

EECS 452 Practice Midterm Exam

Solutions

Fall 2014

Name: _____ unique name: _____

Sign the honor code:

I have neither given nor received aid on this exam nor observed anyone else doing so.

Scores:

#	Points
Section I	/40
Section II	/30
Section III	/30
Total	/100

NOTES:

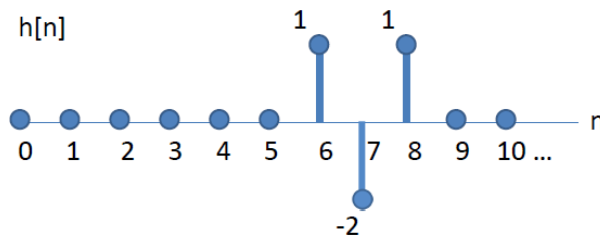
- Open book, open notes.
- There are **8** pages including this one.
- Calculators are allowed, but no PDAs, Portables, Cell phones, etc.
- You have 120 minutes for the exam.
- **Be sure to show work and explain what you've done when asked to do so.** You will not receive partial credit without showing work.
- Unless otherwise specified all signed numbers are two's complement numbers.

Section I -- Short answer 40 points

1) Convert the Hexadecimal value 0xC3 into base 10 assuming it is: [4]

- a) An unsigned binary number: 195
- b) A 2's complement number: -61
- c) An unsigned Q6 number: 3.0469
- d) A signed (two's complement) Q6 number -0.9531

2) A FIR filter has the impulse response shown below [6]



a) Find and plot the magnitude and phase of this filter's transfer function? [3]

$$H(z) = z^{-6} - 2z^{-7} + z^{-8} = z^{-7}(z + z^{-1} - 2).$$

Therefore

$$H(f) = e^{-j2\pi f 7} (e^{j2\pi f} + e^{-j2\pi f} - 2) = e^{-j2\pi f 7} (2\cos(2\pi f) - 2) = 4\sin^2(\pi f)e^{-j2\pi f 7 + j\pi}$$

Plot the magnitude $4\sin^2(\pi f)$ over f and see that it is a bandpass filter characteristic with zero magnitude at $f=0$ and $f=1/2$ Hz.

Plot the magnitude over f and see that it is a bandpass filter characteristic with zero magnitude at $f=0$ and $f=1/2$ Hz.

b) Is this filter linear phase? If so what is its group delay? [3]

Yes. The filter is linear phase since $\arg(H(f))=2\pi f 7+ \pi$. Group delay is $-d/df$ ($\arg(H(f))/2\pi=7$, which can also be seen directly from the plot for $h[n]$).

- 3) If you add together the following Q7 two's complement numbers (without saturation) from left to right will there ever be overflow? If so when does it occur and will it affect the final result? Here is the addition: $0x81+0xFD+0x10$. [3]

$$0x81+0xFD=10000001+11111101=01111110 \text{ (overflow)}$$

$$0x81+0xFD+0x10=01111110+00010000=10001110 \text{ which is } -0.8906 \text{ in decimal}$$

The result of adding these numbers in infinite precision (decimal) is also -0.8906 ; the same result as obtained above. Thus overflow did not affect the final result. This is simply confirmation of the two's complement overflow property discussed in Lecture 3.

- 4) A waveform is sampled using a sample rate of 48 kHz. What is minimum size of the FFT required (minimum number of points in the FFT) to achieve a frequency spacing of less than 100Hz in the DFT spectrum of the samples? [6]

The DFT analysis frequencies are F_s/N . Therefore $F_s/N < 100$ implies that $N > F_s/100 = 480$ samples. Hence 512 would work.

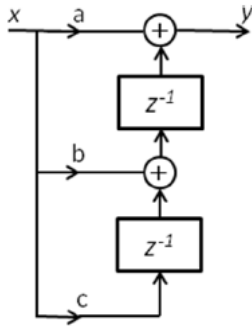
- 5) A compensated quantizer has an input range from $-8V$ to $+8V$. Assuming the signal amplitudes are distributed uniformly over any quantization interval, what is the minimum number of bits required to guarantee that the maximum quantization error is less than $0.01V$? What are the fewest bits required to guarantee that the root mean squared quantization error is less than $0.01V$? [6]

The max quantization error is $\Delta/2$ which (Lec 6) is given by $2V_{ref}/2^{\{B\}}$ where $V_{ref}=8$. Therefore for $\Delta/2 < 0.01V$ require: $2^{\{B\}} > 8/0.01=800$, or $B > \log_2(800)-1=9.6$. At least 10 bits are needed. The RMSQE is given by $\sqrt{\Delta^2/12} = \Delta/2 \sqrt{1/3}$. Hence for this to be less than $0.01V$ we require $(8/2^{\{B\}})\sqrt{1/3} < 0.01$ or $B > \log_2(800*\sqrt{1/3}) = 8.85$. Hence require at least 9 bits.

