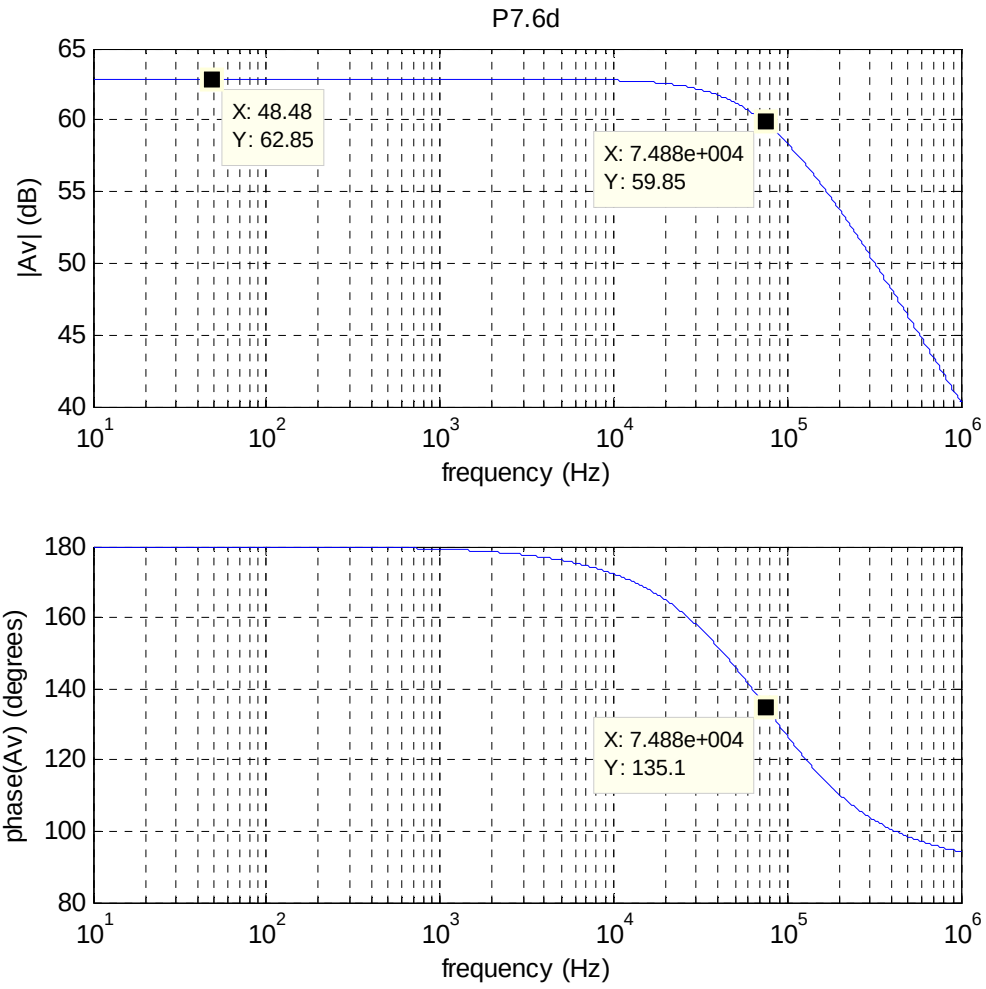
f) plots attached.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% #1
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

R1 = 100e3;
R2 = 300e3;
Ri = 2e3;
Re = 13e3;
R7 = 100e3;
Bf = 100; % current gain, A/A
Af = Bf/(Bf+1);
Vt = 0.026; % thermal voltage @ room temp.
Ic = 0.25e-3;

gm = Ic/Vt;
rpi = Bf/gm;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Midband gain
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Rin = (1/R1 + 1/R2 + 1/(rpi+(1+Bf)*Re))^-1;
RL = (1/Re + 1/R7)^-1;
Gain = (Rin/(Ri+Rin))*((1+Bf)*RL/(rpi+(1+Bf)*RL))

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% SCTC - fL
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

C1 = 4.7e-6;
C3 = 10e-6;
Re_eq1 = Ri+Rin;
Re_eq3 = R7 + ((1/Re + gm)^-1);

FL = (1/(Re_eq1*C1) + 1/(Re_eq3*C3))/(2*pi) %Hz

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% #2
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

C3 = ((20*pi - (1/(Re_eq1*C1)))^-1)/Re_eq3


clc;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% #4
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Vcc = 12;
R1 = 10e3;
R2 = 30e3;
Ri = 250;
rx = 350;
Re1 = 200;
Re2 = 1100;
Rc = 4.3e3;
R3 = 47e3;
Vbeon = 0.7;
```

```
Bf = 100; % current gain, A/A
Af = Bf/(Bf+1);
Vt = 0.026; % thermal voltage @ room temp.

