EECS 401 **Midterm 1** **Feb. 9, 1998**

1 1/2 hours

Closed book. You may have **one side of a page of notes**. **Calculators** are permitted.

You may use without rederivation the results derived in class, the homework, the text, or distributed handouts, unless you are specifically asked to derive them.

Sign the **honor code pledge**. ("I have neither given nor received aid on this exam, nor have I concealed any honor code violations.")

Estimated points for each part of each problem are marked in square brackets. While grading, I sometimes find that changes are needed.

Testmanship:

- Some questions are short answer, for example, you may be asked to state a result. In such cases no explanation is required.
- Some questions say "show, prove or justify". This means you will be graded on how you demonstrate the needed fact.
- Some questions say "find the ..." or "what is the ..." . Although these don't require you to show your work or explain or justify your approach, if you make a mistake and I can see that you have a correct or partially correct approach, you can get partial credit. So it is recommended that you show your work and explain your approach (briefly).
- Suggestion: If there is something like a formula you need but cannot derive or recall, introduce some notation to represent it, state what it represents, and express the answer in terms of it, or describe what you would do if you knew the formula. This also applies if you cannot answer a part of a question upon which a later part depends.
- Don't try to cram too much on a page. Give yourself room to work and to add to or modify your answers.
- If you use a blue book, start each problem on a new "left" page, leaving the "right" page open for additions, modifications and checks. If you don't use a blue book, start each problem on a new page (so you leave room to modify your answer to the previous problem) and don't carry the answer from one page to the back of the same page (because you cannot see both sides of a page at once).

1. Short answer questions

[5] (a) **State** the formula for the conditional probability of event B given event A.

[5] (b) **State** Bayes rule. (Two versions were given in class; either will do)

[10] (c) **State** the axioms of probability.

[5] (d) **State**, mathematically, what it means for three events A, B, C to be independent.

2. E, F and G are events such that

$P(E \cap F) = 0.2$
$P(E \cap G) = 0.6$
$P(F \cap G) = 0$
$P(F \cup G) = 1$
$P(F) = 0.25$

[7] (a) **Find** the probability of E.

[7] (b) **Find** the probability of $E \cup F$

[6] (c) **Are** E and F independent? **Justify your answer.**

3. Suppose A, B and C are events such that

A and B are independent

B and C are independent

$P(A) = P(B) = P(C) \neq 0$

Which of the following must necessarily be true? (In case of mistakes, you can get partial credit if you justify your answer and your justification is partially correct.)

[5] (a) A and B are disjoint.

[5] (b) A and C are independent.

[5] (c) $P(A \cup B) = 2P(A) - (P(A))^2$

[5] (d) $P(A|B) = P(B|A)$.

4. In a certain community 36% of the families own a dog, and 22% of the families that own a dog also own a cat. In addition, 15% of the families who do not own a dog own a cat.

[10] (a) **Find** the probability that a randomly selected family owns both a dog and a cat.

[10] (b) **Find** the probability that a randomly selected family owns a dog given that it owns a cat.

5. Suppose that on the average 3% of people who make airplane reservations do not show up for their flight. Knowing this, an airline accepts 103 reservations for a plane that seats 100. **Find** the probability that when the plane is ready to leave, more passengers with reservations show up than there are seats. (If you are short of time, you can get substantial partial credit by giving just a correct expression for the probability, as opposed to the final numerical value.)

[15]