Homework Set 1 Solutions **EECS 401** **Friday, Jan. 21, 2000**

This is the "final" version. Revisions to the 1/14/00 draft are marked with asterisks: *******.
Problems from Leon-Garcia's book

1. 1-11, p. 22. See Problems 1-7 and 1-9 for definitions of sample mean and sample variance.

   a) Sample mean: $\mathbf{\langle x\rangle_{20} = 8.55}$, because n = 20 and $\langle x\rangle_n = \frac{1}{n}\sum_{i=1}^{n} x_i$ where $x_1,\ldots,x_n$ is the data.

   Sample variance: $\mathbf{\langle v^2\rangle_{20} = 29.05}$, because $\langle v^2\rangle_n = \frac{1}{n}\sum_{i=1}^{n}(x_i - \langle x\rangle_m)^2$

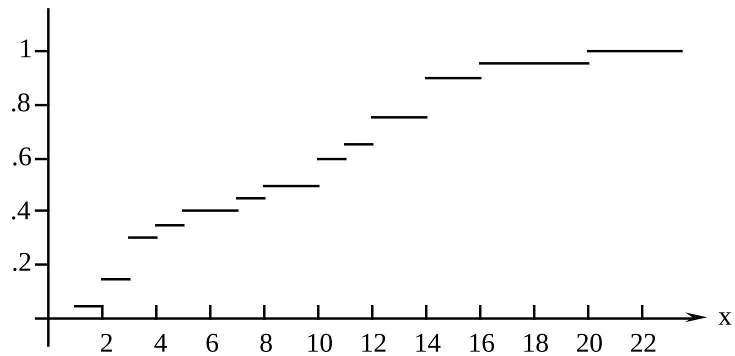
   b) The event "interarrival time is greater than 10 ms" occurs 8 times out of 20.

   Relative frequency = $\mathbf{\frac{2}{5}}$

******* c) outcomes in sequence

outcomes in sequence	1	2	3	4	5	7	8	10	11	12	14	16	20
frequency	$\frac{1}{20}$	$\frac{2}{20}$	$\frac{3}{20}$	$\frac{1}{20}$	$\frac{1}{20}$	$\frac{1}{20}$	$\frac{1}{20}$	$\frac{2}{20}$	$\frac{1}{20}$	$\frac{2}{20}$	$\frac{3}{20}$	$\frac{1}{20}$	$\frac{1}{20}$
empirical distributionn function values	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{6}{20}$	$\frac{7}{20}$	$\frac{8}{20}$	$\frac{9}{20}$	$\frac{10}{20}$	$\frac{12}{20}$	$\frac{13}{20}$	$\frac{15}{20}$	$\frac{18}{20}$	$\frac{19}{20}$	$\frac{20}{20}$

   plot of empirical distribution function:



2. 2-1, p. 73

   a) sample space: $\mathbf{S = \{1,2,3,4,5,6\}}$

   b) $\mathbf{A = \{2,4,6\}}$

   c) $\mathbf{A^c = \{1,3,5\}}$**.** This is the event "odd number of dots".

   (d) Probability law: $\mathbf{P(A) = \frac{|A|}{6}}$ **, for any subset A of S**, where |A| denotes the size of A.

   Another sufficient answer is: $\mathbf{P(i) = \frac{1}{6}}$ **, i = 1,...,6**.

3. 2-3, p. 73

   a) sample space: $\mathbf{S = \{2,3,4,5,6,7,8,9,10,11,12\}}$

   b) $\mathbf{A = \{2,4,6,8,10,12\}}$

   c) event "sum = 2" corresponds to {(1,1)}
   event "sum = 3" corresponds to {(1,2),(2,1)}

event "sum= 4" corresponds to {(1,3),(3,1),(2,2)}
and in general for 1 <k ≤ 12

event "sum = k" corresponds to $\displaystyle {}_{j=1}^{\min(6,k-1)} \{(j,k-j)\}$

