SOLUTIONS TO THE FINAL EXAM

Problem 1:(10 points)
Consider a long \( p^+ - n \) diode made from GaAs with area 1 mm\(^2\). The following parameters characterize the diode which is used as a LED:

\[
V_{bi} = 1.3 \text{ V} \\
\tau_r = 5 \times 10^{-9} \text{ s} \quad N_d = 10^{17} \text{ cm}^{-3}; \quad D_p = 30 \text{ cm}^2/\text{s} \\
\tau_{nr} = 10^{-9} \text{ s}
\]

where \( \tau_r \) and \( \tau_{nr} \) are the radiative and non-radiative lifetimes for carriers in the diode.

The diode is forward biased at 1.2 V (assume that all current is due to hole injection into the n-side).

(i) Calculate the current flowing in the diode.

(ii) What is the wavelength of the light emitted?

(iii) Calculate the optical power produced in the diode.

In this problem we first calculate the ideal current prefactor, \( I_0 \), and then the non-ideal prefactor, \( I_{GR}^0 \). The carrier lifetime is from the inverse sum rule \( 8.33 \times 10^{-10} \text{ s} \). The diffusion length is

\[
I_p = \sqrt{30 \times 8.33 \times 10^{-10}} = 1.58 \times 10^{-4}
\]

\[
p_n = \frac{3.39 \times 10^{12} \text{ cm}^{-6}}{10^{17} \text{ cm}^{-3}} = 3.39 \times 10^{-5} \text{ cm}^{-3}
\]

\[
I_0 = (1.6 \times 10^{-16})(10^{-2})\left(\frac{30}{1.58 \times 10^{-4}}\right) \times 3.39 \times 10^{-5} = 1.03 \times 10^{-20} \text{ A}
\]

The depletion width at a forward bias of 1.2 V with the built-in voltage given as 1.3 V is

\[
W = 3.82 \times 10^{-6} \text{ cm}
\]

This gives

\[
I_{GR}^0 = \frac{eAWn_i}{2\tau} = 5.62 \times 10^{-12} \text{ A}
\]

The total current is dominated by the ideal part at 1.2 V forward bias and is

\[
I_{tot} = I_0 \exp(1.2/0.026) + I_{GR}^0 \exp(1.2/0.052)
\]

\[
= 1.14 + 5.91 \times 10^{-2} \text{ A}
\]
Out of the injected current only one sixth causes radiative emission. The rest goes into non-radiative processes. The radiative part of the current is then

\[ I_{rad} = 0.19 \ \text{A} \]

The wavelength of light is given by the photon energy which is equal to the bandgap, \( E_g \) of GaAs. Since the bandgap is 1.43 eV the wavelength is

\[ \lambda = 0.86 \ \mu m \]

The power is given by

\[ P = I_{rad} \frac{E_g}{e} \]

The power emitted is

\[ P = 0.19 \times \frac{1.43 \times 10^{-19}}{1.6 \times 10^{-19}} = 0.27 \ \text{W} \]

**Problem 2:** (12 points)

Consider an n-p-n Si-BJT at 300 K with the following parameters:

- \( N_{de} = 10^{18} \ \text{cm}^{-3} \)
- \( N_{ab} = 10^{17} \ \text{cm}^{-3} \)
- \( N_{dc} = 10^{16} \ \text{cm}^{-3} \)
- \( D_b = 20.0 \ \text{cm}^2/\text{s} \)
- \( L_b = 15.0 \ \mu \text{m} \)
- \( D_e = 10.0 \ \text{cm}^2/\text{s} \)
- \( L_e = 5.0 \ \mu \text{m} \)
- \( W_b = 0.5 \ \mu \text{m} \)

*The non-radiative lifetime for carriers in the EBJ depletion region is 10^{-8} \ \text{s}.*

(i) Calculate the base transport factor (\( B \)) and current gain (\( \beta \)) of the BJT in the forward active mode when the EBJ is forward biased at 0.1 V and the BCJ is reverse biased at 5.0 V.

(ii) Calculate the current gain of the BJT in the forward active mode when the EBJ is forward biased at 0.8 V and the BCJ is reverse biased at 5.0 V.

You may assume that the base is very small compared to \( L_b \) and the reverse bias collector current is zero.

In this problem we have to consider the ideal and non-ideal part of the forward current in the EBJ. The non-ideal current and the hole injection current into the emitter are the base current. The electron current injected into the base from the emitter multiplied by the base transport factor is the collector current.

