

EECS 452 *Midterm* Closed book part

Fall 2010

Name: _____ unique name: _____

Sign the honor code:

I have neither given nor received aid on this exam nor observed anyone else doing so.

Scores:

#	Points
Closed book	
Page 2	/12
Page 3	/12
Open book	/76
Total	/100

NOTES:

- There are **3** pages including this one.
- On the *closed book* section you may not have books, notes, or calculator. Nothing but a writing utensil.
- Don't spend too much time on any one problem.
- You have about 120 minutes for the exam. You probably shouldn't spend more than 15-20 minutes on this section.

Closed book portion

1. Say you took a 16-point DFT of the following signals with $f_s=800\text{Hz}$ starting at time $t=0$. What would be what the non-zero DFT outputs would be (magnitude and phase)?

[3 points each]

a. $2*\sin(2\pi*200t + \pi/2)$

$X(4) = 16@0$

$X(12) = 16@0$

b. $2*\cos(2\pi*150t - 1) - 2$

$Y(0) = 32@pi$

$Y(3) = 16@-1$

$X(13) = 16@1$

All angles in radians.

2. Draw a block diagram which corresponds to the following transfer function. **[6 points]**

$$H(z) = \frac{z^{-1} + 1}{1 + z^{-1}}$$

3. *Fill in the blank or circle the best answer.* Provide all numbers in decimal. [12 points, -1 per wrong or blank answer, minimum 0]
- If we multiply an 8-bit $Q7$ value by an 8-bit $Q7$ value the result would be a 16-bit Q 14 number.
 - 4 is the smallest number (closest to negative infinity) that can be exactly represented by a signed 12-bit $Q9$ number. $31/4$ is the largest number (closest to positive infinity) that can be exactly represented by a signed 6-bit $Q2$ number.
 - In signed 5-bit two's complement $Q3$, what is the value of 11110? -0.25.
 - Say you took an N -point DFT of a signal with a sample frequency of f_s . The frequency spacing of the DFT outputs would be f_s/N .
 - We say a filter is has *linear phase* if the group delay is a constant / proportional to the input frequency / zero / inside the unit circle.
 - All FIR filters with real-valued coefficients have linear phase / are stable / are low-pass.
 - A 10Hz cos wave has a period of 100 ms.
 - Say you have an ideal A to D converter which converts values from 8V to -8V to a 4-bit value. In the ideal case, the worst-case quantization error is \pm 0.5 Volts.
 - FIR filters tend to require more / fewer / the same number of stages to implement a given filter when compared to an IIR filter.
 - FIR filters can easily / cannot / can with great difficulty implement a typical RC analog filter.

EECS 452 *Midterm* Open book part

Fall 2010

Name: _____ unique name: _____

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Scores:

#	Points
Open book	
Page 2	/8
Page 3	/7
Page 4	/11
Page 5	/12
Page 6	/9
Page 7	/12
Page 8	/6
Page 9	/11
Total	/76

NOTES:

- There are **9** pages including this one.
- On the *open book* section you may use calculators, books and notes, but no PDAs, Portables, Cell phones, etc.
- Don't spend too much time on any one problem.
- You have about 120 minutes for the entire exam.
- Some questions are much harder than others...

1) DFT/FFT

a. Say you have a 60-point DFT which is processing data sampled at 600Hz.

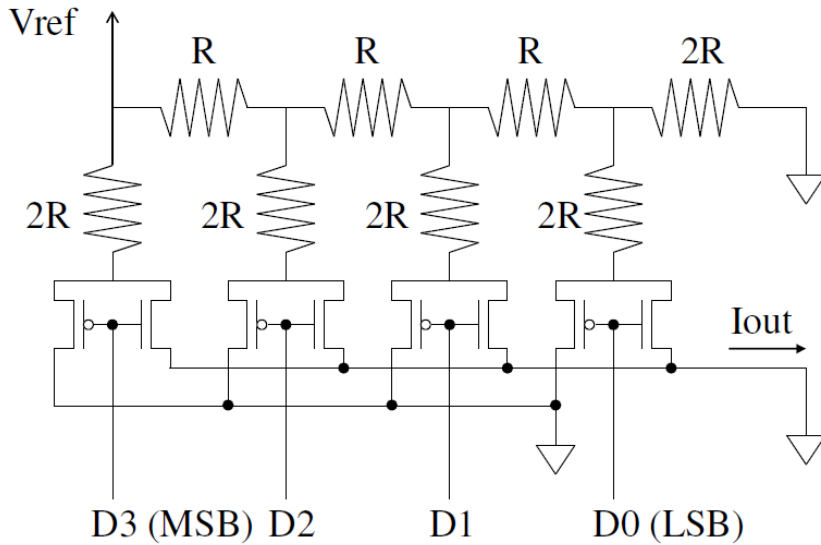
- i. What frequency is associated with $X(2)$? 20Hz [1]
- ii. Why would it be difficult to use the FFT algorithm we discussed in class? Your answer must be 20 words or less. [2]

We discussed a radix-2 FFT algorithm. That requires the number of inputs be a power of 2.

