Midterm — October 12, 2003

Write and sign the honor pledge:

1. (2 pts) Print your name on every sheet.
2. (10 pts) Write the equations that define the discrete Fourier transform (DFT) and the inverse discrete Fourier transform (IDFT). Do not use the letter W in your answer.

   Forward transform:

   Inverse transform:

3. (10 pts) I have a set of 1000 consecutive samples of a waveform. The sample rate is 8000 Hz. I form the DFT. What is the frequency step in Hz between DFT values?

4. (10 pts) I have a data set of 500 sample values of a 1000 Hz sinewave. These were obtained using a sample rate of 8 kHz. Using MATLAB I take the DFT with the result being in the `X` array. I use `X=fftshift(X)` to reposition the data.

   (a) What is the frequency in Hz associated with the `X(1)` value ?

   (b) What is the frequency in Hz associated with the `X(500)` value?

5. (4pts) A waveform is sampled using a sample rate of 8 kHz. How many samples are required in order to have a frequency of 2 Hz in the DFT of the sample set?

6. (8 pts)

   (a) Write a line of MATLAB script that uses `kron` to take the vector [1 2 3 4] and create the vector [1 0 0 2 0 0 3 0 0 4 0 0].

   (b) Write a line of MATLAB script that uses `kron` to take the vector [1 2 3 4] and create the vector [1 1 1 2 2 2 3 3 3 4 4 4].

   (c) Write a line of MATLAB script that uses `kron` to take the vector [1 2 3 4] and create the vector [1 2 3 4 1 2 3 4 1 2 3 4].

(d) Write a line of MATLAB script that uses `kron` to take the vector [1 2 3 4] and create the vector [1 2 3 4 0 0 0 0 1 2 3 4].

7. (8 pts)

Assume use of a 16-bit computer word. Given the binary bit pattern:

0000 1111 0000 0000

(a) what is the value if we interpret this an unsigned integer?

(b) what is the value if we interpret this as a two's complement integer?

(c) what is the value if we interpret this as a Q8 unsigned integer?

(d) what is the value if we interpret this as a Q15 two's complement value?

8. (8 pts)

Assume use of a 16-bit computer word. Given the binary bit pattern:

1111 0010 0000 0000

(a) what is the value if we interpret this an unsigned integer?

(b) what is the value if we interpret this as a two's complement integer?

(c) what is the value if we interpret this as a Q8 unsigned integer?

(d) what is the value if we interpret this as a Q15 two's complement value?

9. (9pts) During the semester we designed IIR filters based on analog prototypes. To do this we used MATLAB's FDAtool. FDAtool offers a choice of four filter prototypes. These are: Butterworth, Chebyshev type 1, Chebyshev type 2, and elliptic. These prototypes have differing transfer function characteristics in terms of monotonicity and ripple.

   For lowpass use what are the distinguishing monotonicity and ripple characteristics of each prototype's transfer function?

   Butterworth
   - passband characteristic?
   - stopband characteristic?

   Chebyshev type 1
   - passband characteristic?
   - stopband characteristic?

   Chebyshev type 2
   - passband characteristic?
   - stopband characteristic?

   Elliptic
   - passband characteristic?
   - stopband characteristic?
   - transition band characteristic?

10. 12pts When writing hand optimized functions for use by C we need know the calling conventions used by the C compiler. The IIR functions that were used in lab used a call of the form:

```
short iircas5(DATA* x, DATA* h, DATA* r, DATA* db, ushort nh, ushort nx);
```

There are 6 parameter values that are passed. Specify the registers in which they are passed:

(a) `*x` __________

(b) `*h` __________

(c) `*r` __________

(d) `*db` __________

(e) `nh` _________

(f) `nx` _________

11. (4pts) Given a function, $f(x)$ what condition must it satisfy if it is linear?

12. (8pts) We have a symmetric FIR digital filter that we believe has a delay of 72 sample times. The sample rate is 48000 Hz. The nominal center frequency of the filter is 1500 Hz.

(a) What is the band of frequencies centered on the nominal center frequency, in Hz, over which the filter phase will change by 360 degrees?

(b) (2pts) Will the phase be increasing or decreasing as the frequency increases?

