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Fall 2001
EECS 320

Solutions to Homework 5

Problem 1 An abrupt silicon p-n diode at 300 K has a doping of $N_a = 10^{18}$ cm^{-3}, $N_d = 10^{15}$ cm^{-3}. Calculate the built-in potential and the depletion widths in the n and p regions.

Note that the depletion width falls primarily on the lightly doped side.

We assume that we can use the Boltzmann approximation on which the equations in the text are based. We have

$$\begin{aligned}
V_{bi} &= \frac{k_B T}{e} \, \ell n \, \frac{n_n}{n_p} \\
n_p &= \frac{n_i^2}{p_p} = \frac{(2.25 \times 10^{20} \text{ cm}^{-6})}{(10^{18} \text{ cm}^{-3})} = 2.25 \times 10^2 \text{ cm}^{-3} \\
V_{bi} &= (0.026 \text{ volt}) \, \ell n \, \left(\frac{10^{15}}{2.25 \times 10^2} \right) \\
&= 0.757 \text{ volt}
\end{aligned}$$

The p-side depletion width is

$$\begin{aligned}
W_p(V_{bi}) &= \left\{ \frac{2(11.9 \times 8.85 \times 10^{-14} \text{ F/cm})(0.757\ V)}{(1.6 \times 10^{-19} \text{ C})} \right. \\
& \quad \left. \left[\frac{10^{15} \text{ cm}^{-3}}{(10^{18} \text{ cm}^{-3})(10^{18} + 10^{15} \text{ cm}^{-3})} \right] \right\}^{1/2} \\
&= 9.89 \times 10^{-8} \text{ cm} = 9.89 \text{ Å!} \\
W_n(V_{bi}) &= W_p(V_{bi}) \times \frac{N_a}{N_d} = 9.89 \times 10^{-5} \text{ cm} \\
& \qquad = 0.989\ \mu\text{m}
\end{aligned}$$

Essentially all the depletion is on the lightly doped n-side.

Problem 2 Consider a p^+n Si diode with $N_a = 10^{18}$ cm^{-3} and $N_d = 10^{16}$ cm^{-3}. The hole diffusion coefficient in the n-side is 10 cm^2/s and $\tau_p = 10^{-7}$ s. The device area is 10^{-4} cm^2. Calculate the reverse saturation current and the forward current at a forward bias of 0.7 V at 300 K.

In order to calculate the current we need the values of L_p and p_n.

$$
\begin{aligned}
p_n &= \frac{n_i^2}{n_n} = \frac{2.25 \times 10^{20}\ \text{cm}^{-6}}{10^{16}\ \text{cm}^{-3}} = 2.25 \times 10^4\ \text{cm}^{-3} \\
L_p &= \sqrt{D_p \tau_p} = \left[(10\ \text{cm}^2/\ \text{s})(10^{-7}\ \text{s})\right]^{1/2} = 1.0 \times 10^{-3}\ \text{cm}
\end{aligned}
$$

The current is now (the current is controlled by hole injection),

$$
I = I_p(W_n) = \frac{eAD_p p_n}{L_p}\left(e^{eV/k_B T} - 1\right)
$$

The reverse saturation current is,

$$
\begin{aligned}
I_o &= \frac{(1.6 \times 10^{-19}\ \text{C})(10^{-4}\ \text{cm}^2)(10\ \text{cm}^2/\ \text{s})(2.25 \times 10^4\ \text{cm}^{-3})}{(1.0 \times 10^{-3}\ \text{cm})} \\
&= 3.6 \times 10^{-15}\ \text{A}
\end{aligned}
$$

The forward current at V = V_f = 0.7 V is

$$
\begin{aligned}
I(V = 0.4V) &= 3.6 \times 10^{-15}\left[\exp\left(\frac{0.7}{0.026}\right) - 1\right] \\
&= 1.77\ mA
\end{aligned}
$$

Problem 3 A GaAs LED has a doping profile of $N_a = 10^{17}\ \text{cm}^{-3}, N_d = 10^{18}\ \text{cm}^{-3}$ at 300 K. The minority carrier time is $\tau_n = 10^{-8}$ s; $\tau_p = 5 \times 10^{-9}$ s. The electron diffusion coefficient is 100 cm^2s^{-1} while that of the holes is 20 cm^2s^{-1}. Calculate the ratio of the electron-injected current (across the junction) to the total current.

