EECS 452 Midterm — October 29, 2008

Write out and sign the honor pledge:

Unless otherwise indicated, multiple part questions have the shown points split uniformly between the parts.

1. (2 pts) Print your name on *each and every* sheet.
2. (8 pts) Write down the equations for the forward and inverse Discrete Fourier Transform. Indicate which is which. Use the "standard" definitions.

   Forward:

   $$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N} \quad \text{where } k = 0, 1, 2, \ldots, N-1.$$

   Inverse:

   $$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j2\pi kn/N} \quad \text{where } n = 0, 1, 2, \ldots, N-1.$$

3. (8 pts)

   Relate the frequency spacing Δf between DFT values corresponding to indices k and $k+1$, where $0 \le k < N-1$ and N is the number of samples used to form the DFT, to the sample rate f_s, and N.

   $\Delta f = \frac{f_s}{N}$

   Also relate Δf to the data set duration, T seconds.

   $\Delta f = \frac{1}{T}$

4. (2 pts each)

   The following questions deal with the TI hardware and software tools.

(a) How many A/D channels does the AIC23 support? 2

(b) The analog line input to the AIC23 has a nominal gain of? 1/2

(c) How many bits are there in a C5510 C char? 16

(d) How many bits are there in a C5510 unsigned int? 16

(e) How many bits are there in a C5510 long? 32

(f) How many bits are there in a C5510 float? 32

(g) How many bits are there in a C5510 double? 32

(h) What is the name of the linker section used by the C compiler to place the executable code? .text

(i) What is the name of the linker section used by the C compiler to place uninitialized data values? .bss

(j) What is the maximum possible number of values in an array of integers? $2^{16} = 65536$

(k) What is the build option entry needed to specify the processor version to the C compiler? 5510:2.2

(l) The file extension of the file used to specify the memory organization to the linker is? .cmd

(m) The C5510 has how many 16-bit words contained on-chip? 160K

(n) The C5510 stack grows toward the higher or the lower memory addresses? lower

(o) The C5510 possesses two 17 -bit multipliers.

(p) In lab we normally operate the C5510 at a 200 MHz clock rate.

(q) The name of the manufacturer of the C5510 is? Texas Instruments

5. (6 pts) The C5510 supports three addresses spaces. These are

program

data

I/O

6. True (T) or false (F) (2 pts each):

F The AIC23 only supports two sample rates, these are 8 kHz and 48 kHz.

- T One should operate a DSP processor at as slow a clock rate as possible as determined by the application being implemented.
- F The TI C compiler generates code for the C5510's floating point unit.
- F The C5510's PLL is used to convert the 20 MHz USB clock to the CPU's 200 clock.
- T An advantage of the two's complement number representation is that the same add/sub hardware is used as is with unsigned numbers.
- F Two's complement multiplication has the advantage that it can be accomplished using the a multiplier designed to implement unsigned multiplication.
- T 32-bit fixed point values have the potential to give results more accurate than those obtained using 32-bit floating point.
- T FPGAs allow performance increases over DSP devices because they readily support parallel operation.
- F The class projects need not make any use of digital signal processing.
- F The MATLAB fftshift function forms the fft and then re-orders the result so that 0 Hz is centered in the output array.
- F All FIR filters have a linear phase response.
- F All IIR filters have a linear phase response.
- T The FPGA boards used in lab include off-FPGA memory.

7. (2 pts each)

   The following questions deal with the Xilinx hardware and software tools.

   (a) Then nominal equivalent number gates on the Spartan-3 used in lab is? 10^6

   (b) The number of bits in a Block Ram is? 18K

   (c) What is the file extension used by the file that maps signals into physical pins? .ucf

   (d) The name of the manufacturer of the Spartan-3 Starter Board is? Digilent Inc.

   (e) The clock frequency normally used with the S3SB is 50 MHz.

