Homework 8

This homework is due on November 15.

Note: The final exam for this course is regularly scheduled for Dec. 21 from 10:30am — 12:30 pm. In addition to this time I would like to offer another exam time to accommodate students who may want to take the exam earlier for various reasons. I will offer another exam on Friday Dec. 14 from 6:30 — 8:30 pm. This time is chosen so there is no conflict with any other exam.

The finals will be open book/notes and the last lecture for new material will be on Dec. 6. The last class on Dec. 11 will be used for review. Those of you who want to take the final on Dec. 14 should have enough time to prepare. Those who want to wait can take the finals on Dec. 21.

In a few weeks I will ask you to select the date you want for your finals.

Problem 1: In a pnp silicon transistor at 300 K, the base doping is $5 \times 10^{16}$ cm$^{-3}$. The base width is 1.0 $\mu$m and $L_b = 10.0$ $\mu$m. What is the total minority carrier charge in the base (a) at $V_{EB} = 0.5$ V; $V_{BC} = 1.0$ V, (b) at $V_{EB} = 0.7$ V; $V_{BC} = 2.0$ V. The area of the device is $10^{-2}$ cm$^2$.

Problem 2: An npp transistor at 300 K has an area of 1 mm$^2$, base width of 1.0 $\mu$m, and dopings of $N_{de} = 10^{18}$ cm$^{-3}$, $N_{ab} = 10^{17}$ cm$^{-3}$, $N_{dc} = 10^{16}$ cm$^{-3}$. The minority carrier lifetimes are $\tau_E = 10^{-7}$; $\tau_B = 10^{-4}$ s. Calculate the collector current in the active mode for (a) $V_{BE} = 0.5$ V, (b) $I_E = 2.5$ mA, and (c) $I_B = 5 \mu$ A. The base diffusion coefficient is $D_b = 20$ cm$^2$s$^{-1}$.

Problem 3: Plot the dependence of the base transport factor in a bipolar transistor as a function of $W_b/L_b$ over the range $10^{-2} \leq W_b/L_b \leq 10$. Assume that the emitter efficiency is unity. How does the common-emitter current gain vary over the same range of $W_b/L_b$?

Problem 4: The mobility of holes in silicon is 100 cm$^2$/Vs. It is required
that a BJT be made with a base width of 0.5 μm and base resistivity of no more than 1.0 Ω-cm. It is also desired that the emitter injection efficiency be at least 0.999. Calculate the emitter doping required. The various device parameters are

\[
\begin{align*}
L_b &= 10 \, \mu m \\
L_e &= 10 \, \mu m \\
D_e &= 10 \, \text{cm}^2/\text{s} \\
D_b &= 20 \, \text{cm}^2/\text{s}
\end{align*}
\]

What is the current gain \( \beta \) of the device? Assume \( W_{bn} = W_b \).

**Problem 5** The punch-through voltage of a Ge \( pnp \) bipolar transistor is 20 V. The base doping is \( 10^{18} \, \text{cm}^{-3} \), and the emitter and collector dopings are \( 10^{18} \, \text{cm}^{-3} \). Calculate the zero bias base width. If \( \tau_B = 10^{-6} \, \text{s} \), what is the \( \alpha \) of the transistor at a 10 V reverse bias across the collector-base junction at 300 K? The hole diffusion coefficient in the base is 40 cm²s⁻¹.

**SOME IMPORTANT ISSUES DISCUSSED THIS WEEK**

**THE BIPOLAR TRANSISTOR**

The bipolar junction transistor (BJT) is a very versatile device that finds widespread use in both digital and analog applications. The device is a back to back arrangement of two \( n-p \) diodes creating a \( n-p-n \) or \( p-n-p \) structure. The three doped regions form the emitter-base-collector of the device. The device is based on the ability of a very small change in the base current to produce a large change in the collector current. As a result the device can be used to amplify signals or to switch the state of the device from ON to OFF.

To understand how the device operates we have examined the forward active mode of the device. In this mode the emitter-base junction (EBJ) is forward biased while the base-collector junction (BCJ) is reverse biased.

The forward biased EBJ causes electrons to be injected from the emitter to the base and holes to be injected from the base into the emitter. The electron current is denoted as \( I_{En} \) and the hole current is \( I_{Ep} \). Note that from the \( p-n \) diode theory we know that

\[
I_{En} \propto n_{40}
\]

where \( n_{40} \) is the equilibrium electron concentration in the p-base. Similarly the hole injection current is

\[
I_{Ep} \propto p_{40}
\]
where \( p_{b0} \) is the hole density in the emitter. If we choose \( n_{b0} \) to be much larger than \( p_{b0} \), we can ensure that

\[
I_{En} \gg I_{Ep}
\]

This requires that the emitter doping be much larger than the base doping.