- 6) In Lab 6 we needed to perform an FFT on collected audio data and collect audio data at the same time. How can we use interrupts and two buffers to help us perform FFTs on data without missing data samples? [5]

We can use interrupts to collect data samples and store them in one of the buffers while using the other buffer as input to our FFT function. Then once the sample buffer is full we switch the two buffers.

- 7) Write the transfer function of the following filter. [5]



This is an all zero filter. $H(z)=Y(z)/X(z)= a+b z^{-1}+c z^{-2}$

- 8) A filter has impulse response of the form $h[n]=(0.9)^n$ for $n=0,2,4,\dots$ and $h[n]=0$ for $n=1,3,5,\dots$. Draw the pole zero constellation and classify the filter as to FIR vs IIR, LP vs HP vs BP vs BS (band stop). [5]

The pole zero constellation has no zeros and two poles, one at -0.9 and one at 0.9 . To see this find the transfer function from the Z-transform of $h[n]$:

$$H(z)=\sum_{n=0}^{\infty} h[n] z^{-n}=\sum_{n=0}^{\infty} h[2n] z^{-2n}=\sum_{n=0}^{\infty} (0.81 z^{-2})^n=1/(1-0.81 z^{-2})$$

Where we have used the fact the $h[2n+1]=0$ and we have applied the geometric series formula. This is an IIR filter and it is neither a lowpass, highpass or bandpass.

Section II – longer answers 30 points

1) Consider the discrete time signal $x[n] = \sin(2\pi f_1 n + \phi) + \cos(2\pi f_2 n - \phi)$, $n=0, \dots, 7$. [10]

a) What is the DFT of $x[n]$ if $\phi=60$ degrees, $f_1=1/8$ and $f_2=1/4$? Carefully plot the magnitude and phase being sure to label and quantify the ranges of the axes. [5]

DFT is comprised of 8 frequencies total as it is $N=8$ point. f_1 and f_2 are equal to the DFT analysis frequencies $1/N$ and $2/N$. Hence the DFT will have four non-zero components each having magnitude $N/2$ located at DFT indices (bins) 1, 2, 6, 7. Use the identity $\sin(x) = \cos(x - \pi/2)$ to see that the phase of these components will be the following (60 degrees is equal to $\pi/3$):

Bin 1: $-\pi/2 + \pi/3 = -\pi/6$

Bin 2: $-\pi/3$

Bin 6: $\pi/3$

Bin 7: $\pi/6$

b) If $x[n]$ corresponded to a continuous-time signal $x(t)$ sampled at 20kHz what frequencies (Hz) are present in the original signal $x(t)$? [3]

The frequencies are $F_1 = F_s/8 = 2500\text{Hz}$ and $F_2 = F_s/4 = 5000\text{Hz}$.

c) Is there spectral leakage in this example? Why or why not? [2]

No spectral leakage here as the sinusoidal frequencies are equal to k/N for $k=1, 2$.

2) A 32 bit PIPO register is implemented using 32 D flip flops. Assuming that the average number of bit transitions per second are equal to half the clock frequency for all bits, what is the total power dissipation of this register and how does it depend on the logic voltage and the clock frequency? [10]

The power dissipation of one flip flop was given in lecture as $P_{ff} = (1 + 3\alpha/2) C f_{clk} V_d^2$ where α is the average number of bit transitions per clock cycle. Let q be the average number of bit transitions per second, which you are given: $q = f_{clk}/2$. Now since $\text{transitions/sec} = (\text{transitions/cycle}) \times (\text{cycle/sec})$ we have $q = \alpha f_{clk}$ and therefore $\alpha = 1/2$. As in the lecture notes we make the additional assumption that the capacitance seen by each flip flops is identical. Thus, the total power of the PIPO register is

$$P_{tot} = 32 * P_{ff} = 32(1 + 3/4) C f_{clk} V_d^2 = 56 C f_{clk} V_d^2$$

The power scales linearly in the clock frequency and quadratically in the logic voltage.