13. (8 pts) I sample the waveform $127\cos(2\pi 800t + \pi/3)$ using a sample rate of 300 Hz. Next I take one second's worth of samples and form the DFT.

   (a) At what frequency should I expect to see this waveform's negative frequency component?

   (b) At what frequency should I expect to see this waveform's positive frequency component?

14. (10 pts) You are given a digital filter described using a biquadratic transfer function of the form

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

and a sample rate of f_s Hz.

   (a) What is the value of $H(z)$ at 0 Hz?

(b) What is the value of $H(z)$ at $f_s/2$ Hz?

15. (8 pts) I sample a real valued noise waveform where the samples are independent of each other, have zero mean and variance σ^2. Given two samples n[1] and n[2]

    (a) What is the expected value of $(n[1] + n[2])^2$

    $$\mathcal{E}\left\{(n[1] + n[2])^2\right\} =$$

    (b) What is the expected value of $(n[1] + n[2])^3$

    $$\mathcal{E}\left\{(n[1] + n[2])^3\right\} =$$

16. (8 pts) In modeling quantization noise we made four assumptions that allowed us to simplify our model. These assumptions have been found to reasonably model reality when working with complex waveforms such as speech and music. The four assumptions were:

    (a)

    (b)

    (c)

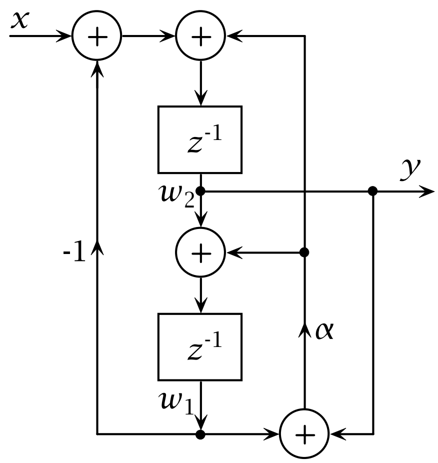

Figure 1: A biquad filter section.

(d)

17. (12pts) The block diagram shown in Figure 1 is a biquad filter section.

    (a) Write the state variable matrices, **A**, **B**, **C** and **D**, for the biquad section shown in Figure 1.

(b) Draw the block diagram of the transposed version of the biquad section shown in Figure 1.

18. (9 pts)

    (a) For "simple" oversampling A/D converter how many bits of equivalent performance are obtained per each doubling of the input sample rate? By equivalent performance I mean how many additional bits (perhaps fractional) would need to be added to a "normal" A/D converter in order to provide the same quantization noise to signal signal-to-noise ratio.

    (b) For the first order sigma-delta A/D converter how many bits of equivalent performance are obtained per each doubling of the input sample rate?

    (c) For the second order sigma-delta A/D converter how many bits of equivalent performance are obtained per each doubling of the input sample rate?

19. (4 pts) Assume a B-bit A/D converter. The analog input voltages can go plus and minus. The converter generates digital output values using the two's complement number format.

    The most negative input analog input is $-V_m$ volts (mid-tread) and produces digital output value -2^{B-1}.

    What is the voltage change (Δ) at the input that produces precisely a one count change in the digital output?

20. (4 pts) The quantization error of the output of an amplitude quantizer was modelled as being uniformly distributed. Using this assumption an expression was derived that related the variance, σ_e^2, of the quantization error to the quantizer step size, Δ. What was this relation?

$$\sigma_e^2 =$$

21. (4 pts) An expression was derived in lecture that relates the signal-to-noise ratio of a signal waveform and the quantization noise to the number of the bits in an amplitude quantizer.

    Adding one bit to the size of the quantizer results in a change of how many dBs in the signal-to-noise ratio?

22. (6 pts) Several times this semester the equation form of Parseval's theorem for the DFT was encountered, both in homework and in lecture. Write down the equation that describes Parseval's theorem and indicate whether what you wrote corresponds to the standard definition of the DFT or to the alternate form that I claim to prefer.

23. (10 pts) Draw the block diagram of the transposed direct form two (TDF2) biquad section. Remember to label the coefficients.