% DC bias

Vb = R1*Vcc/(R1+R2)
Ve = Vb-Vbeon
Ie = Ve/(Re1+Re2);
Ic = Af*Ie
Vc = Vcc - (Ic*Rc) % check for FAR mode

gm = Ic/Vt;
rpi = Bf/gm;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Midband gain
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Rin = (1/R1 + 1/R2 + 1/(rpi+rx+(1+Bf)*Re1))^-1;
Rout = (1/Rc + 1/R3)^-1;
Gain = -(gm/(1+gm*Re1))*(Rout) * (Rin/(Ri+Rin))

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% SCTC - fL
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

C1 = 5e-6;
C2 = 1e-6;
C3 = 4.7e-6;
Re_eq1 = Ri+Rin;
Re_eq2 = R3+Rc;
Re_eq3 = Re1 + (1/gm);

FL = (1/(Re_eq1*C1) + 1/(Re_eq2*C2) + 1/(Re_eq3*C3))/(2*pi) %Hz

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% OCTC - fH
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Cu = 1e-12;
ft = 200e6;
Cpi = (gm/ft) - Cu

GS=1/Ri+1/R1+1/R2;
Rs=1/GS;
Rpi_eq = 1/(1/rpi+(1+gm*Re1)/(Rs+Re1))

Rin = 1/(1/Ri + 1/R1 + 1/R2 + 1/(rpi+rx+(1+Bf)*Re1));
Rout = 1/(1/Rc + 1/R3);
GM = gm/(1+gm*Re1);
Ru_eq = Rin + Rout*(1+GM*Rin);

FH = 1/(2*pi*(Rpi_eq*Cpi + Ru_eq*Cu))
```

```
Bf = 100; % current gain, A/A
Af = Bf/(Bf+1);
Vt = 0.026; % thermal voltage @ room temp.

% DC bias

Vb = R1*Vcc/(R1+R2)
Ve = Vb-Vbeon
Ie = Ve/(Re1+Re2);
Ic = Af*Ie
Vc = Vcc - (Ic*Rc) % check for FAR mode

gm = Ic/Vt;
rpi = Bf/gm;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Midband gain
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Rin = (1/R1 + 1/R2 + 1/(rpi+rx+(1+Bf)*Re1))^-1;
Rout = (1/Rc + 1/R3)^-1;
Gain = -(gm/(1+gm*Re1))*(Rout) * (Rin/(Ri+Rin))

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% SCTC - fL
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

C1 = 5e-6;
C2 = 1e-6;
C3 = 4.7e-6;
Re_eq1 = Ri+Rin;
Re_eq2 = R3+Rc;
Re_eq3 = Re1 + (1/gm);

FL = (1/(Re_eq1*C1) + 1/(Re_eq2*C2) + 1/(Re_eq3*C3))/(2*pi) %Hz

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% OCTC - fH
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

Cu = 1e-12;
ft = 200e6;
Cpi = (gm/ft) - Cu

GS=1/Ri+1/R1+1/R2;
Rs=1/GS;
Rpi_eq = 1/(1/rpi+(1+gm*Re1)/(Rs+Re1))

Rin = 1/(1/Ri + 1/R1 + 1/R2 + 1/(rpi+rx+(1+Bf)*Re1));
Rout = 1/(1/Rc + 1/R3);
GM = gm/(1+gm*Re1);
Ru_eq = Rin + Rout*(1+GM*Rin);

FH = 1/(2*pi*(Rpi_eq*Cpi + Ru_eq*Cu))
```

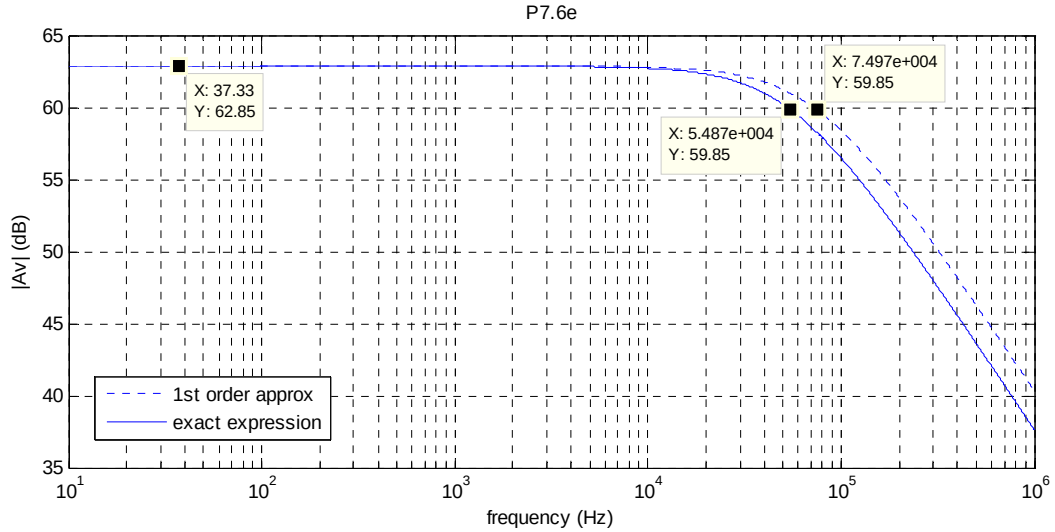
```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% #6
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