8. (2 pts each)

   When working in VHDL we divide up a design into logical units called *entities*. In a sense these are the logic design equivalents of software functions.

(a) The purpose of the ___port___ portion of an entity is to define and make externally visible signals/connections.

(b) The purpose of the ___architecture___ portion of an entity is to define internal signals and to describe how these are used to accomplish the purpose(s) of the entity.

(c) Within an entity sequential statements need to be placed into a section coded called ___process___.

9. (4 pts) One method of forming the two's complement of a binary number is to ___complement the bits___ and ___add 1___.

10. (2 pts) What needs to be placed between the carry in and the carry out of a one-bit adder to make it into a bit-serial adder?

    ___A stage of delay (register, latch, etc.)___.

11. (2 pts) If we multiply a $Q5$ value by a $Q9$ value ___14___ is the Q number of the result.

12. (4 pts) When discarding low bits of a binary bit it is best to round first. The two methods of rounding that we considered are named

    ___two's complement___ and ___convergent___ rounding.

13. (4 pts) Because addition and subtraction in a fixed point computer use a fixed word size there are two situations where one can have arithmetic overflow. These are

    (a) ___when adding two positive numbers___

    (b) ___when subtracting two negative numbers___

14. (8 pts)

    The key property of linear system that we almost constantly exploit when processing complicated waveforms is:

    The *property of superposition*. This allows us to decompose an input into the sum of separate waveforms. Determine the response of the system to each individual waveform. Finally to add the individual responses to find the response to the original composite waveform.

15. (4 pts)

    Point by point multiplication of two equal size DFTs corresponds to what in the time domain?

    Cyclic convolution.

16. (8 pts)

    A DDS is used to generate samples of cosine and sine waveforms for use in an I/Q modulator. The radio system this is to be used in is to at least tune the frequency range from 100 kHz through 80 MHz in less than or equal to 0.1 Hz steps. The clock rate used to generate sample values is 200 MHz. Assuming a binary phase accumulator what is the smallest number bits, N, used in the phase accumulator that will accomplish this?

$$\Delta f = \frac{2 \times 10^8}{2^N} \leq 0.1 \Rightarrow N \geq \log_2(2 \times 10^9) = 1 + \log_2(10^9) = 30.897$$

$$N = 31 \text{ bits.}$$

    Recall that $\log_a(x) = \log_b(x)/\log_b(a)$.

17. (4 pts each)

    An amplitude 1000 sine wave data set is generated using MATLAB. The frequency is 125 Hz. The sample rate is 16000 Hz. The MATLAB function `fft` is used to take the DFT of a 4096 sample data set.

    For the resulting output what is

    (a) The frequency spacing between adjacent array values in Hz?

    125/32 = 3.90625 Hz

    (b) The MATLAB index associated with +125 Hz.

    33

    (c) The MATLAB index associated with -125 Hz.

    4065

(d) The magnitude (`abs( )`) of the largest value.

1000*4096/2 = 2048000

18. (4 pts)

If the waveform used in the above problem has phase angle θ what is the magnitude of the difference in angle between the -125 Hz and +125 Hz lines in the FFT output?

2θ

19. (4 pts)

The main cause of spectral leakage in a DFT output is?

Several answers are possible, they pretty much boil down to that the waveform being transformed does not have a period that is an exact integer fraction of the duration of the waveform being transformed. This can result in a step discontinuity. It means that the frequency content of the waveform does not entirely upon the DFT analysis frequencies, etc.

20. (2 pts each)

Assume the use of a B-bit word size.

(a) What is the value of the largest unsigned integer value that can be represented?

$2^B - 1$

(b) What is the value of the largest positive two's complement integer value that can be represented?

$2^{B-1} - 1$

(c) What is the value of the most negative two's complement integer value that can be represented?

-2^{B-1}

(d) What is the value of the largest unsigned Q(B-1) value that can be represented?

$2 - 2^{-B+1}$

(e) What is the value of the largest positive Q(B-1) value that can be represented?