24. (4 pts) Not counting DMA data transfers the C5510 can simultaneously perform up to

_______ memory reads and _______ memory writes.

25. (2 pts each)

    (a) The amount of DARAM present on the C5510 is ________ Kwords.
    (b) The amount of SARAM present on the C5510 is __________ Kwords.
    (c) The C5510DSK CPU minimum cycle time is _______ ns.
    (d) The AIC23 codec A/D and D/A word size is ________ bits.
    (e) The C5510 has _______ auxiliary registers.
    (f) The C5510 has _______ temporary registers.
    (g) The technical name for the norm that we used to scale for overflow in our IIR filter designs is the "__________ norm".
    (h) The name of the SigLab program that we used in lab to measure transfer functions is _________.
    (i) The C5510 always fetches _______ bytes from program memory every clock interval.
    (j) C5510 instructions vary in length and use from 1 to _______ bytes.

26. (6 pts) The C5510 supports three address spaces these are named

    (a)

    (b)

    (c)

27. (4 pts) The transfer function of a biquad section is

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}.$$

To have a stable filter section there are limits which the a_1 and a_2 values must meet. These are:

$$< a_1 <$$

and

$$< a_2 <$$

28. (6 pts) In the realtime FFT lab exercise we used three window functions to demonstrate the use of windowing to reduce leakage. The names of these three windows are:

(a)

(b)

(c)

29. (8 pts) What is the transfer function of the filter shown in Figure 2?

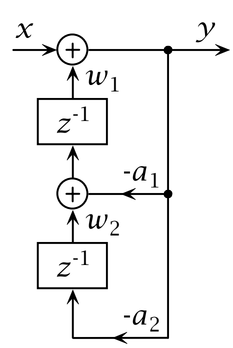

Figure 2: Filter block diagram.

30. (8 pts) In lab exercise 4 you worked with a direct digital frequency synthesizer that used a table containing 256 samples of a cosine wave. A 48 kHz clock was used to determine the rate at which samples were sent to the D/A converter. An unsigned 32-bit word was used as a phase accumulator. Every 1/48000 a value was added into this accumulator. The top 8 bits of the phase accumulator were used to address a value in the cosine array. The value added to the accumulator was termed the frequency tuning value, FTV.

    (a) What is the frequency tuning value that will result in an output frequency as close as possible to 1000 kHz?

    (b) What is the smallest increment in Hz that can be made in the output frequency?

31. (10 pts) When addressing or doing arithmetic calculations using assembly language on the C5510 our results are affected by the settings of bits in the status words. What are the functions of the following bits?

    (a) SXM

    (b) M40

(c) CPL

(d) FRCT

(e) C54CM

32. (8 pts) If we multiply a 16-bit Q14 format number by a 16-bit Q12 number on the TI C5510 with the FRCT bit set to 1.

    (a) The result in the high-low portion of the accumulator is a m-bit Qn number. What are the values of m and n?

    (b) The result is moved from the high word of the accumulator. What is the Qn value for the moved 16-bit value?

33. (8 pts) Figure 3 shows the pole and zero locations for an Elliptic filter design. The letters `a` through `d` identify the upper half pole pair positions and the letters `r` through `u` identify the upper half plane zero pair positions.

    To implement an IIR filter as a cascade of biquad sections we learned some commonly accepted guidelines regarding matching poles and zeros and ordering the resulting sections.

    (a) Which zero pairs should be paired with which pole pairs to form biquad sections?

    (b) When cascading biquad sections how should the sections be ordered going from the filter input to output? Identify the sections using the letters identifying the poles.