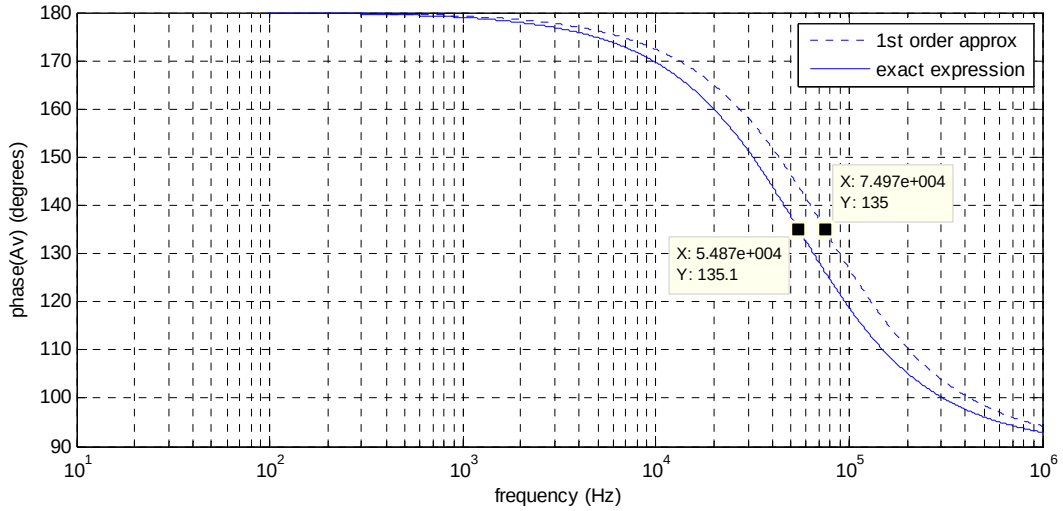



```
Rb = 1e3;
Rc = 50e3;
Bf = 100;
Cu = 2e-12;
Cje = 0; %no depletion cap, just consider diffusion cap for B-E junction
Ic = 1e-3;
Tf = 500e-12;
Vt = 0.026;

gm = Ic/Vt;
rpi = Bf/gm;
Cpi = gm*Tf;

Av = -gm*Rc*(rpi/(Rb+rpi))
CA = Cpi + Cu*(1-Av);
CB = Cu*(1-(1/Av));
RAeq = (1/Rb + 1/rpi)^-1;
RBeq = Rc;

fH = (1/(RAeq*CA + RBeq*CB))/(2*pi)

f = logspace(1,6,10000);
```

```
Gain = Av./(1+(j*f./fH));
subplot(2,1,1);
semilogx(f,20*log10(abs(Gain)));
grid on
xlabel('frequency (Hz)');
ylabel('|Av| (dB)');
title('P7.6d');
subplot(2,1,2);
semilogx(f,angle(Gain)*180/pi);
grid on
xlabel('frequency (Hz)');
ylabel('phase(Av) (degrees)');
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% #6e
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```





```
Gain_e = 1./(Rb.*(((1+j*2*pi.*f.*Rc*Cu)./(Rc.*(j*2*pi.*f.*Cu-
gm))).*(gm+((1/rpi)+j*2*pi.*f.*Cpi)+(1/Rb))+(1/Rc)));
subplot(2,1,1);
semilogx(f,20*log10(abs(Gain)),':');
hold on
semilogx(f,20*log10(abs(Gain_e)));
hold off
grid on
xlabel('frequency (Hz)');
ylabel('|Av| (dB)');
title('P7.6e');
legend('1st order approx','exact expression');
subplot(2,1,2);
semilogx(f,angle(Gain)*180/pi,':');
hold on
semilogx(f,angle(Gain_e)*180/pi);
hold off
grid on
xlabel('frequency (Hz)');
ylabel('phase(Av) (degrees)');
legend('1st order approx','exact expression');
```