- iii. Say the data passed into the DFT were from a 15Hz sinusoid and $x(0)=0$ while $x(5)=1$. Would $X(0)$ be positive, negative or zero? You must clearly justify your answer to get any points. A picture might be useful... [5]

15Hz clearly isn't one of the frequencies represented by the FFT, so we expect leakage. We know that the sinusoid will manage exactly 1.5 periods and that at time 0 we are seeing the start of the sinusoid going above zero. Therefore we expect that the "unfinished" part of the sinusoid is above zero. So the DC component, $X(0)$, is positive.

Answer the following questions about the ideal D2A converter shown



b. Assuming $V_{ref}=2V$ and $R=4k\Omega$, if $D[3:0]=0111$, what is I_{out} ? [2]

$I_{in}=0.5mA$. $7/16*0.5mA$ is $0.21875mA$.

c. If we were going to make an 8-bit converter using this R/2R ladder scheme, how many transistors would we need? [2]

16

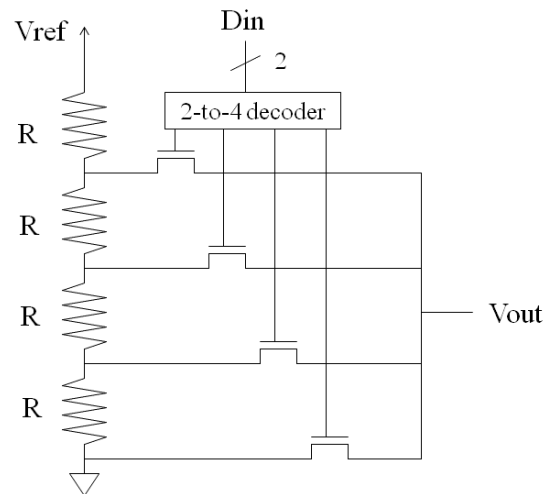
2) Consider the traditional voltage divider DAC (a version of which is seen on the right)

a. How many transistors would you need for an 8-bit converter using this scheme? [1]

2^8 or 256 (not including the decoder)

b. For a 4-bit version of this DAC, what value would you expect to have at V_{out} if $V_{ref}=4V$, $R=4k\Omega$, and $D_{in}=0110$? [2]

1.5V



4. Consider the following DDS code. You may assume it is intended for a machine where an int is a 32-bit number.

```
#include<stdio.h>
int array[50]={
    0,      4106,      8148,      12062,      15785,
    19259,  22430,  25247,  27666,  29648,
    31163,  32186,  32702,  32702,  32186,
    31163,  29648,  27666,  25247,  22430,
    19259,  15785,  12062,  8148,  4106,
    0,      -4106,     -8148,    -12062,   -15785,
    -19259, -22430, -25247, -27666, -29648,
    -31163, -32186, -32702, -32702, -32186,
    -31163, -29648, -27666, -25247, -22430,
    -19259, -15785, -12062, -8148, -4106};

void main()
{
    int counter1=20,counter2=0;
    int total;
    while(1)
    {
        total=array[counter1/10]+array[counter2/10];
        counter1=(counter1+7)%500;
        counter2=(counter2+14)%500;
        printf("%d\n",total);
    }
}
```

- a. Assuming “array” is intended to be an approximation of a unit amplitude sine function, what Q value is being used by “array”? [2]

Q15

- b. What would be the second number printed by this program? What value does it correspond to given your answer to part “a”? Be sure to pay attention to how counter1 and counter2 are initialized. [2]

4106 + 8148 = 12254

- c. Assuming $f_s=1\text{MHz}$, what are the frequency components of the output? Provide your answer in terms of amplitude, phase and frequency for each component. [7]

#1) Amplitude 1, frequency 14 KHz, phase $1/25*(2\pi)$

#2) Amplitude 1, frequency 28 KHz, phase 0.

(Note: phase above is relative to a sin wave, as long as the phase difference was correct you could do it relative to anything, many did phase wrt cos() for example.)

5. Short answer.

- a. Explain, though words and a simple illustration, why having ‘linear phase’ is a desirable property for filters. [3]

Non-linear phase is the same as saying non-constant delay. This means that the input waveform could get distorted in the time domain. Ideal picture would be a square wave input with a distorted wave after the non-linear phase filter.

