Solutions to Homework 4

Problem 1: Consider a pure Si sample on which light of intensity 1000 W/cm² shines (photon energy is 1.6 eV). The electron-hole recombination time is 100 ns.

(i) Calculate the excess electron and hole densities produced by light;

(ii) Calculate the conductivity of the sample in dark and in light. The electron and hole mobilities are:

\[ \mu_n = 1100 \text{ cm}^2/V.s; \quad \mu_p = 300 \text{ cm}^2/V.s \]

(iii) Calculate the quasi-Fermi levels for electrons and holes in the presence of light.

Use Figure 3.11 for absorption coefficients.

At photon energy 1.6 eV the value for \( \alpha \sim 10^3 \text{ cm}^{-1} \). We have, for the generation rate

\[ G_L = \frac{\alpha P_{\text{ph}}}{h\omega} = \frac{(10^3 \text{ cm}^{-1}) (1000 \text{ Wcm}^{-2})}{(1.6 \times 1.6 \times 10^{-19} \text{ J})} = 3.9 \times 10^{24} \text{ cm}^{-3}\text{s}^{-1} \]

The excess carrier density this generation creates is

\[ \Delta n = \Delta p = G_L \tau = (3.9 \times 10^{24}) (2 \times 10^{-9}) = 7.81 \times 10^{15} \text{ cm}^{-3} \]

The dark conductivity is \( (\sigma = ne\mu_n + pe/\mu_p) \)

\[ \sigma_{\text{dark}} = (1.5 \times 10^1 \text{cm}^{-3})(1.6 \times 10^{-10} \text{C})(1400 \text{cm}^2/V.s) = 3.36 \times 10^{-6} \text{ (\Omega cm)}^{-1} \]

The conductivity in light is

\[ \sigma_{\text{light}} = (7.81 \times 10^{15} \text{cm}^{-3})(1.6 \times 10^{-10} \text{C})(1400 \text{cm}^2/V.s) = 1.75 \text{ (\Omega cm)}^{-1} \]

We see that the conductivity has increased by 6 orders of magnitude.

\[ E_{F_n} = E_c + k_BT \ln \frac{n}{N_c} = E_c - 0.21 \text{ eV} \]

\[ E_{F_p} = E_v - k_BT \ln \frac{p}{N_v} = E_v + 0.186 \text{ eV} \]
**Problem 2:** A photodetector uses n-type silicon doped at $10^{16}$ cm$^{-3}$ as its active region. Calculate the **dark conductivity** of the detector (i.e., conductivity when no light is shining on the detector). Light with intensity $10^{-3}$ W/cm$^2$ shines on the device. Calculate the conductivity in the presence of light.

\[
\begin{align*}
\mu_n &= 1000 \text{ cm}^2/\text{Vs} \\
\mu_p &= 400 \text{ cm}^2/\text{Vs} \\
\alpha &= 10^9 \text{ cm}^{-1} \\
\tau_r &= 10^{-7} \text{s}
\end{align*}
\]

In the dark the conductivity is (dominated by electrons)

\[
\sigma = ne\mu_n + pe\mu_p \approx ne\mu_n
\]

\[
= \left(1.0 \times 10^{16} \text{ cm}^{-3}\right) \left(1.6 \times 10^{-19} \text{ C}\right) \left(1000 \text{ cm}^2/\text{V}\cdot\text{s}\right)
\]

\[= 1.6 \left(\Omega\cdot\text{cm}\right)^{-1}\]

To find the excess carrier density we need to get the photon energy. We are given the absorption coefficient. From Fig. 3.11 (or the previous problem) we see that $h\omega \sim 1.6 \text{ eV}$. The excess carrier density in the presence of light is

\[
\Delta n = \Delta p = G_L\tau = 3.9 \times 10^{11} \text{ cm}^{-3}
\]

The new conductivity is essentially the same as the dark conductivity.

The problem shows why a pure material should be used for photodetectors otherwise the conductivity is similar in light and dark. If, for example, we had used a pure sample we would get an order of magnitude change in light for this problem.

**Problem 3:** An important material for very high speed photodetectors is low temperature GaAs which is grown at low temperatures and has a large density of trap states. A sample is found to have an electron lifetime of 1.0 ps at 300 K.

Assume that a trap can be represented by an area with a radius of 3 Å. Calculate the trap density in the sample.