$1 - 2^{-B+1}$

(f) What is the value of the most negative two's complement Q(B-1) value that can be represented?

-1

21. (4 pts)

What is the main reason why high order filter transfer functions are implemented using biquad cascades?

Often there is an extremely wide range of coefficient values in the polynomial such that the arithmetic is not readily implementable. The biquad coefficient values are bounded to have magnitudes less than 2, for a_1 and 1 for a_2. This greatly simplifies doing the arithmetic in a fixed point environment.

22. (2 pts)

For the 8-section elliptic filter used in lab what was the major performance advantage that the TDF2 form had over the DF2 form?

The gains from the input to the internal delay stages were less than 1. There were no overflow problems for sine wave inputs. This was not the case for the DF2.

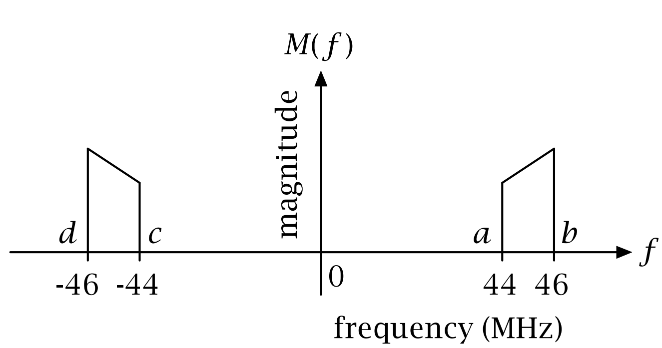
23. (2 pts) The filter preceding an A/D converter is termed an

anti- aliasing filter.

24. (2 pts) The filter following a D/A converter is terms an

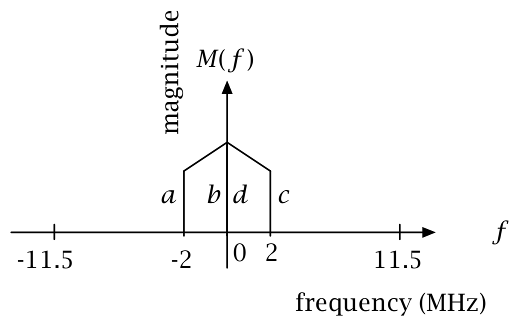
anti- imaging filter.

25. (10 pts) A useful aspect of sampling is to alias a bandlimited signal to lower frequencies. This is accomplished by using sample rates lower than or equal to Nyquist.



Sketch and label the spectrum after sampling using the following sampling rates:

(a) f_s = 23 MHz.



i. a = -2 MHz.

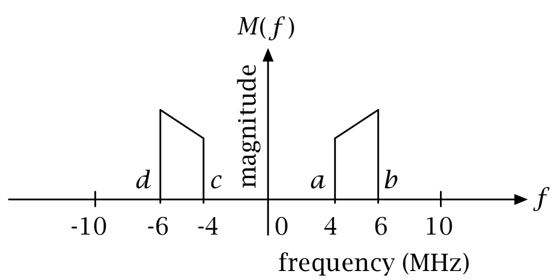
ii. b = 0 MHz.

iii. c = 2 MHz.

iv. d = 0 MHz.

v. The Nyquist range is from -11.5 to 11.5 MHz.

(b) f_s = 20 MHz.



i. a = 4 MHz.

ii. b = 6 MHz.

iii. c = -4 MHz.

iv. d = -6 MHz.

v. The Nyquist range is from -10 to 10 MHz.

26. (4 pts each)

Why are *anti-alias* and *anti-image* filters used (i.e., their purpose)?

An anti-alias filter removes unwanted high frequency signal content so that it is not aliased to lower frequencies when the signal is sampled.

An anti-image filter removes the high frequency images of a waveform when a digitized waveform is reconstructed using a D/A converter.