3) Using one complex FFT to compute two real FFTs

- a) Let $\{x[n]\}$ and $\{y[n]\}$ be sets of N samples, i.e. $n=0, \dots, N-1$, of real signals $x(t)$ and $y(t)$ sampled at identical sampling rates f_s . Explain how you can compute the DFTs $X[k]$ and $Y[k]$ simultaneously with a single FFT. [5]

First form the complex signal $z[n]=x[n]+jy[n]$. Next recall that if $Z[k] = \text{DFT}(z[n])$ then $\text{DFT}(z^*[n])=Z^*[N-k]$. Note: unlike for the case of real valued inputs to the FFT, for a complex valued input $z[n]$ the FFT output $Z[k]$ does not satisfy the conjugate symmetry relation, i.e. $Z[N-k]$ is not equal to $Z^*[k]$. Now, as we can represent $x[n]=(z[n]+z^*[n])/2$ and $y[n]=(z[n]-z^*[n])/(2j)$, by linearity of the FFT we obtain the FFTs of x and y from the FFT of z by the formulas: $\text{DFT}(x[n])= (Z[k]+Z^*[N-k])/2$ and $\text{DFT}(y[n])= (Z[k]-Z^*[N-k])/(2j)$.

- b) Now let $\{x[n]\}$ and $\{y[m]\}$ be sets of samples of different lengths, i.e., $n=0, \dots, N_x-1$ and $m=0, \dots, N_y-1$. Show how you can compute their DFTs with a single FFT by using zero padding. How do you interpret the DFT frequencies for $X[k]$ and $Y[k]$ in terms of Herzian frequency if the sampling rates on the two signals are not identical? [5]

Without loss of generality assume $N_y > N_x$ and define the zero padded version of $x=\{x[n]:n=0, \dots, N-1\}$ as $x_{\text{padded}}=\{x[0], x[1], \dots, x[N_x-1], 0, \dots, 0\}$ where there are N_y-N_x zeros appended to x . Now y and x_{padded} are of the same length and their $N=N_y$ point DFT's ($Y[k]$ and $X_{\text{padded}}[k]$) can be computed simultaneously using the method of part (a).

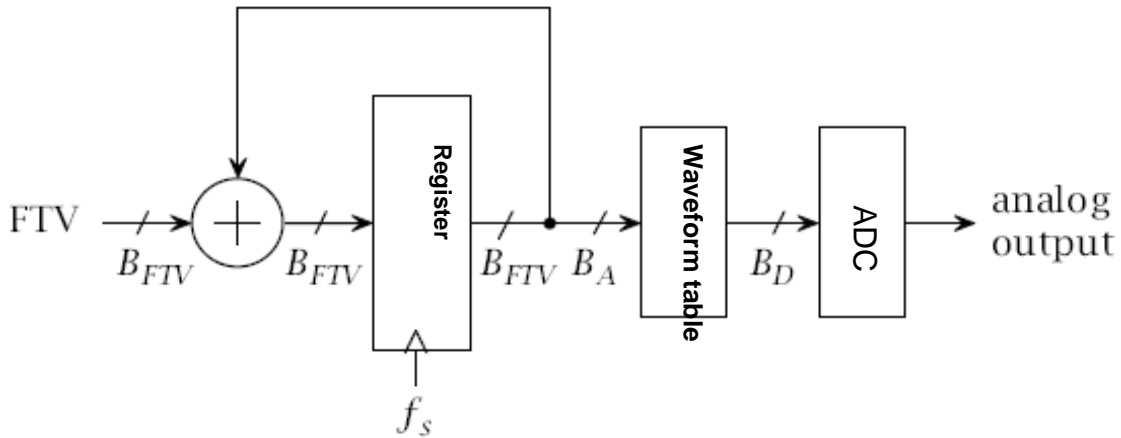
The interpretation of the DFT frequencies of $Y[k]$ is as usual: the k -th DFT index of Y corresponds to Hz frequency kf_y/N_y where f_y is the sampling frequency used to obtain the samples $\{y[n]\}$. On the other hand, the k -th index of the DFT $X_{\text{padded}}[k]$ will correspond to Hz frequency $k f_x/N_y$. To see this recall that the DFT $X_{\text{padded}}(k)$ is obtained as N_y uniform samples of the frequencies in the DTFT $X_{\text{padded}}(f)$ of $x_{\text{padded}}[n]$

$$X_{\text{padded}}[k]= X_{\text{padded}}(f_k) = \sum_{n=0}^{N_y-1} x_{\text{padded}}[n] e^{-j2\pi f_k n}$$

