EECS 401 **Final Exam** **April 29, 1998**

2 hours: 8 AM to 10 AM

Closed book. You may have **three sides of a page of notes**.
Calculators are permitted.

Write your answers in a **blue book.**

You may use without rederivation the results derived in class, the homework, the text, or distributed handouts, unless you are specifically asked to derive them.

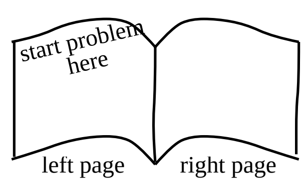
Sign the **honor code pledge**. ("I have neither given nor received aid on this exam, nor have I concealed any honor code violations.")

When you turn in your blue book, **staple your three sides of a page of notes** inside the back cover of the blue book. If you did not use such notes, indicate this in writing after the honor code pledge.

Estimated points for each part of each problem are marked in square brackets. While grading, I sometimes find that changes are needed.

Testmanship:

- Some questions are short answer, for example, you may be asked to state a definition or fact. In such cases no explanation is required.
- Some questions say "find the ..." or "what is the ..." . Although these don't require you to show your work or explain or justify your approach, if you make a mistake and I can see that you have a correct or partially correct approach, you can get partial credit. So it is recommended that you show your work and explain your approach (briefly).
- Suggestion: If there is something like a formula you need but cannot derive or recall, introduce some notation to represent it, state what it represents, and express the answer in terms of it, or describe what you would do if you knew the formula. This also applies if you cannot answer a part of a question upon which a later part depends.
- Don't try to cram too much on a page. Give yourself room to work and to modify your answers.
- Start each problem on a new "left" page of the blue book, leaving the "right" page open for additions, modifications, checks.



1. **Give** a brief precise definition or formula for each of the following. (No explanations needed.)

[5] (a) Random variable X is *continuous* . (You do not have to define "random variable".)

[5] (b) Discrete random variables X and Y are *identical* . (You do not have to define "discrete random variable".)

[5] (c) Continuous random variables X, Y and Z are *independent*.. (You do not have to define "continuous random variable".)

[5] (d) Random variables X and Y are *uncorrelated*.

[5] (e) *Bayes rule* for conditional probability densities.

[5] (f) The *autocorrelation function* of a random process $X(t)$.

2. Stores A, B and C have 50, 75 and 100 employees and, respectively, 50, 60 and 70% of these are women. Resignations are equally likely among all employees, regardless of gender. One employee resigns at random, and this is a woman. **What is** the probability that she works in store C?

[15]

3. A man and a woman agree to meet at a certain location at about 12:30 PM. The man arrives at a time that is uniformly distributed between 12 and 1 PM and the woman independently arrives at a time that is uniformly distributed between 12:15 and 12:45. **Find** the probability that they arrive within 5 minutes of each other?

[15]

4. On a passenger plane carrying 225 people, a maximum of 35,000 pounds is allocated for the weight of the passengers. The weights of airline passengers have a mean value of 150 pounds and a standard deviation of 50 pounds. **Find** the probability that the passenger weight limit of 35,000 pounds will be exceeded. (You may use an approximate method.)

[20]

5. Let (X_1,X_2,X_3) be jointly Gaussian random variables with zero means and covariance matrix

$$C = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

[5] (a) **Find** the correlation coefficient for X_1 and X_2.

[10] (b) **Find** $E\left[(X_1+X_2+X_3)^2\right]$

6. Consider the continuous-time random process

$$X(t) = \cos 2\pi Wt, \quad -\infty < t < \infty,$$

where W is a random variable that is uniformly distributed on the interval [0,1].

[5] (a) **Find** the mean function of $X(t)$

[5] (b) **Is** $X(t)$ widesense stationary? **Justify** your answer.

[10] (c) **Find** the probability that from time 0 to 3, the random process does not equal zero.

7. A stationary, Gaussian, continuous-time random process $X(t)$ has zero mean and power spectral density function

$$S_X(f) = 5, \quad -\infty < f < \infty .$$

This random process is the input to a linear time-invariant system with frequency response

$$H(f) = e^{-|f|}, \quad -\infty, < f < \infty .$$

Let $Y(t)$ denote the output of the system.

[5] (a) **Find** the power in the **input** random process $X(t)$.

[5] (b) **Find** the mean function of the **output** random process $Y(t)$.

[10] (c) **Find** the power in the **output** random process $Y(t)$.

[10] (d) **Find** the probability that the **output** at time 3 is at least 4.

Note: This is one of those problems where if there is something you need to get the answer but cannot determine, you can get partial credit by clearly indicating what it is that you need and showing how you would use it to compute the answer. For example, use a symbol or symbols to represent the unknown quantity and express the answer terms of the symbol(s).

[145 points total]