- b. Provide an example of a filter application where a digital filter cannot be used. Explain why. [3]

Say we have an input with arbitrarily high frequencies and we want to filter everything out above (say) 1KHz. We’d need to sample arbitrarily fast. Other valid answers would include A) When we don’t have power available for an active bit of hardware. B) When the power output requirements of the system are too high (filtering in the context of power-lines for example). The one is more “not impossible, but stupid”, but we took it anyways.

- c. What would be the result of sampling an 8 kHz sine-wave at (Note: this question was clarified during the exam)

- i. 4kHz? [2]

You’d observe a DC signal.

- ii. 6kHz? [2]

You’d observe a 2kHz sine-wave.

- iii. 10kHz? [2]

You’d observe a 2kHz sine wave (well negative 2kHz) if you are watching the phase)

- d. Draw a block diagram which corresponds to the following difference-equation. [3]
 $y[n]=2*x[n] + x[n-2] - y[n-3]$

No answer provided.

- e. Consider the following difference equation
 $y[n] = x[n] + x[n-2] + 0.8 y[n-1]$

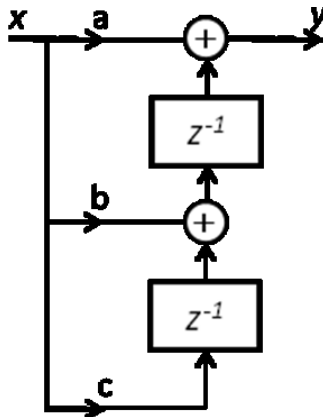
- i. Write the transfer function that corresponds to that difference equation. [3]

$H(z)=(1+z^{-2})/(1-0.8z^{-1})$

- ii. Is the corresponding filter stable? Justify your answer [3]

It is. The two poles are at 0 and .8, both inside of the unit circle.

6. Write a function, `short trans (short x, short *w)` which takes a single input `x`, and returns the `y` value. Assume $a=1.5$, $b=-.5$ and $c=1$. You may also assume that the array `w` is of any fixed size you find useful. All inputs are in 16-bit Q14 and the output is also to be in 16-bit Q14. If the output is determined to be outside of the range of a 16-bit Q14 you should saturate. You are to write this for a 16-bit machine (that is, ints are 16 bits). [12]



```

short trans (short x, short *w)
{
    //Choosing to use Q14 for constants
    short a= 3 <<13;
    short b=-1 <<13;
    short c= 2 <<13;
    long y;

    y= (long)x*a + (long)w[0]<<14; // Now in Q28
    w[0]= ((long)x*b+0x2000 >> 14) + w[1];
    w[1]= ((long)x*c+0x2000 >> 14);
    if(y>0x1FFFFFFF) y=0x1FFFFFFF;
    if(y<0xE0000000) y=0xE0000000;
    return((short) (y+0x2000)>>14); //Q28 shifted 14 the left is Q14
}

```

Note: There are still significant overflow issues here, but given the question this is the best we can do. Don't be shocked if the final exam asks about overflow issues with respect to a similar (or identical) question.

7. It is known that the group delay of a given linear-phase filter is 0.02 seconds. Assuming the following frequencies are in the pass-band region, what would be the expected phase of each signal? Provide your answers in degrees in the range of 0 to -360 and show your work. [6]

a) 10Hz -72 degrees

b) 100Hz 0 degrees (or -360 given the wording above)

c) 75Hz -180 degrees

8. IIR biquad filters

- a. Why do we generally break down an IIR filter into biquad sections (2 zeros and 2 poles per section) rather than some other size? Specifically why not 1 zero and 1 pole per section? How about 4? [4]

Having only one zero and pole would mean we'd need to deal with complex numbers (the complex conjugates couldn't be paired). Having four would increase the range in values the coefficients could take and would cause greater potential for quantization and overflow issues.

- b. Explain how we group poles and zeros into biquad sections. Why do we group them in the way that we do? [3]

In case we've grouped the biquad sections by choosing the pole closest to the unit circle and pair it to the zero closest to itself (Cartesian distance). We then select the remaining pole closest to the unit circle and pair it with the closest remaining zero and so on. We group them this way in an attempt to reduce the maximum gain of the biquad pair by putting those with the largest maximum gain (highest "quality") with those that have a zero closest (in some combination of angle and sharpness). This should reduce internal overflow.

- c. Explain how we order the biquad sections. Why do we pick that ordering? [2]

We order with those poles farthest from the unit circle going first, next closest second, etc. The argument is that in doing so the higher "quality" poles will be largely cancelled out by those biquad sections in front of them, reducing overflow and quantization issues. The last biquad, for example, will produce the expected filter output which generally is free of spikes (if it is unit gain).

- d. Why do we normalize the filter coefficients? [2]

To reduce the risk of internal overflow by evenly distributing the gain among the biquad sections.