Write out and sign the honor pledge:

Unless otherwise indicated, multiple part questions have the shown points split uniformly between the parts.

1. (2 pts) Print your name on each and every sheet.

2. (8 pts) Write down the equations for the forward and inverse Discrete Fourier Transform. Indicate which is which. Use the “standard” definitions.

Forward:

\[ X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N} \quad \text{where } k = 0, 1, 2, \ldots, N - 1. \]

Inverse:

\[ x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j2\pi kn/N} \quad \text{where } n = 0, 1, 2, \ldots, N - 1. \]

3. (8 pts) Relate the frequency spacing \( \Delta f \) between DFT values corresponding to indices \( k \) and \( k + 1 \), where \( 0 \leq k < N - 1 \) and \( N \) is the number of samples used to form the DFT, to the sample rate \( f_s \), and \( N \).

\[ \Delta f = \frac{f_s}{N} \]

Also relate \( \Delta f \) to the data set duration, \( T \) seconds.

\[ \Delta f = \frac{1}{T} \]

4. (2 pts each) The following questions deal with the TI hardware and software tools.
5. (6 pts) The C5510 supports three addresses spaces. These are

- program
- data
- I/O

6. True (T) or false (F) (2 pts each):

   F  The AIC23 only supports two sample rates, these are 8 kHz and 48 kHz.
One should operate a DSP processor at as slow a clock rate as possible as determined by the application being implemented.

The TI C compiler generates code for the C5510's floating point unit.

The C5510's PLL is used to convert the 20 MHz USB clock to the CPU's 200 clock.

An advantage of the two's complement number representation is that the same add/sub hardware is used as is with unsigned numbers.

Two's complement multiplication has the advantage that it can be accomplished using the a multiplier designed to implement unsigned multiplication.

32-bit fixed point values have the potential to give results more accurate than those obtained using 32-bit floating point.

FPGAs allow performance increases over DSP devices because they readily support parallel operation.

The class projects need not make any use of digital signal processing.

The MATLAB fftshift function forms the fft and then re-orders the result so that 0 Hz is centered in the output array.

All FIR filters have a linear phase response.

All IIR filters have a linear phase response.

The FPGA boards used in lab include off-FPGA memory.

7. (2 pts each)
The following questions deal with the Xilinx hardware and software tools.

(a) Then nominal equivalent number gates on the Spartan-3 used in lab is? 10^6

(b) The number of bits in a Block Ram is? 18K

(c) What is the file extension used by the file that maps signals into physical pins? .ucf

(d) The name of the manufacturer of the Spartan-3 Starter Board is? Digilent Inc.

(e) The clock frequency normally used with the S3SB is 50 MHz.

8. (2 pts each)
When working in VHDL we divide up a design into logical units called entities. In a sense these are the logic design equivalents of software functions.
(a) The purpose of the **port** portion of an entity is to define and make externally visible signals/connections.

(b) The purpose of the **architecture** portion of an entity is to define internal signals and to describe how these are used to accomplish the purpose(s) of the entity.

(c) Within an entity sequential statements need to be placed into a section coded called **process**.

9. (4 pts) One method of forming the two's complement of a binary number is to **complement the bits** and **add 1**.

10. (2 pts) What needs to be placed between the carry in and the carry out of a one-bit adder to make it into a bit-serial adder?

   A stage of delay (register, latch, etc.)

11. (2 pts) If we multiply a **Q5** value by a **Q9** value, **14** is the **Q** number of the result.

12. (4 pts) When discarding low bits of a binary bit it is best to round first. The two methods of rounding that we considered are named **two's complement** and **convergent** rounding.

13. (4 pts) Because addition and subtraction in a fixed point computer use a fixed word size there are two situations where one can have arithmetic overflow. These are

   (a) **when adding two positive numbers**

   (b) **when subtracting two negative numbers**

14. (8 pts)

   The key property of linear system that we almost constantly exploit when processing complicated waveforms is:

   The *property of superposition*. This allows us to decompose an input into the sum of separate waveforms. Determine the response of the system to each individual waveform. Finally to add the individual responses to find the response to the original composite waveform.
15. (4 pts)
Point by point multiplication of two equal size DFTs corresponds to what in the time domain?

