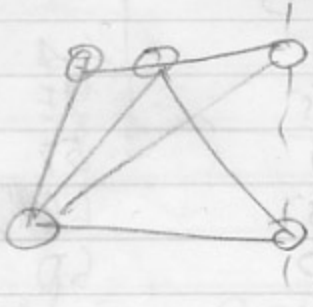
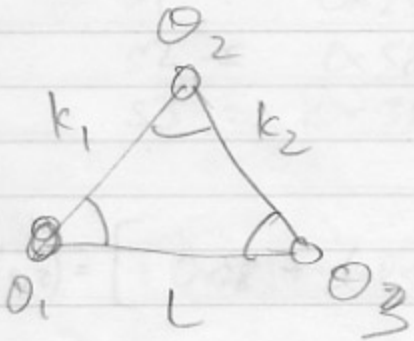
Servo angles calculation



$L^2 = r^2 + z^2$



k_1, k_2 fixed lengths
L depends on (r, z)

Using cosine law,

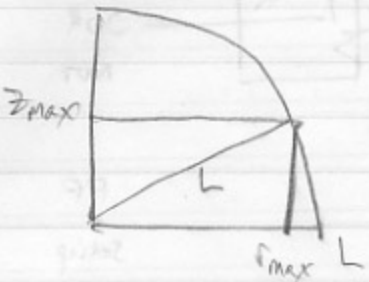
$$L^2 = k_1^2 + k_2^2 - 2k_1k_2\cos\theta_2$$
$$k_1^2 = L^2 + k_2^2 - 2k_2L\cos\theta_3$$
$$k_2^2 = k_1^2 + L^2 - 2k_1L\cos\theta_1$$

$$\Rightarrow \cos\theta_1 = \frac{L^2 + k_1^2 - k_2^2}{2k_1L}$$

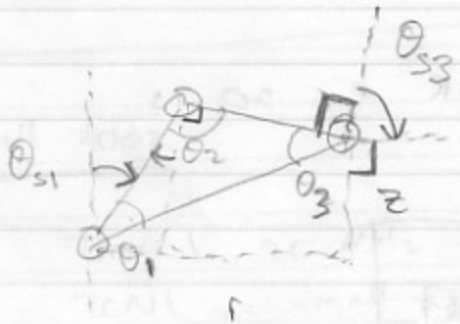
$$\cos\theta_2 = \frac{k_1^2 + k_2^2 - L^2}{2k_1k_2}$$

$$\cos\theta_3 = \frac{L^2 + k_2^2 - k_1^2}{2k_2L}$$

Max $L = k_1 + k_2 \Rightarrow 180° \ \theta_2$



$\therefore \quad r_{max}^2 + z_{max}^2 = L^2$



$$\boxed{\theta_{s2}} = \theta_2 - 90°$$
$$= \cos^{-1}\left(\frac{k_1^2 + k_2^2 - r^2 - z^2}{2 k_1 k_2}\right) - 90°$$

$$\theta_{s3} + \theta_3 + \cos^{-1}\left(\frac{r}{z}\right) = 180°$$

$$\Rightarrow \boxed{\theta_{s3}} = 180° - \cos^{-1}\left(\frac{r}{z}\right) - \theta_3$$
$$= 180° - \cos^{-1}\left(\frac{r}{z}\right) - \cos^{-1}\left(\frac{r^2+z^2+k_2^2-k_1^2}{2k_2\sqrt{r^2+z^2}}\right)$$

$$\theta_{s_1} + \theta_1 + \cos^{-1}\left(\frac{z}{r}\right) = 90°$$

$$\Rightarrow \boxed{\theta_{s_1}} = 90° - \cos^{-1}\left(\frac{z}{r}\right) - \cos^{-1}\left(\frac{r^2+z^2+k_1^2-k_2^2}{2k_1\sqrt{r^2+z^2}}\right)$$