4. 2-4, p. 73

a) sample space **S** = **{(1,1), (2,1), (2,2), (3,1), (3,2), (3,3), (4,1), (4,2), (4,3), (4,4), (5,1), (5,2), (5,3), (5,4), (5,5), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}**

b) **{(4,1), (4,2), (4,3), (4,4)}**

c) **{(3,3), (4,3), (5,3), (6,3)}**

d) **{(6,6)}**

5. 2-6, p. 73

a) sample space: $\mathbf{S = \{(F,F), (F,R), (F,K), (R,F), (R,R), (R,K), (K,F), (K,R), (K,K)\}}$

b) event = $\mathbf{\{(F,F), (F,R), (R,F), (R,R)\}}$

6. 2-7, p. 73

a) sample space: $\mathbf{S = \{(1,2,3), (2,1,3), (3,1,2), (1,3,2), (2,3,1), (3,2,1)\}}$

b) $A_1 = \{(1,2,3), (1,3,2)\}$, $A_2 = \{(1,2,3), \{3,2,1)\}$, $A_3 = \{(1,2,3), (2,1,3)\}$

c) $A_1 \cap A_2 \cap A_3 = \{(1,2,3)\}$

which can be interpreted as "the number of each ball drawn matches the number of the draw"

d) $A_1 \quad A_2 \quad A_3 = \{(1,2,3), (1,3,2), (3,2,1), (2,1,3)\}$

which can be interpreted as "at least one of the balls drawn matches the number of the draw"

*** e) $(A_1 \quad A_2 \quad A_3)^c = \{(3,1,2), (2,3,1)\}$

which can be interpreted as "none of the balls drawn matches the number of the draw"

7. 2-15, p. 74 (Use the axioms, not Venn diagrams, though Venn diagram may suggest what to do.)

a) sample space: $\mathbf{S = \{(T_1,T_2): 0 \le T_1 < T_2 \le 24\}}$

b) $\mathbf{A}$ = {awake at 9} = $\{ T_1 \le 9 \text{ and } T_2 \ge 9\}$ = { wake up before 9 } ∩ {go to sleep after 9}

$\mathbf{= \{ (T_1,T_2): 0 \le T_1 < 9 \text{ and } 9 < T_2 \le 24 \}}$

c) the amount of time the student is awake is $T_2 - T_1$. the amount of time the student is asleep is $24-(T_2-T_1)$. Therefore,

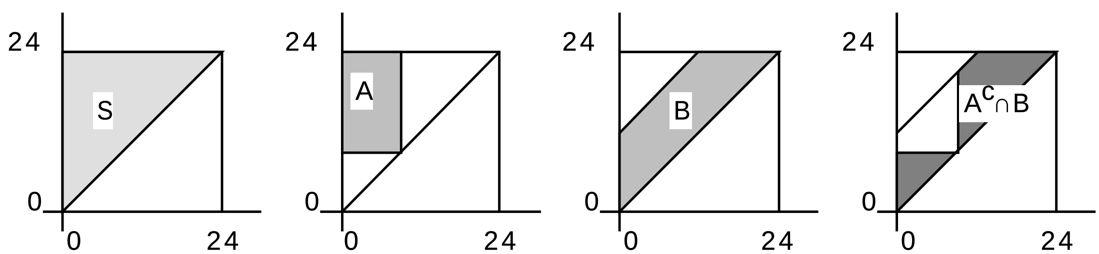
$\mathbf{B}$ = { student asleep more than he is awake } = $\{ (T_1,T_2) \quad S : 24-(T_2-T_1) > T_2-T_1 \}$

$= \{ (T_1,T_2) \quad S : T_2-T_1 < 12 \} = \mathbf{\{ (T_1,T_2) \quad S : T_2 < T_1 + 12 \}}$

c) $A^c = \{ (T_1,T_2) \quad S : (T_1,T_2) \quad A \} = \{ (T_1,T_2) : 0 \le T_2 < T_2 \le 24, \text{ and } (T_1 > 9 \text{ or } T_2 < 9)\}$