Cyclic convolution.

16. (8 pts)
A DDS is used to generate samples of cosine and sine waveforms for use in an I/Q modulator. The radio system this is to be used in is to at least tune the frequency range from 100 kHz through 80 MHz in less than or equal to 0.1 Hz steps. The clock rate used to generate sample values is 200 MHz. Assuming a binary phase accumulator what is the smallest number bits, \( N \), used in the phase accumulator that will accomplish this?

\[
\Delta f = \frac{2 \times 10^8}{2^N} \leq 0.1 \Rightarrow N \geq \log_2 (2 \times 10^9) = 1 + \log_2 (10^9) = 30.897
\]

\( N = 31 \) bits.

Recall that \( \log_a(x) = \frac{\log_b(x)}{\log_b(a)} \).

17. (4 pts each)
An amplitude 1000 sine wave data set is generated using MATLAB. The frequency is 125 Hz. The sample rate is 16000 Hz. The MATLAB function \texttt{fft} is used to take the DFT of a 4096 sample data set.

For the resulting output what is

(a) The frequency spacing between adjacent array values in Hz?

\[
125/32 = 3.90625 \text{ Hz}
\]

(b) The MATLAB index associated with +125 Hz.

33

(c) The MATLAB index associated with -125 Hz.

4065
(d) The magnitude \( \text{abs}( ) \) of the largest value.

\[
1000 \times 4096 / 2 = 2048000
\]

18. (4 pts)
If the waveform used in the above problem has phase angle \( \theta \) what is the magnitude of the difference in angle between the -125 Hz and +125 Hz lines in the FFT output?

\[
2\theta
\]

19. (4 pts)
The main cause of spectral leakage in a DFT output is?
Several answers are possible, they pretty much boil down to that the waveform being transformed does not have a period that is an exact integer fraction of the duration of the waveform being transformed. This can result in a step discontinuity. It means that the frequency content of the waveform does not entirely upon the DFT analysis frequencies, etc.

20. (2 pts each)
Assume the use of a \( B \)-bit word size.

(a) What is the value of the largest unsigned integer value that can be represented?
\[
2^B - 1
\]

(b) What is the value of the largest positive two's complement integer value that can be represented?
\[
2^{B-1} - 1
\]

(c) What is the value of the most negative two's complement integer value that can be represented?
\[
-2^{B-1}
\]

(d) What is the value of the largest unsigned Q(B-1) value that can be represented?
\[
2 - 2^{-B+1}
\]

(e) What is the value of the largest positive Q(B-1) value that can be represented?
\[
1 - 2^{-B+1}
\]
(f) What is the value of the most negative two’s complement Q(B-1) value that can be represented?

\[-1\]

21. (4 pts)

What is the main reason why high order filter transfer functions are implemented using biquad cascades?

Often there is an extremely wide range of coefficient values in the polynomial such that the arithmetic is not readily implementable. The biquad coefficient values are bounded to have magnitudes less than 2, for \(a_1\) and 1 for \(a_2\). This greatly simplifies doing the arithmetic in a fixed point environment.

22. (2 pts)

For the 8-section elliptic filter used in lab what was the major performance advantage that the TDF2 form had over the DF2 form?

The gains from the input to the internal delay stages were less than 1. There were no overflow problems for sine wave inputs. This was not the case for the DF2.

23. (2 pts) The filter preceding an A/D converter is termed an anti-_________ aliasing_________ filter.

24. (2 pts) The filter following a D/A converter is termed an anti-_________ imaging_________ filter.

