EECS 401 **Midterm 2** **April 1, 1998**

1 1/2 hours

Closed book. You may have **two sides of a page of notes**. **Calculators** are permitted.

Write your answers in a **blue book.**

You may use without rederivation the results derived in class, the homework, the text, or distributed handouts, unless you are specifically asked to derive them.

Sign the **honor code pledge**. ("I have neither given nor received aid on this exam, nor have I concealed any honor code violations.")

When you turn in your blue book, **put your two sides of a page of notes** in the back of the blue book. If you did not use such notes, indicate this in writing after the honor code pledge.

Estimated points for each part of each problem are marked in square brackets. While grading, I sometimes find that changes are needed.

Testmanship:

- Some questions are short answer, for example, you may be asked to state a definition or fact. In such cases no explanation is required.
- Some questions say "find the ..." or "what is the ..." . Although these don't require you to show your work or explain or justify your approach, if you make a mistake and I can see that you have a correct or partially correct approach, you can get partial credit. So it is recommended that you show your work and explain your approach (briefly).
- Suggestion: If there is something like a formula you need but cannot derive or recall, introduce some notation to represent it, state what it represents, and express the answer in terms of it, or describe what you would do if you knew the formula. This also applies if you cannot answer a part of a question upon which a later part depends.
- Don't try to cram too much on a page. Give yourself room to work and to modify your answers.
- Start each problem on a new "left" page of the blue book, leaving the "right" page open for additions, modifications, checks.

1. **Give** a brief precise definition or formula for each of the following. (No explanation is needed.)

[5] (a) A continuous random variable.

[5] (b) A discrete random variable

[5] (c) The joint cdf (cumulative distribution function) for random variables X and Y.

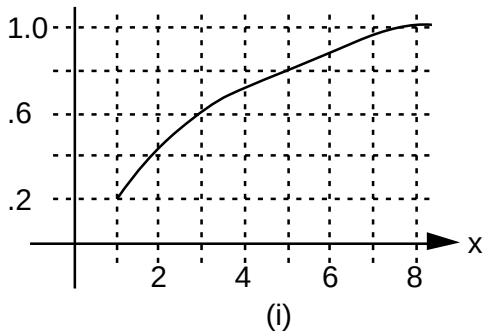
[5] (d) Independence of two discrete random variables X and Y.

[5] (e) The Gaussian density with mean 3 and variance 5.

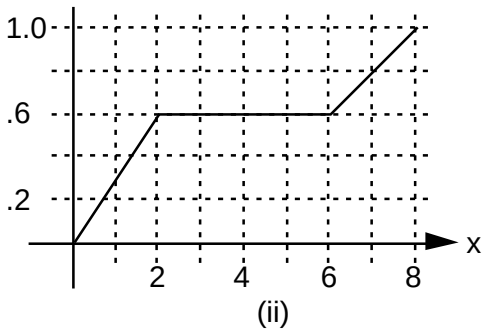
2.

[18] (a) For each of the three cdf's shown below, **state** the type of the random variable to which it corresponds, and **find** the probability that the outcome of the random variable is greater or equal to 3 and less than 5.

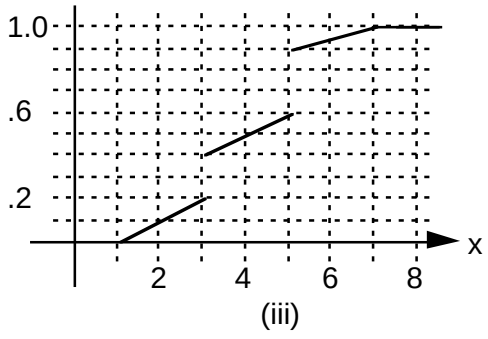
[7] (b) **Find** the expected value of the random variable whose cdf is given in (ii).



(i)



(ii)



(iii)

3. A random variable X is uniformly distributed on the interval [-3,3]. Let Y = g(X), where $g(x) = e^{-|x|}$.

[5] (a) **What** kind of random variable is Y?

[10] (b) **Find** the pdf (probability density function) of Y.

[5] (c) **Find** the probability that $Y \geq 1/2$.

[5] (d) **Find** the expected value of Y.

Note: It is possible to answer parts (c) and (d) even if you did not answer parts (a) and (b).

4. Suppose X and Y are jointly continuous random variables with joint pdf

$$f_{XY}(x,y) = \begin{cases} x e^{-(x+y)} , & x \geq 0, \ y \geq 0 \\ 0 , & \text{otherwise} \end{cases}$$

[7] (a) **Are** X and Y independent?

[8] (b) **Find** P(X>3, Y>4).

Note: $\int x e^{-x} dx = - e^{-x} (x+1)$

5. A random variable X is exponentially distributed with expected value 10. Let Y be a random variable whose conditional pdf given X = x is

$$f_{Y|X}(y|x) = \begin{cases} \frac{1}{10} e^{(x-y)/10} , & y \geq x \\ 0 , & \text{else} \end{cases}$$

[15] (a) **Find** the conditional pdf of X given Y = y.

[5] (b) **Find** the conditional expected value of X given Y = y.

[110 points total]