Study Problems **EECS 401** **Friday, April 14, 2000**

Not for handing in. These are problems that would have been on a homework that covered the last week of class material. They are as important for preparing for the final exam as a typical homework set.

Solutions will be available on the web and outside my office (Rm 4215) on Wednesday, April 19.

Readings on power spectral density and linear filters: Sections 7.1 pp. 403-409 (we have studied power spectral density only for continuous-time random processes), Section 7.2 pp. 413-419.

Things to Practice: Working with mean functions, autocorrelation functions, power spectral densities, wide-sense stationary random processes, and Gaussian random processes. Calculating the mean, autocorrelation function and power spectral density of the output of a filter when the input is a wide-sense stationary probability. Calculating the probability of an event at the output of a filter when the input is a wide-sense stationary Gaussian random process.

1. The autocorrelation function of a widesense stationary random process {X(t)} is

$$R_X(\tau) = a\, e^{-b\tau^2}$$

   (a) Find its power spectral density. (Hint: the integral of a Gaussian probability density is one.)

   (b) Find the mean of this random process.

   (c) Find the (average) power of this random process.

   (d) How much power does this random process have in the frequency band [2,3]? (Hint: Q-function tables are useful.)

2. A widesense stationary random process {X(t)} has power spectral density

$$S_X(f) = \frac{6f^2}{1+f^4}$$

   Find the power of the random process.

3. Let {X(t)} and {Y(t)} be independent widesense stationary random processes with power spectral densities $S_X(f)$ and $S_Y(f)$, respectively. Find the power spectral density of the random processes {U(t)} and {V)t)} defined by

   (a) U(t) = a + b X(t)

   (b) V(t) = X(t) + Y(t)

4. White noise {X(t)} with power spectral density η is applied to an ideal low pass filter with frequency response

$$H(f) = \begin{cases} 1, & |f| \le W \\ 0, & |f| > W \end{cases}$$

   (a) Find and sketch the autocorrelation function of {Y(t)}, the random process at the output of the filter.

   (b) Let $Z_n = Y(n/2W)$ be the sample of the ouput taken at times n/2W. Find the autocorrelation function of the discrete-time random process $\{Z_n\}$ and comment on what you find.

5. A wide-sense stationary Gaussian random process {X(t)} with mean 0 and autocorrelation function $R_X(\tau) = 3e^{-|\tau|}$ is the input to a filter (linear time-invariant system) with impulse response $h(t) = e^{-t}$, $t \ge 0$ and $h(t) = 0$ for $t < 0$.

   (a) Find the probability that the output of the filter is less than 3 at time 5 .

   (b) Find an expression for the power of the output of the filter in the frequency band [1,2]. You may leave your answer as an integral.

6. 7.8, p. 451

7. 7.19, p. 452, except to simplify the problem a bit, let $Y(t) = \frac{1}{T}\int_{-T/2}^{T/2} X(t')\, dt'$