25. (10 pts) A useful aspect of sampling is to alias a bandlimited signal to lower frequencies. This is accomplished by using sample rates lower than or equal to Nyquist.
Sketch and label the spectrum after sampling using the following sampling rates:

(a) $f_s = 23$ MHz.
   i. $a = -$2 MHz.
   ii. $b = 0$ MHz.
   iii. $c = 2$ MHz.
   iv. $d = 0$ MHz.
   v. The Nyquist range is from $-11.5$ to $11.5$ MHz.

(b) $f_s = 20$ MHz.
   i. $a = 4$ MHz.
   ii. $b = 6$ MHz.
   iii. $c = -4$ MHz.
   iv. $d = -6$ MHz.
   v. The Nyquist range is from $-10$ to $10$ MHz.

26. (4 pts each)
Why are *anti-alias* and *anti-image* filters used (i.e., their purpose)?

An anti-alias filter removes unwanted high frequency signal content so that it is not aliased to lower frequencies when the signal is sampled.

An anti-image filter removes the high frequency images of a waveform when a digitized waveform is reconstructed using a D/A converter.

27. (8 pts)
Write the definition of group delay indicating the units involved:

$$\tau_g = -\frac{1}{2\pi} \frac{d\theta}{df}$$

where $\tau_g$ is seconds, $\theta$ is in radians and $f$ is in Hz (cycles-per-second).
28. (8 pts) A set of measurements is made in order to estimate the group delay though a reasonably broad band measurement system.

<table>
<thead>
<tr>
<th>Hz</th>
<th>phase (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>-120</td>
</tr>
<tr>
<td>1250</td>
<td>90</td>
</tr>
<tr>
<td>1500</td>
<td>-60</td>
</tr>
<tr>
<td>1750</td>
<td>150</td>
</tr>
</tbody>
</table>

What is the group delay? Hint: plot the data then think about it.

Plot the points first. Next assume that the data is not aliased. In that case the phase must have negative slope. The second and third points lie within a -180 to 180 range. Sketch that line, the the other two follow.

The extended phase shift between the first two points is -150 degrees. The extended phase shift between the second and third points is -150 degrees. The extended phase shift between the third and fourth points is -150 degrees.

\[ \tau_g = \frac{-1}{360} \frac{-150}{250} \approx 1.667 \text{ms}. \]

29. (4 pts each)

There were three methods shown in lecture describing how one can use a DFT to take the inverse DFT. Given a set of samples \( x[n], n = 0, 1, 2, \ldots, N-1 \) write the corresponding DFT as \( X[k], k = 0, 1, 2, \ldots, N-1 \).

Write the equations for the two methods that I indicated in lecture that are particularly easy to implement for using the DFT to determine the \( x[n] \) values given the \( X[k] \) values. Use the notation \( \text{DFT}(\ ) \) rather than writing out the summations.

\[ \text{DFT}(X[k]) = N x[N - n] \quad \text{where } n = 0, 1, 2, \ldots, N - 1. \]

\[ \text{DFT}(X^*[k]) = N x^*[n] \quad \text{where } n = 0, 1, 2, \ldots, N - 1. \]

30. (4 pts)

Two methods were discussed (briefly) in lecture for using an \( N \) value FFT to implement a \( P \) value FIR filter where \( N >> P \). The two methods were
Overlap-and-add and Overlap-and-save.

31. (8 pts)
What is the transfer function of the filter shown in Figure 1?

![Figure 1: Filter block diagram.](image)

We can break the unit into the cascade of two systems. Let the first have output \( Y_1 \). For the first delay stage we have its \( Y_1 = X + z^{-1}Y_1 \) giving

\[
\frac{Y_1}{X} = \frac{1}{1 - z^{-1}}
\]

The transfer function of the second part is trivially \( 1 - z^{-R} \). The composite transfer function is the product of the two individual transfer functions:

\[
\frac{Y}{X} = \frac{1 - z^{-R}}{1 - z^{-1}}
\]

32. (2 pts each)
For the IIR lab exercise we considered the use of four filter types offered us by MATLAB.

(a) Which filter type (name) did we not use because it required an excessive number of poles?

