Homework Set 8 **EECS 401** **Due: Monday, March 20, 2000,**
in class before lecture begins.

Things to practice: working with conditional pmf's, pdf's and cdf's, and the Big Four.

1. 4.30, p. 259

2. 4.32, case (iii), p. 259

3. 4.35, p. 260 (Hint: First, find $F_T(t)$ using the law of total probability. A big part of this problem is figuring out exactly what the problem statement provides.)

4. A random variable X is exponentially distributed with expected value 10. Let Y be a random variable whose conditional pdf given X = x is

$$f_{Y|X}(y|x) = \begin{cases} \frac{1}{10} e^{(x-y)/10}, & y \geq x \\ 0, & \text{else} \end{cases}$$

Find the conditional pdf of X given Y = y.

5. Joe buys his gasoline at the local cut-rate gas station. The station sells two grades of gas, A and B, but the station owner won't tell which you get. Joe is concerned because his car mileage varies uniformly between 20 and 32 miles per gallon with brand A and between 16 and 26 miles per gallon with brand B. Suppose Joe knows the probabilities of the gas station having brands A and B are .3 and .7, respectively. Let X be a random variable representing Joe's gas mileage.

   (a) Find the cumulative distribution function of X.

   (b) Joe has found that his gas mileage is at least 24 . Given this, determine the probability that he is using brand B.

6. A gambler brings X dollars to a casino where X is a random variable with density

$$f_X(x) = \begin{cases} \frac{x}{80{,}000}, & 0 \leq x \leq 400 \\ 0, & \text{otherwise} \end{cases}$$

   After a night of gambling the gambler leaves the casino with Y dollars, where Y is uniformly distributed between 0 and X .

   (a) Given the gambler leaves the casino with less than \$200 dollars, find the probability that he brought less than \$200.

   (b) Find the probability that his loss was less than \$100.

   (c) Find the probability that his loss was exactly \$75.

   (d) Find the density of Y.

7. Suppose X and Y are jointly continuous, independent random variables. Show that

$$P(Y \leq X) = \int_{-\infty}^{\infty} F_Y(y)\, f_X(y)\, dy$$