Part of the electron injection current \( I_{En} \) recombines with holes in the base while a part makes it to the collector. Since the BCJ is reverse biased, there is almost no collector current other than the one due to injection from the emitter. If \( B \) is the fraction of the electron injection that makes it to the collector (this factor is called the base transport factor) we have

\[
I_C = BI_{En}
\]

We now have

\[
I_E = I_{En} + I_{Ep}
\]

\[
I_C = BI_{En}
\]

and the base current is

\[
I_B = I_{Ep} + (1 - B)I_{En}
\]

The current gain is (denoting the gain by \( \beta \))

\[
\beta = \frac{I_C}{I_B} = \frac{BI_{En}}{I_{Ep} + (1 - B)I_{En}}
\]

The quantity

\[
\gamma_e = \frac{I_{En}}{I_E}
\]

is called emitter efficiency. In terms of emitter efficiency we have

\[
\beta = \frac{B\gamma_e}{1 - B\gamma_e}
\]

We see that for high current gain we must have \( B \) and \( \gamma_e \) close to 1.

Another useful definition is the current transfer ratio, \( \alpha \) defined as

\[
\frac{I_C}{I_E} = \frac{BI_{En}}{I_{En} + I_{Ep}} = B\gamma_e
\]

Using the formalism developed in the context of \( p-n \) diodes we have calculated the BJT \( I-V \) characteristics, i.e. the relation between biasing and emitter, collector and base currents.

For the forward active mode if we ignore the current flow in the reverse biased BCJ and assume that the base width is small we get

\[
I_E = -\frac{eAD_n n_{b0}}{W_{en}} \left[ \exp \left( \frac{eV_{BE}}{k_BT} \right) - 1 \right] - \frac{eAD_e p_{e0}}{L_e} \left[ \exp \left( \frac{eV_{BE}}{k_BT} \right) - 1 \right]
\]

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Here the first part is due to electron injection from the emitter into the base and the second part is due to the hole injection from the base into the emitter. The collector current is

\[ I_C = \frac{eAD_b n_{i0}}{W_{bn}} \left[ \exp \left( \frac{eV_{BE}}{k_BT} \right) - 1 \right] - \frac{eAD_h n_{i0} W_{in}}{2I_b^2} \left[ \exp \left( \frac{eV_{BE}}{k_BT} \right) - 1 \right] \]

The first part represents \( I_{En} \), the electron current injected from the emitter while the second term represents the part lost through recombination with holes in the base. The base current is

\[ I_B = \frac{eAD_e p_{i0}}{L_e} \left[ \exp \left( \frac{eV_{BE}}{k_BT} \right) - 1 \right] + \frac{eAD_h n_{i0} W_{in}}{2I_b^2} \left[ \exp \left( \frac{eV_{BE}}{k_BT} \right) - 1 \right] \]

The first part represents the hole current injected from the base into the emitter and the second part represents the hole current recombining with electrons injected from the emitter.

The base current should be small compared to the collector current for high current gain in the device.

From the equations given above we can see that the device performance parameters, emitter efficiency \( \gamma_e \), base transport factor, \( B \) and current gain, \( \beta \) are given by the following:

\[ \gamma_e = \frac{D_b n_{i0} / W_{bn}}{D_b n_{i0} / W_{bn} + D_e p_{i0} / L_e} \sim 1 - \frac{D_e p_{i0} W_{in}}{D_h n_{i0} L_e} \]

This requires that we have \( W_{bn} \ll L_e \) and \( p_{i0} \ll n_{i0} \).

\[ B = \frac{I_C}{I_{En}} = 1 - \frac{W_{bn}^2}{2I_b^2} \]

and the current gain is

\[ \beta = \frac{B \gamma_e}{1 - B \gamma_e} \]

If the base transport factor is unity (i.e. the base is very narrow) we have

\[ \beta = \frac{n_{i0} D_b L_e}{p_{i0} D_e W_{in}} \]

If the emitter efficiency is unity we get

\[ \beta = \frac{2I_b^2}{W_{bn}^2} \]

The BJT when well made provides a very high gain, excellent input-output isolation, very good high speed switching and high frequency operation. It is
also a high current device which is very useful to drive other electronic and optoelectronic devices.

However, the device also has some problems. It is a vertical device that requires great care in contacting a buried base region. It is also a device that requires minority charge flow. This can make the device have a slow response if the design is not carried out carefully. The following design issues are of importance:

i) Device doping: The values of $N_{de}$, $N_{ab}$, $N_{dc}$ for an n-p-n BJT can be altered within limits set by technology. The limit on $N_{de}$ is also set by bandgap shrinkage which is important for doping above $10^{19}$ cm$^{-3}$.

ii) Device dimensions: The width of the base is an important design parameter. The width should be wide enough that the base resistance is not too large. Otherwise the base resistance may overwhelm the device response.

The emitter thickness should also be as small as possible. A thick emitter adds to the device series resistance.

● Device Optimization: As in any other semiconductor device, one has to tweak many different device parameters to reach a desired performance. Usually the design requirements for one demand conflict with the design requirements for another demand. The basis device demands can be characterized as:

i) Current gain ($\beta$): This is of great importance since the ratio of $I_C$ and $I_B$ determines the gain in the device.

ii) Input-Output Isolation: This requires that the device output conductance be very low. This is important so that parameters external to the device (i.e. parameters controlled by the devices being driven by the BJT) would not control the device response.

iii) Device Breakdown: The device should be designed so that it does not suffer breakdown at very low voltages.

iv) Device Switching Speed: The device should be able to switch from the cutoff state (OFF-state) to the saturation state (ON-state) in a very short time.

v) Small Signal Response of the Device: The device should operate with high gain at as high a frequency as possible.