- Butterworth

(b) Which filter type (name) has equiripple only in the passband?

- Chebyshev Type 1

(c) Which filter type (name) has equiripple only in the stop band?

- Chebyshev Type 2

(d) Which filter type (name) has equiripple in both the passband and the stopband?

- Elliptic
(e) Which filter type (name) has the smallest transition band for a given order?

Elliptic

(f) Which filter type (name) has the maximally flat passband?

Butterworth

33. (8 pts) What is the transfer function of the filter shown in Figure 2?

$$w_2 = bz^{-1}v$$

$$w_1 = az^{-1}v + bz^{-2}v$$

$$v = x + w_1 = x + az^{-1}v + bz^{-2}v$$

Solving gives

$$\frac{V(z)}{X(z)} = \frac{1}{1 - az^{-1} - bz^{-2}}$$

$$y = cv + dz^{-1}v + ez^{-2}v$$

Solving

$$\frac{Y(z)}{V(z)} = c + dz^{-1} + ez^{-2}$$

Combining

$$\frac{V(z) Y(z)}{X(z) V(z)} = \frac{Y(z)}{X(z)} = \frac{c + dz^{-1} + ez^{-2}}{1 - az^{-1} - bz^{-2}}$$

34. (8 pts)

Draw the block diagram of the transposed version of the biquad section shown in Figure 2.
The three adders in (a) might also be combined into a single five input adder.

35. (8 pts)

Figure 3 shows the pole and zero locations for a Elliptic filter design similar to the one we used for the IIR lab exercise. The letters a through d identify the upper half pole pair positions and the letters r through u identify the

Figure 3: Elliptic filter pole-zero locations in the z-plane.
upper half plane zero pair positions.

To implement our cascade of biquad sections, in order to minimize the potential for overflow, we followed some commonly accepted guidelines regarding matching poles and zeros and ordering the resulting sections.

(a) Which zero pairs should be paired with which pole pairs to form biquad sections?

a-r, b-s, c-t, d-t

(b) When cascading biquad sections how should the sections be ordered going from the filter input to output? Identify the sections using the letters identifying the poles.

d c b a

36. (8 pts) In modeling quantization noise we made four assumptions that allowed us to simplify our model. These assumptions have been found to reasonably model reality when working with complex waveforms such as speech and music. The four assumptions were:

(a) The signal and quantization noise are independent.

(b) The noise samples are individually independent.

(c) The quantization noise is uniformly distributed.

(d) The statistics do not vary with time.

37. (6 pts) We have been modeling noise as having zero average value and variance $\sigma^2$. If we form the DFT as below of $N$ complex noise values, $e[n]$

$$E[k] = \frac{1}{N} \sum_{n=0}^{N-1} e[n]e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1$$
what are the expected values of

\[ E[k] = 0 \]

\[ E[k]E^*[k] = \sigma^2 / N \]

\[ E[k]E^*[k'] \text{ where } k \neq k' = 0 \]

38. (2 pts)

The lifetime of a patent starting at the date of filing is \underline{20 years}.

39. (4 pts) The grace period from first public disclosure of an invention and the date of filing for a patent on that invention is \underline{1 year}. After this period of time the invention is no longer patentable.

40. (8 pts)

A common method used to scale filter coefficients is shown in Figure 4. It is desired that the transfer function between \( x \) and \( y \) be the same after scaling as when \( k = 1 \). What are the values of \( r \) and \( s \) needed to make this so?

![Scaling problem block diagram.](image-url)
\[
Y = rsX - s \left( a_1 z^{-1} / k + a_2 z^{-2} / k \right) Y
\]

\[
Y \left[ 1 + (sa_1 / k) z^{-1} + (sa_2 / k) z^{-2} \right] = rsX
\]

\[
\frac{Y}{X} = \frac{rs}{1 + (sa_1 / k) z^{-1} + (sa_2 / k) z^{-2}}
\]

Thus \( s = k \) and \( r = 1/k \).