Homework Set 9 **EECS 401** **Due: Friday, March 30, 2000,**
in class before lecture begins.

Things to practice: conditional expectation, estimation rules, correlation and covariance, uncorrelated vs. independence, the distribution of multiple random variables (big 4 and chain rule), functions of multiple random variables, jointly Gaussian random variables

1. Joe buys his gasoline at the local cut-rate gas station. The station sells two grades of gas, A and B, but the station owner won't tell which you get. Joe is concerned because his car mileage varies uniformly between 20 and 32 miles per gallon with brand A and between 16 and 26 miles per gallon with brand B. Suppose Joe knows the probabilities of the gas station having brands A and B are .3 and .7, respectively. Let X be a random variable representing Joe's gas mileage. (This is same descriptions as Prob. 5 of previous homework.)

   Find the expected value of X WITHOUT integrating x times the density of X.

2. A gambler brings X dollars to a casino where X is a random variable with density

$$f_X(x) = \begin{cases} \frac{x}{80,000}, & 0 \le x \le 400 \\ 0, & \text{otherwise} \end{cases}$$

   After a night of gambling the gambler leaves the casino with Y dollars, where Y is uniformly distributed between 0 and X . (This is same descriptions as Prob. 6 of previous homework.)

   (a) Find the expected value of Y WITHOUT integrating x times the density of Y.

   (a) Given the gambler leaves the casino with $200 dollars, find the expected number of dollars that he brought.

3. 4.39, p. 260

4. 4.41, p. 261 (don't use the chain rule; you are, in effect, rederiving it for 3 random variables.)

5. Let W = X + 3 Y - Z, where X, Y and Z be uncorrelated Gaussian random variables with means $m_X = 1$, $m_Y = 2$ and $m_Z = 3$, and variances $\sigma_X^2 = 1$, $\sigma_Y^2 = 2$, $\sigma_Z^2 = 3$.

   Find $P(W \ge 1)$. Hint: W is a linear combination of Gaussian random variables.

6. 4.51, p. 262

7. 4.61, p. 263

8. 4.67, p. 263

9. Let X,Y be continuous random variables with joint density

$$f_{XY}(x,y) = \begin{cases} x+y, & 0\le x\le 1, 0\le y\le 1 \\ 0, & \text{else} \end{cases}$$

   (a) Find the best linear estimate for Y given X=x.

   (b) Find the best overall estimate for Y given X =x.