27. (8 pts)

Write the definition of group delay indicating the units involved:

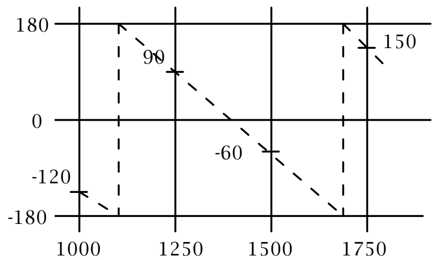
$$\tau_g = -\frac{1}{2\pi}\frac{d\theta}{df}$$

where τ_g is seconds, θ is in radians and f is in Hz (cycles-per-second).

28. (8 pts) A set of measurements is made in order to estimate the group delay though a reasonably broad band measurement system.

Hz	phase (degrees)
1000	-120
1250	90
1500	-60
1750	150

What is the group delay? Hint: plot the data then think about it.



Plot the points first. Next assume that the data is not aliased. In that case the phase must have negative slope. The second and third points lie within a -180 to 180 range. Sketch that line, the the other two follow.

The extended phase shift between the first two points is -150 degrees. The extended phase shift between the second and third points is -150 degrees. The extended phase shift between the third and fourth points is -150 degrees.

$$\tau_g = \frac{-1}{360}\frac{-150}{250} \approx 1.667\text{ms}.$$

29. (4 pts each)

There were three methods shown in lecture describing how one can use a DFT to take the inverse DFT. Given a set of samples $x[n], n = 0, 1, 2, \ldots, N-1$ write the corresponding DFT as $X[k], k = 0, 1, 2, \ldots, N-1$.

Write the equations for the two methods that I indicated in lecture that are particularly easy to implement for using the DFT to determine the $x[n]$ values given the $X[k]$ values. Use the notation DFT( ) rather than writing out the summations.

$$\text{DFT}(X[k]) = Nx[N-n] \quad \text{where } n = 0, 1, 2, \ldots, N-1.$$

$$\text{DFT}(X^*[k]) = Nx^*[n] \quad \text{where } n = 0, 1, 2, \ldots, N-1.$$

30. (4 pts)

Two methods were discussed (briefly) in lecture for using an N value FFT to implement a P value FIR filter where $N >> P$. The two methods were

Overlap-and-add and Overlap-and-save .

31. (8 pts)

What is the transfer function of the filter shown in Figure 1?

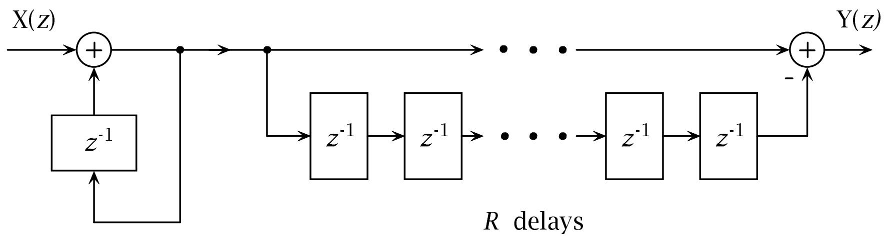


Figure 1: Filter block diagram.

We can break the unit into the cascade of two systems. Let the first have output Y_1. For the first delay stage we have its $Y_1 = X + z^1 Y_1$ giving

$$\frac{Y_1}{X} = \frac{1}{1 - z^{-1}}$$

The transfer function of the second part is trivially $1 - z^{-R}$. The composit transfer function is the product of the two individual transfer functions:

$$\frac{Y}{X} = \frac{1 - z^{-R}}{1 - z^{-1}}$$

32. (2 pts each)

For the IIR lab exercise we considered the use of four filter types offered us by MATLAB.

(a) Which filter type (name) did we not use because it required an excessive number of poles?

Butterworth

(b) Which filter type (name) has equiripple only in the passband?

Chebyshev Type 1

(c) Which filter type (name) has equiripple only in the stop band?