The minority carrier densities in the n- and p-side are

$$
\begin{aligned}
p_n &= \frac{n_i^2}{n_n} = \frac{(3.24 \times 10^{12}\ \text{cm}^{-6})}{(10^{18}\ \text{cm}^3)} = 3.24 \times 10^{-6}\ \text{cm}^{-3} \\
n_p &= \frac{n_i^2}{p_p} = \frac{(3.24 \times 10^{12}\ \text{cm}^{-6})}{(10^{17}\ \text{cm}^{-3})} = 3.24 \times 10^{-5}\ \text{cm}^{-3}
\end{aligned}
$$

The diffusion lengths are,

$$
\begin{aligned}
L_n &= (D_n \tau_n)^{1/2} = (100 \times 10^{-8})^{1/2} = 10^{-3}\ \text{cm} \\
L_p &= (D_p \tau_p)^{1/2} = (20 \times 5 \times 10^{-9})^{1/2} = 3.16 \times 10^{-4}\ \text{cm}
\end{aligned}
$$

The electron injected current to total injected current rate is

$$
\begin{aligned}
\frac{J_n}{J_n + J_p} &= \frac{\frac{D_n n_p}{L_) n}}{\frac{D_n n_p}{L_n} + \frac{D_p p_n}{L_p}} = \frac{3.24}{3.24 + 0.205} \\
&= 0.94
\end{aligned}
$$

We note that due to the asymmetric doping most of the current is carried by electrons injected from the n-side into the p-side. In actual LEDs this is important since if the p-side is on the top (near the surface) electrons injected into the p-side can recombine with hole and the resultant photons have a high probability of escaping into air and being seen. Holes injected into the n-side will also generate photons but these photons may not be able to escape to the outside and create illumination. Thus our device is said to have an injection efficiency of 94%.

Problem 4: The diode of Problem 3 has an area of 1 mm^2 and is operated at a forward bias of 1.2 V. Assume that 50% of the minority carriers injected recombine with the majority charge to produce photons. Calculate the rate of the photon generation in the n- and p-side of the diode.

In the diode of Problem 3 we have the following parameters,

$$\begin{aligned} p_n &= 3.24 \times 10^{-6} \text{ cm}^{-3}; n_p = 3.24 \times 10^{-5} \text{ cm}^{-3} \\ L_n &= 10^{-3} \text{ cm}; L_p = 3.16 \times 10^{-4} \text{ cm} \end{aligned}$$

The electron current injected into the p-side is given by (hole injection is very small as we have noted in the previous problem)

$$\begin{aligned} In(-W_p) &= \frac{eAD_n n_p}{L_n}\left[\exp\left(\frac{eV}{k_B T}\right) - 1\right] \\ &= \frac{(1.6 \times 10^{-19} \text{ C})(10^{-2} \text{ cm}^2)(100 \text{ cm}^2/\text{s})(3.24 \times 10^{-5} \text{ cm}^{-3})}{(10^{-3} \text{ cm})}\left[e^{\left(\frac{1.2}{0.026}\right)} - 1\right] \\ &= 0.574\ A \end{aligned}$$

The rate at which electrons are injected into the p-side is

$$R_{inj} = \frac{I_n}{e} = 3.59 \times 10^{18} \text{ s}^{-1}$$

According to the problem, 50% of these electrons produce photons. Thus the photons emerge with a rate

$$I_{ph} = 1.79 \times 10^{18} \text{ s}^{-1}$$

The power generated (not asked for in the problem) is

$$P = Rate \times \hbar\omega = 0.41\ W$$

Problem 5 Compare the dark currents (i.e., reverse saturation current) in p-n diodes fabricated from GaAs and Si. Assume that all the diodes are doped at $N_d = N_a = 10^{18} \text{ cm}^{-3}$. The material parameters are (300 K):

$$\begin{aligned} \text{GaAs} &: \tau_n = \tau_p = 10^{-8} \text{ s}; D_n = 100 \text{ cm}^2/\text{s}; D_p = 20 \text{ cm}^2/\text{s} \\ \text{Si} &: \tau_n = \tau_p = 10^{-7} \text{ s}; D_n = 30 \text{ cm}^2/\text{s}; D_p = 15 \text{ cm}^2/\text{s} \end{aligned}$$

When p-n diodes are used as light detectors, the dark current is a noise source.

$$\begin{aligned} \text{GaAs :} &\quad n_p = p_n = 1.84 \times 10^{-6} \text{ cm}^{-3} \\ \text{Si :} &\quad n_p = p_n = 5.43 \times 10^{8} \text{ cm}^{-3} \end{aligned}$$

The reverse saturation current density is

$$J_o = e\left(\frac{D_p p_n}{L_p} + \frac{D_n n_p}{L_n}\right)$$

GaAs:

$$\begin{aligned} J_o &= \left(1.6 \times 10^{-19} \text{ C}\right)\left(\frac{20 \times 1.84 \times 10^{-6}}{4.47 \times 10^{-4}} + \frac{100 \times 1.84 \times 10^{-6}}{10^{-3}}\right) \\ &= 4.26 \times 10^{-20} \text{ Acm}^{-2} \end{aligned}$$

Si:

$$\begin{aligned} J_o &= \left(1.6 \times 10^{-19}\right)\left(\frac{15 \times 2.25 \times 10^{2}}{1.22 \times 10^{-3}} + \frac{30 \times 2.25 \times 10^{2}}{1.73 \times 10^{-3}}\right) \\ &= 1.07 \times 10^{-12} \text{ Acm}^{-2} \end{aligned}$$

GaAs has a much smaller reverse current due to the larger bandgap and lower intrinsic carrier density.