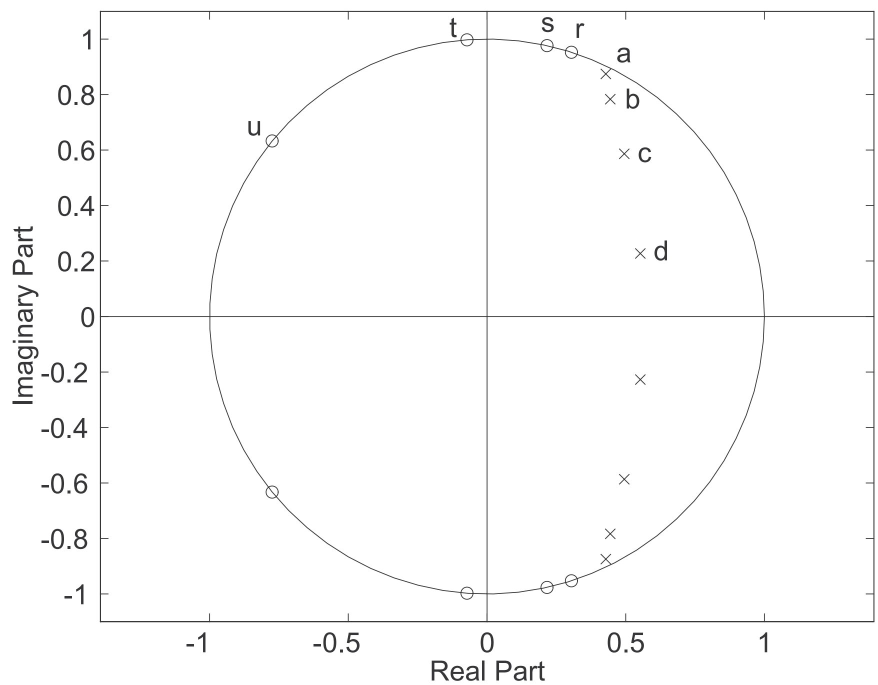

Figure 3: Elliptic filter pole-zero locations in the z-plane.

address	contents
0x010080	0x8080
⋮	⋮
0x010004	0x4444
0x010003	0x3333
0x010002	0x2222
0x010001	0x1111
0x010000	0x0000
0x00FFFF	0xFFFF

	contents
T0	0x0004
XDP	0x010000
XAR0	0x010000

Data memory and registers

Figure 4:

34. (18 pts) Given a memory block and XAR0, XDP, and T0 as shown in Figure 4. Determine the contents of AC0, AR0, and T0 after execution of the following instructions:

	AC0	AR0	T0
(a) `mov *(#x+2),AC0`	________	________	________
(b) `mov @(x-x+1),AC0`	________	________	________
(c) `mov @(x-x+0x80),AC0`	________	________	________
(d) `mov *AR0+,AC0`	________	________	________
(e) `mov *(AR0+T0),AC0`	________	________	________
(f) `mov *AR0(T0),AC0`	________	________	________
(g) `mov *AR0(#-1),AC0`	________	________	________
(h) `mov *AR0(#2),AC0`	________	________	________
(i) `mov *AR0(#0x80),AC0`	________	________	________

35. (12pts) In laboratory exercise 7 we use a 32-bit FFT code from the TI DSPlib. This code does not scale its results. The sample rate is 48000 Hz and the FFT size is 1024 values. Assume an input sinewave at a frequency of 3000 Hz. If this sinewave has amplitude after sampling 0f 16384 (2^{14}) what is:

   (a) the $\log_2$ of the magnitude of the DFT value corresponding to +3000 Hz?

(b) the index of this line (remember the calling program is written in C)?

(c) the index of the DFT value corresponding to -3000 Hz?

36. (8 pts) Using a polynomial fitting program in MATLAB an equiripple approximation was made to $\log_2(m)$ over the range $[0.5, 1]$. The polynomial we found was

$$\log_2(m) \approx 1.26596829m^3 - 4.20745495m^2 + 6.09573793m - 3.15361428.$$

Going from left to right denote the polynomial coefficients as a_3, a_2, a_1, a_0. ($a_3 = 1.26596829$).

The following code segmented was suggested as a way to evaluate this expression.

```
mov     #a<<#16,ac0
mpyr    T0,ac0,ac0          ; T0 has m in Q15
add     #b<<#16,ac0
mpyr    T0,ac0,ac0
add     #c<<#16,ac0
mpyr    T0,ac0,ac0
add     #d<<#16,ac0
```

Relate the a_x coefficients to (by name not value)

a is

b is

c is

d is

A 16 bit word is being used. The value m is in Q15 format. The coefficient values are to be placed into Qn format. What is largest usable value of n can be used?