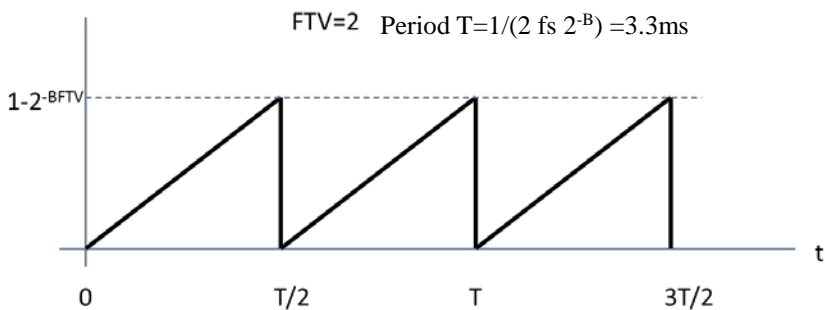
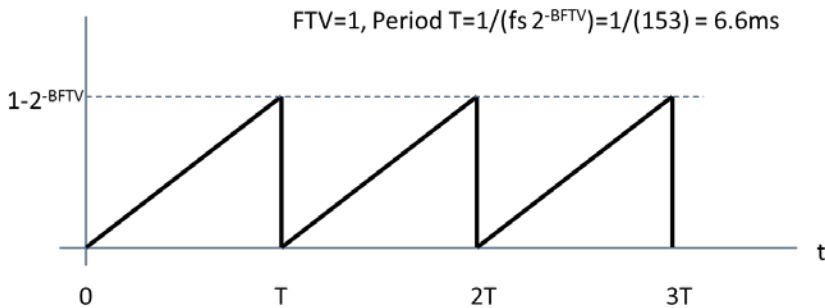
where $f_k = kf_x/N_y$. Now, by definition of $x_{\text{padded}}[n]$, this is equivalent to the DTFT of $x[n]$ evaluated at these N_y frequencies:

$$X_{\text{padded}}[k]= X_{\text{padded}}(f_k) = X(f_k) = \sum_{n=0}^{N_x-1} x[n] e^{-j2\pi f_k n}$$

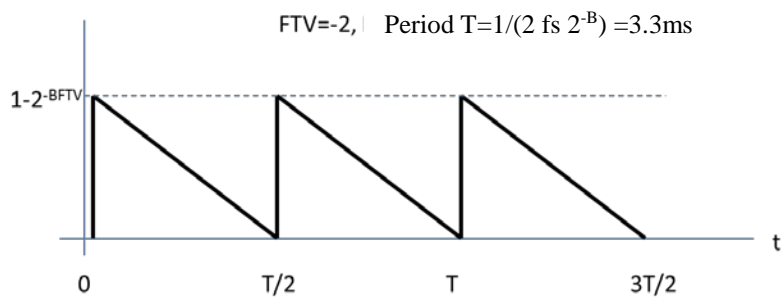
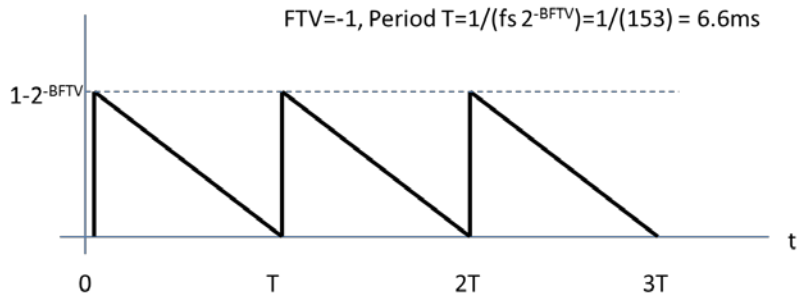
Section III Longer answers 30 points



