Solutions to Homework 1

**Problem 1:** An Aluminum interconnect is used in an IC. The parameters for the interconnect are:

- **Length** = 1 mm
- **Area** = 10 $\mu$m $\times$ 5$\mu$m
- **Atomic mass** = 27
- **Density** = 2.7 g/cm$^3$
- **Valence** = 3
- **Mobility** = 50 cm$^2$/V.s

- Calculate the resistance of the interconnect.

The electron density is

\[
n = 6.023 \times 10^{23} \times \frac{3 \times 2.7}{27} = 1.807 \times 10^{23} \text{ cm}^{-3}
\]

\[
\sigma = ne\mu = 1.807 \times 10^{23} \times 1.6 \times 10^{-19} \times 50
\]

\[
= 1.446 \times 10^6 \text{ (Ω cm)}^{-1}
\]

\[
\rho = 6.92 \times 10^{-7} \text{ Ω cm}
\]

\[
R = \frac{\rho L}{A} = \frac{6.92 \times 10^{-7} \times 0.1}{5.0 \times 10^{-7}} = 0.138 \text{ Ω}
\]

This is quite a small resistance but can still cause RC time constant delays in high speed circuits.

**Problem 2:** Last week Motorola announced the ability to produce a 12 inch diameter GaAs layer grown on a 12 inch silicon substrate.

- Calculate the number of Si atoms that fit on a 12 inch line along the (110) direction.

The distance between atoms along the (110) direction (between atoms at (000) and ($a/2, a/2, 0$)) is

\[
d(110) = \frac{a}{\sqrt{2}} = 3.84 \text{ Å}
\]

The number of atoms in 12 inches is then $7.94 \times 10^8$.

**Problem 3:** Calculate the wavelengths and wavevectors for an electron and a photon with energies of:
(i) 1.0 eV; 
(ii) 1.0 keV; 
The difference in the values for an electron and a photon is very important in understanding optoelectronics.

WE need to use the appropriate expressions relating a particle and a photon energy to the wavelength. For an electron we have

$$\lambda_e = \frac{h}{p} = \frac{h}{\sqrt{2m_0E}}$$

For a photon we have

$$\lambda_{ph} = \frac{hc}{E}$$

With this we have: 

$$E = 1.0 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$\lambda_e = \frac{6.6 \times 10^{-34} \text{ J s}}{\sqrt{2 \times 9.1 \times 10^{-31} \text{ kg} \times 1.6 \times 10^{-16} \text{ J}}} = 1.22 \times 10^{-9} \text{ m} = 12.2 \text{ Å}$$

For a 1.0 keV electron we get

$$\lambda_e = 0.3866 \text{ Å}$$

We see that the higher energy electron has a wavelength that is comparable to atomic spacings in a solid. Such electrons are used to see atoms in a crystal and for drawing very fine lines in microelectronic technology (e-beam lithography).

A similar calculation for photons with the proper relation (given above) gives

$$\lambda_{ph}(1.0 \text{ eV}) = 1.24 \times 10^{-6} \text{ m} = 1.24 \mu m$$

$$\lambda_{ph}(1.0 \text{ keV}) = 12.4 \text{ Å}$$

One electron volt photons are close to the visible photons while a keV photons are called X-Rays.

**Problem 4:** Calculate the number of allowed electron states between energies 1.0 and 1.1 eV for a volume $10^{-4}$ cm$^3$. The energy-momentum relation is

$$E = 1.0 \text{ eV} + \frac{\hbar^2 k^2}{2m_0}; \quad m_0 = 9.1 \times 10^{-31} \text{ kg}$$

The density of states is (using Examples 1.7 and 1.8 of the text)

$$N(E) = 6.8 \times 10^{21} (E - 1.0)^{1/2} \text{ eV}^{-1} \text{ cm}^{-3}$$

with $E$ is units of eV. The number of states between 1.0 eV and 1.1 eV is

$$N = 6.8 \times 10^{21} \times 10^{-4} \int_{1.0}^{1.1} (E - 1.0)^{1/2} dE$$

$$= 6.8 \times 10^{18} \frac{2}{3} [(1.1 - 1.0)^{3/2} - (1.0 - 1.0)^{3/2}]$$

$$= 5.23 \times 10^{17}$$
Problem 5: Plot the Fermi function for energies between 1.0 eV and 1.2 eV. Assume that $E_F = 1.1$ eV. Plot the function for a temperature of (i) 77 K and (ii) 300 K. Also plot the Boltzmann function (which is an approximation to the Fermi function) on the same plots.

Examine how well the Boltzmann approximation works (or does not work). The values of $k_B T$ at 300 K and 77 K are 0.026 eV and 0.067 eV respectively. 

$T=77$ K:

<table>
<thead>
<tr>
<th>$E$ (eV)</th>
<th>Fermi</th>
<th>Boltzmann</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1</td>
<td>1.0</td>
<td>$3 \times 10^6$</td>
</tr>
<tr>
<td>-0.075</td>
<td>0.9999</td>
<td>$7.27 \times 10^4$</td>
</tr>
<tr>
<td>-0.05</td>
<td>0.999</td>
<td>$1.74 \times 10^3$</td>
</tr>
<tr>
<td>0.0</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>0.05</td>
<td>$5.7 \times 10^{-1}$</td>
<td>$5.7 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.075</td>
<td>$1.37 \times 10^{-5}$</td>
<td>$1.37 \times 10^{-5}$</td>
</tr>
<tr>
<td>0.1</td>
<td>$3.3 \times 10^{-7}$</td>
<td>$3.3 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

$T=300$ K:

<table>
<thead>
<tr>
<th>$E$ (eV)</th>
<th>Fermi</th>
<th>Boltzmann</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1</td>
<td>0.979</td>
<td>46.8</td>
</tr>
<tr>
<td>-0.075</td>
<td>0.947</td>
<td>17.9</td>
</tr>
<tr>
<td>-0.05</td>
<td>0.872</td>
<td>6.8</td>
</tr>
<tr>
<td>0.0</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>0.05</td>
<td>0.127</td>
<td>0.146</td>
</tr>
<tr>
<td>0.075</td>
<td>0.053</td>
<td>0.056</td>
</tr>
<tr>
<td>0.1</td>
<td>0.021</td>
<td>0.021</td>
</tr>
</tbody>
</table>

You will notice that the occupation is close to unity below $E_F$ and falls off very fast as the $E-E_F$ increases. The values fall off faster at lower temperatures. This simple fact is used to control electron densities in semiconductor devices. We also see that Boltzmann approximation is quite good if $E-E_F$ is large compared to $k_B T$. 

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