The radiative lifetime in GaAs is 2 ns.

The thermal velocity at 300 K is given by

\[
v_{th} = \sqrt{\frac{3 \times (0.026 \times 1.6 \times 10^{-19} \text{ J})}{(0.067 \times 0.91 \times 10^{-30} \text{ kg})}}
\]

\[= 4.52 \times 10^5 \text{ m/s}
\]

\[= 4.52 \times 10^7 \text{ cm/s}
\]
The trap density needed is given by

\[
N_t = \frac{1}{\tau \nu_h \sigma}
\]

\[
= \frac{1}{(10^{-12} \text{ s})(4.52 \times 10^7 \text{ cm/s})(\pi \times 9 \times 10^{-16} \text{ cm}^2)}
\]

\[= 7.82 \times 10^{18} \text{ cm}^{-3}\]

This is a very high density of traps.

**Problem 4:** Consider a Si sample with length \(L = 1.0 \mu m\). Excess carrier density in the sample has the following boundary values:

\[\delta n(0) = 10^{17} \text{ cm}^{-3}; \quad \delta n(L) = 0\]

Calculate the electron diffusion current in the material at (i) \(x = 0\); (ii) \(x = 0.5 \mu m\); (iii) \(x = 0.9 \mu m\).

\[D_n = 20 \text{ cm}^2/\text{s}; \quad \tau_n = 10^{-7} \text{ s}\]

In this problem the diffusion length is

\[L_n = \sqrt{D_n \tau_n} = 1.41 \times 10^{-3} \text{ cm}\]

The diffusion length is much larger than the sample length and we can use the linear approximation for the charge distribution. As a result in this problem the current flow due to diffusion is essentially unchanged with distance. The current is given by (\(A\) is the device area)

\[I_n = -A \frac{eD_n \delta n(0)}{L_n}\]

This gives

\[I_n = A3200 \text{ A/cm}^2\]

**Problem 5:** Electrons are injected into \(p\)-type GaAs at 300 K. The radiative lifetime for the electrons is 2 ns. The material has \(10^{15}\) impurities with a cross-section of \(10^{-14} \text{ cm}^2\). Calculate the distance the injected minority charge will travel before 50% of the electrons recombine with holes. The diffusion coefficient is 100 \text{ cm}^2/\text{s} and the sample is very thick.

Assume that we make a 0.2 \(\mu m\) thick device from the material described above. Electrons are injected at one end and move across the device through diffusion alone. Calculate the time it takes electrons to travel the thickness of the device. (See Example 3.17 for the needed expression).
For this problem we first need to find the total recombination time. To calculate the non-radiative recombination time we need the thermal velocity of electrons.

\[ v_{th} = \sqrt{\frac{3k_B T}{m^*}} = \sqrt{\frac{3(0.026 \times 1.6 \times 10^{-19} J)}{(0.067 \times 9.1 \times 10^{-31} \text{ kg})}} = 4.52 \times 10^7 \text{ m/s} = 4.52 \times 10^7 \text{ cm/s} \]

The non-radiative time is

\[ \tau_{nr} = \frac{1}{N_i v_{th} \sigma} = \frac{1}{(10^{15} \text{ cm}^{-3})(10^{-14} \text{ cm}^2)(4.52 \times 10^7 \text{ cm/s})} = 2.2 \times 10^{-9} \text{ s} \]

The total recombination time is given by

\[ \frac{1}{\tau_{tot}} = \frac{1}{\tau_r} + \frac{1}{\tau_{nr}} \]

This gives for \( \tau_r = 2 \text{ ns} \), \( \tau_{tot} = 1.05 \text{ ns} \). The diffusion length is now

\[ L_d = \sqrt{D \tau} = \sqrt{(100 \text{ cm}^2/\text{s})(1.05 \times 10^{-9} \text{ s})} = 3.24 \times 10^{-4} \text{ cm} \]

The excess carrier concentration decays as \( \exp(-L/L_d) \). Thus to reach 50% the distance is \( 2.24 \times 10^{-4} \text{ cm} \).

For the thin sample the transit time is

\[ t_{tr} = \frac{d^2}{2D_n} = \frac{(0.2 \times 10^{-4} \text{ cm})^2}{2 \times 100 \text{ cm}^2/\text{s}} = 2.0 \text{ ps} \]