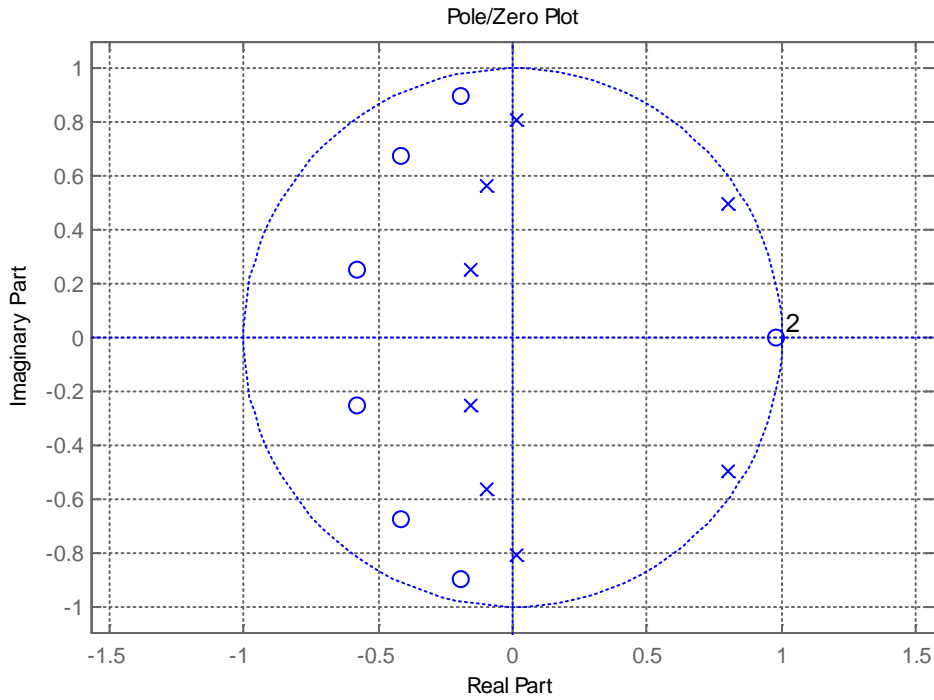
- 1) An arbitrary waveform synthesizer uses the DDS system above to generate a periodic waveform using a waveform table containing samples of a single period of a waveform $s[0], \dots, s[2^B-1]$ where $B=B_A$. Assume that $s[n]$ is a ramp, $s[n]=n \cdot 2^{-B}$ in the following.[10]
- a) Assume a clock rate $f_s = 10\text{MHz}$, $B_A = B_{FTV} = 16$, and $FTV=1$. Also assume an ideal ADC (e.g., cardinal series reconstruction) and plot three or four periods of the analog output over time. Be careful to label your axes and indicate the amplitude and period of the waveform on your plot. Repeat for $FTV=2$. [5]



- b) Assuming the same parameters as above explain what would happen if $FTV=-1$? Plot three or four periods of the waveform. Repeat for $FTV=-2$. [5]



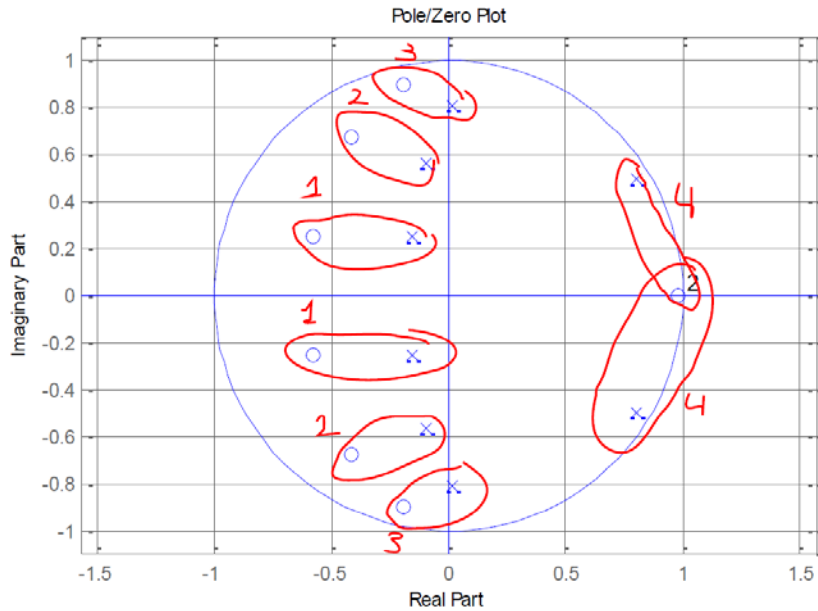
2) A pole zero constellation of a filter is shown below. [10]



a) It is desired to implement this filter using a cascade of biquad's as you did in Labs 5 and 6. How many biquads are required to implement it? [3]

4 biquads are needed.

b) Circle the pole-zero combinations that you would use for for each biquad in order to minimize the likelihood of overflow in your finite precision filter implementation. Label each combination 1,2,... corresponding to the order (from left to right) in which you would cascade the biquads. [4]



- c) Would you need to normalize any of the denominator coefficients of any of the biquads in order to implement this filter in 16 bit Q15?. If so which biquad(s) would you need to normalize in this manner? [3]

Yes. The angles theta of the pole pair nearest the unit circle has magnitude less than $\pi/3$. Hence the biquad coefficient a_1 associated with this pole will be $a_1 = 2r \cos(\theta)$ which is greater than 1. You can also look directly at the condition $\text{Re}(\text{pole}) > 1/2$ as a criterion for when normalization is necessary – and come to the same conclusion. Will need to normalize a_1 by dividing by 2 as discussed in class.