Chebyshev Type 2

(d) Which filter type (name) has equiripple in both the passband and the stopband?

Elliptic

(e) Which filter type (name) has the smallest transition band for a given order?

Elliptic

(f) Which filter type (name) has the maximally flat passband?

Butterworth

33. (8 pts) What is the transfer function of the filter shown in Figure 2?

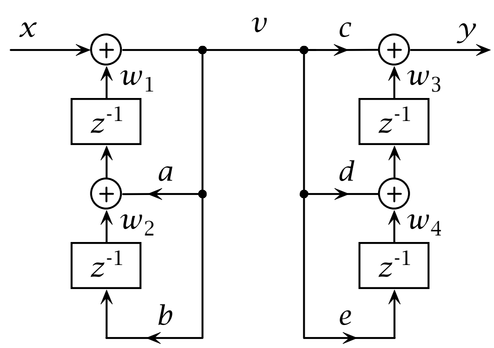


Figure 2: Filter block diagram.

$$w_2 = bz^{-1}v$$

$$w_1 = az^{-1}v + bz^{-2}v$$

$$v = x + w_1 = x + az^{-1}v + bz^{-2}v$$

Solving gives

$$\frac{V(z)}{X(z)} = \frac{1}{1 - az^{-1} - bz^{-2}}$$

$$y = cv + dz^{-1}v + ez^{-2}v$$

Solving

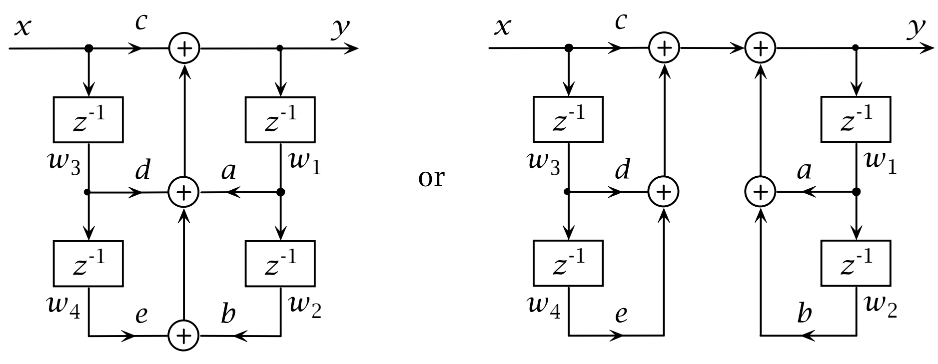
$$\frac{Y(z)}{V(z)} = c + dz^{-1} + ez^{-2}$$

Combining

$$\frac{V(z)}{X(z)}\frac{Y(z)}{V(z)} = \frac{Y(z)}{X(z)} = \frac{c + dz^{-1} + ez^{-2}}{1 - az^{-1} - bz^{-2}}$$

34. (8 pts)

Draw the block diagram of the transposed version of the biquad section shown in Figure 2



(a) (b)

The three adders in (a) might also be combined into a single five input adder.

35. (8 pts)

Figure 3 shows the pole and zero locations for a Elliptic filter design similar to the one we used for the IIR lab exercise. The letters a through d identify