We find that for the EBJ the built-in voltage is 0.877 V. The depletion width of the EBJ for a forward bias of 0.1 V is found to be

\[ W(0.1V) = 1.06 \times 10^{-5} \ \text{cm} \]
The depletion width for a forward bias of 0.8 V is
\[ W(0.8V) = 3.34 \times 10^{-6} \, \text{cm} \]

The non-ideal current in the EBJ at a forward bias of 0.1 V is found to be
\[ I_{GR} = A \times 8.32 \times 10^{-7} \, \text{A} \]

where \( A \) is the area of the BJT.

To calculate the base transport factor we need to calculate the depletion width into the base side. This is found to be (at a 5.0 V reverse bias at the BCJ)
\[ \Delta W_b = 8.29 \times 10^{-6} \, \text{cm} \]

The neutral base width is now 0.417 \( \mu \text{m} \). The base transport factor is
\[ B = 1 - \frac{W_b^2}{2L_b} = 0.9996 \]

We now find for the 0.1 V forward bias
\[ I_C \sim A \times 8.24 \times 10^{-9} \, \text{A} \]

The base current is essentially the generation recom‌bination non-ideal current and is
\[ I_B = A \times 8.22 \times 10^{-7} \, \text{A} \]

The current gain is then
\[ \beta(0.1 \text{ V}) = \frac{8.24 \times 10^{-9}}{8.22 \times 10^{-7}} \sim 10^{-2} \]

The gain is less than unity because of the high non-ideala current.

At high forward bias the non-ideal current part becomes negligible and the ideal part dominates. In this case we get
\[ \beta(0.8 \text{ V}) = \frac{4.05 \times 10^{3}}{18.52} \sim 220 \]

This result is similar to what is indicated on Fig. 7.12 of the text.

Problems 3. (8 points)
Consider an n-MOSFET made from Si doped p-type at \( N_a = 10^{16} \, \text{cm}^{-3} \).

The other parameters for the device are the following:

\[ \begin{align*}
V_{fb} & = -1.0 \text{V} \\
\mu_n & = 500 \, \text{cm}^2\text{V}^{-1}\text{s}^{-1} \\
\text{Gate Length} & = 2.0 \, \mu\text{m} \\
\text{Gate Width} & = 20.0 \, \mu\text{m} \\
\text{d} & = 500 \, \text{Å} \\
\end{align*} \]
i) Calculate the threshold voltage of the device.

ii) Calculate the electron densities (in units of electrons per cm$^2$) in the channel on the source and drain side of the gate when the gate bias is $V_T + 1.5V$ and $V_{DS} = 1.0V$.

iii) Calculate the saturation current in the channel for the gate bias specified above.

The threshold voltage is obtained from Eqn. 9.13 of the text and is found to be

$$V_T = 0.4 \, V$$

To find the charge in the channel we use Eqn. 9.32 of the text. At the source side we get

$$n_s = \frac{Q_s}{e} = \frac{C_{ox}}{e} \times 1.5 = 6.47 \times 10^{11} \, cm^{-2}$$

At the drain the value is

$$n_s = \frac{Q_s}{e} = \frac{C_{ox}}{e} \times 0.5 = 2.1 \times 10^{11} \, cm^{-2}$$

The saturation current is

$$I_D(sat) = \frac{\mu_n ZC_{ox} (V_{GS} - V_T)}{2L} (V_{GS} - V_T)^2$$

$$= 0.388 \, mA$$

**Problem 4:** (10 points)

Consider an n-MOSFET at room temperature made from Si doped p-type.

To characterize the device $C - V$ measurements are done for the MOS capacitor. It is found from the low frequency measurements that the maximum capacitance per unit area is $2.07 \times 10^{-7} \, F/cm^2$.

The following drain current is measured in the linear regime at $V_{DS} = 0.1 \, V$:

$$V_{GS} = 1.1 \, V; \quad I_D = 37.26 \, \mu A$$

$$V_{GS} = 1.3 \, V; \quad I_D = 62.1 \, \mu A$$

The other parameters for the device are the following:

$$Gate \ Length = 2.0 \, \mu m$$

$$Gate \ Width = 20.0 \, \mu m$$

(i) Calculate the oxide thickness.

(ii) Calculate the mobility of electrons in the channel.

(iii) Calculate the threshold voltage.
(iv) Calculate the channel current in saturation when the gate bias is 1.5 V. The maximum capacitance of a MOS structure is just $C_{ox}$. This gives

$$d_{ox} = \frac{3.9 \times 8.84 \times 10^{14}}{2.07 \times 10^{-7}} = 1.66 \times 10^{-6} \text{ cm}$$

Using the equations for linear region we find that the mobility is (using Eqn. 9.43 of the text)

$$\mu_n = 600 \text{ cm}^2 / \text{V.s}$$

and

$$V_T = 0.8 \text{ V}$$

The saturation current is found to be

$$I_D(sat) = 3.04 \times 10^{-4} \text{ A}$$