$\mathbf{A^c \cap B = B - A =}$ **{student is asleep at 9 and sleeps for more than 12 hours}**

sketches:



8. Find a probability model for the experiment described in Prob. 2-17, p. 75.
Then do Problem 2-17

The sample space is $S = \{(1,1), (1,2),...,(1,6),(2,1),...,(2,6),(3,1),...,(6,6)\}$, and the probability of each elementary event $\{(i,j)\}$ is 1/36.

a) $A_2 = \{ (1,1) \}$, $\mathbf{P(A_2)} = P(\{(1,1)\}) = \mathbf{\frac{1}{36}}$

$A_3 = \{ (1,2), (2,1) \}$, $\mathbf{P(A_3)} = P(\{(1,2)\}) + P(\{2,1\}) = \frac{1}{36} + \frac{1}{36} = \mathbf{\frac{2}{36}}$

$A_4 = \{ (1,3), (3,1), (2,2) \}$, $\mathbf{P(A_4)} = 3 \times \frac{1}{36} = \mathbf{\frac{1}{12}}$, $\mathbf{P(A_5)} = 4 \times \frac{1}{36} = \mathbf{\frac{1}{9}}$

Similarly, $\mathbf{P(A_6) = \frac{5}{36}}$, $\mathbf{P(A_7) = \frac{6}{36}}$, $\mathbf{P(A_8) = \frac{5}{36}}$, $\mathbf{P(A_9) = \frac{4}{36}}$,

$\mathbf{P(A_{10}) = \frac{3}{36}}$, $\mathbf{P(A_{11}) = \frac{2}{36}}$, $\mathbf{P(A_{12}) = \frac{1}{36}}$

b) B = {two outcomes are different}. We could list the members of this set, but it's easier to list the elements of the complement:

B^c = {two outcomes are the same} = $\{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$P(B^c) = \frac{6}{36}$. And therefore $\mathbf{P(B)} = 1 - P(B^c) = 1 - \frac{6}{36} = \mathbf{\frac{30}{36}}$

9. 2-19, p. 75

There are many ways to do this problem. Here's one.

{exactly one of the events A or B ocurrs} = { x S : (x is in A but not B) or (x is in B but not A)}

$= (A - B) \quad (B - A)$.

$P(A-B) = P(A) - P(A \cap B)$ (because $A-B$, $A \cap B$ are disjoint , and $(A-B) \quad (A \cap B) = A$, so by Axiom 3: $P(A-B)+P(A \cap B) = P(B)$)

Similarly, $P(B-A) = P(B) - P(A \cap B)$. Finally, since $(A - B)$ and $(B - A)$ are disjoint

$$\mathbf{P((A - B) \quad (B - A))} = P(A - B) + P(B - A) = P(A) - P(A \cap B) + P(B) - P(A \cap B)$$

$$= \mathbf{P(A) + P(B) - 2P(A \cap B)}$$

10. 2-29, p. 76

Let T denote the lifetime. Then $A = \{T > 5\}$ and $B = \{T > 10\}$.

a) $\mathbf{P(A \cap B)} = P(\{T > 5\} \cap \{T>10\}) = P(\{T>10\}) = \mathbf{e^{-10}}$

$\mathbf{P(A \quad B)} = P(\{T > 5\} \quad \{T>10\}) = P(\{T>5\}) = \mathbf{e^{-5}}$

b) {lifetime greater than 5 but less than or equal to 10} = $\{5 < T \leq 10\}$

Now, $\{T > 5\} = \{5 < T \leq 10\} \quad \{T > 10\}$ and the two events on the righthand side are disjoint, so $P(\{T > 5\}) = P(\{5 < T \leq 10\}) + P(\{T > 10\})$. Therefore,

$$\mathbf{P(\{5 < T \leq 10\})} = P(\{T > 5\}) - P(\{T > 10\}) = \mathbf{e^{-5} - e^{-10}}$$