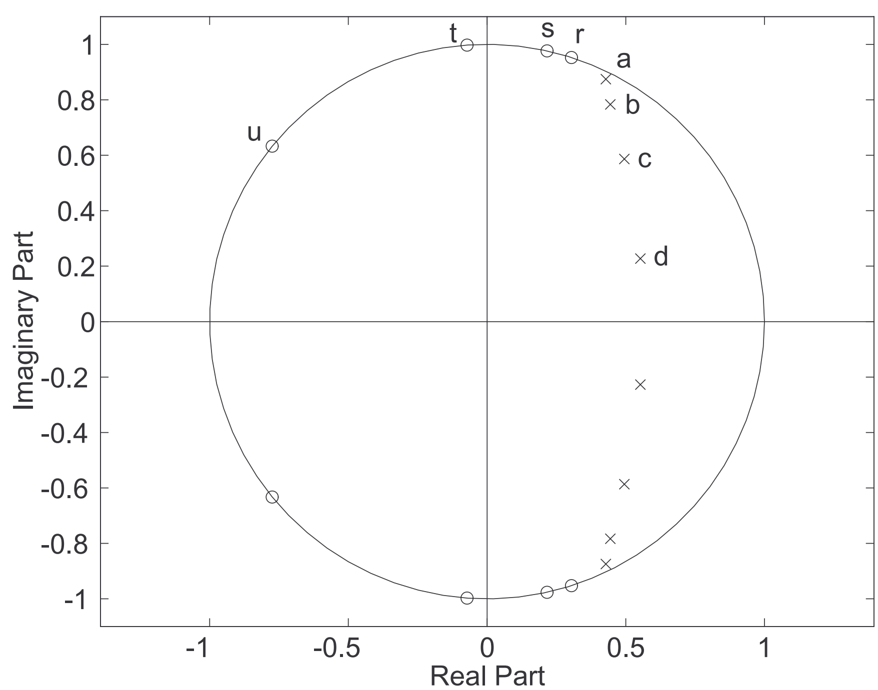


Figure 3: Elliptic filter pole-zero locations in the z-plane.

the upper half pole pair positions and the letters r through u identify the upper half plane zero pair positions.

To implement our cascade of biquad sections, in order to minimize the potential for overflow, we followed some commonly accepted guidelines regarding matching poles and zeros and ordering the resulting sections.

(a) Which zero pairs should be paired with which pole pairs to form biquad sections?

a-r, b-s, c-t, d-t

(b) When cascading biquad sections how should the sections be ordered going from the filter input to output? Identify the sections using the letters identifying the poles.

d c b a

36. (8 pts) In modeling quantization noise we made four assumptions that allowed us to simplify our model. These assumptions have been found to reasonably model reality when working with complex waveforms such as speech and music. The four assumptions were:

(a) The signal and quantization noise are independent.

(b) The noise samples are individually independent.

(c) The quantization noise is uniformly distributed.

(d) The statistics do not vary with time.

37. (6 pts) We have been modeling noise as having zero average value and variance σ^2. If we form the DFT as below of N complex noise values, $e[n]$

$$E[k] = \frac{1}{N}\sum_{n=0}^{N-1} e[n]e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1$$

what are the expected values of

$$E[k] = 0$$

$$E[k]E^*[k] = \sigma^2/N$$

$$E[k]E^*[k'] \text{ where } k \neq k' = 0$$

38. (2 pts)

The lifetime of a patent starting at the date of filing is 20 years.

39. (4 pts) The grace period from first public disclosure of an invention and the date of filing for a patent on that invention is 1 year. After this period of time the invention is no longer patentable.

40. (8pts)

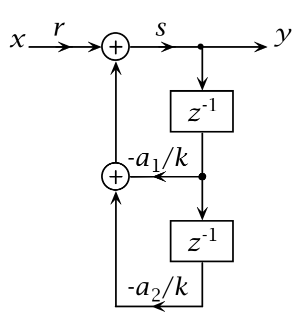


Figure 4: Scaling problem block diagram.

A common method used to scale filter coefficients is shown in Figure 4. It is desired that the transfer function between x and y be the same after scaling as when $k = 1$. What are the values of r and s needed to make this so?

$$Y = rsX - s\left(a_1 z^{-1}/k + a_2 z^{-2}/k\right) Y$$

$$Y\left[1 + (sa_1/k)z^{-1} + (sa_2/k)z^{-2}\right] = rsX$$

$$\frac{Y}{X} = \frac{rs}{1 + (sa_1/k)z^{-1} + (sa_2/k)z^{-2}}$$

Thus $s = k$ and $r = 1/k$.