EECS 452 Midterm Closed book part
Winter 2009

Name: ______________________________ unique name: _______________

Sign the honor code:

I have neither given nor received aid on this exam nor observed anyone else doing so.

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NOTES:
- There are 3 pages including this one.
- On the closed book section you may not have books, notes, but you may have a calculator. Nothing but a writing utensil and calculator is allowed.
- Don’t spend too much time on any one problem.
- You have about 120 minutes for the exam. You probably shouldn’t spend more than 30 of it
1. Say you took a 16-point DFT of the following signals with $f_s=3200\text{Hz}$ starting at time $t=0$. What would be what the non-zero DFT outputs would be (magnitude and phase)?

**[4 each]**

a. $2\sin(2\pi*400t+\pi)$

$$X[2]=16 \text{ at } 90^\circ$$
$$X[14]=16 \text{ at } -90^\circ$$

b. $2\cos(2\pi*800t+\pi/4)+2$

$$X[0]=32 \text{ at } 0^\circ$$
$$X[4]=16 \text{ at } 45^\circ$$
$$X[12]=16 \text{ at } -45^\circ$$

2. Write down the equations for the forward and inverse Discrete Fourier Transform. Indicate which is which. Use the “standard” definitions. **[3 each]**

No answer provided.
3. Fill in the blank or circle the correct answer. Provide all numbers in decimal. [11 points, -1 per wrong or blank answer, minimum 0]

a. If we multiply a 16-bit Q15 value by a 16-bit Q15 value the result would be a ___32____-bit Q__30____ number.

b. An 8-bit unsigned number (Q0) can represent all integers from 0 to ___255__ (inclusive)

c. A 7-bit 2’s complement number can represent all integers from ____-64_____ to ___63____ (inclusive).

d. A 6-bit Q4 2’s complement number has a range from ___-2_______ to ____31/16_______ and a resolution of ____1/16_______.

e. We say a system is stable if: _bounded inputs result in bounded outputs_

f. In 5-bit two’s complement Q2, what is the value of 11011? ___-1.25__

g. For a given task you’d expect an **FIR / IIR** filter to have a higher degree.

h. All FIR filters have **linear phase / no non-zero poles / at least a degree of 8**.

i. A waveform is sampled using a sample rate of 44 KHz. If you desire a frequency spacing of 0.5 Hz in the DFT of the sample set, you’ll need ___88000_______ samples.

j. Consider the figure below. It is of an **IIR / FIR** filter in **Direct Form I / Modified Direct Form I / Direct Form II, / transposed Direct Form II?**
EECS 452 Midterm Open book part
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NOTES:
- There are 8 pages including this one.
- On the open book section you may use calculators, books and notes, but no PDAs, Portables, Cell phones, etc.
- Don’t spend too much time on any one problem.
- You have about 120 minutes for the entire exam.
- Some questions are much harder than others…
1) DDS [10]
   a) Say you are developing an application that needs to use Direct Digital Synthesis to generate sine waves with the following properties:
      - Frequency of the output needs to range from 100KHz or lower to 1MHz or higher.
      - Frequency spacing of the output must be no more than 5 KHz.
   Assuming that \( f_s = 10 \text{MHz} \), what is the minimum number of bits that must be kept in the “Register” (found in the figure below)? Clearly show your work or no credit will be given. Also, circle your final answer. [7]

\[
5\text{KHz steps starting a base of } 10\text{MHz means } 2000 \text{ different values. The } 100\text{KHz and } 1\text{MHz are largely irrelevant here. So we have } \log_2(2000) = 10.966 \text{ or } 11 \text{ bits (rounding up)}. 
\]

b) Say that for the above design you will have outputs ranging from -2 to 2 Volts and you wish to have the resolution of the output be .05V or less. What is the minimum number of bits you’d need for \( B_D \)? Show your work and circle your answer. [3]

\[
4/.05=80. \quad \log_2(80)=6.3 \text{ or } 7 \text{ bits.}
\]
Short answer [11]

c) What is the main reason(s) why high-order filter transfer functions are implemented using biquad cascades? [4]

- High-order recursive filters can be highly sensitive to quantization of their coefficients, and can easily become unstable.

d) What is the transfer function of the filter shown below? [4]

\[
\begin{align*}
    x & \quad w_1 \\
    z^{-1} & \quad w_2 \\
    a & \quad w_3 \\
    \frac{c + dz^{-1} + ez^{-2}}{1 - az^{-1} - bz^{-2}} & \quad w_4 \\
    b & \quad e
\end{align*}
\]

e) When working in VHDL we divide up a design into logical units called entities. In a sense these are the logic design equivalents of software functions. [1 each]

i) The purpose of the **All answers taken** portion of an entity is to define and make externally visible signals/connections.

ii) The purpose of the **All answers taken** portion of an entity is to define internal signals and to describe how these are used to accomplish the purpose(s) of the entity.

iii) Within an entity any sequential statements need to be placed into a **process** block.
2) Say you have a band-limited signal with frequency centered at 36MHz with a bandwidth of 10MHz. What is the lowest frequency you could sample this signal at and (in theory) not lose any information? Show your work. [7] 

20.5MHz

3) It is known that the group delay of a given linear-phase filter is 0.01 seconds. Assuming the following frequencies are in the pass-band region, what would be the expected phase of each signal? Provide your answers in radians in the range of (-π to π] and show your work. [9 points, 3 each]

a) 1000Hz. 0
b) 150Hz π
c) 75Hz π/2
4) Match the following Zero/Pole graphs with their transfer functions. [2 each]

A) _____ X _______
B) _____ W _______
C) _____ Y _______
D) _____ Z _______
5) Consider a C function called `biquad` which has the following prototype:

```c
short biquad(short x, short a, short b, short c, short d, short e, int w[]);
```

You are to write C code which implements the filter shown below. The return value is to be “y”. You should assume that “short” values are 16-bit Q15 numbers and ints are 32-bit Q31 numbers. `w[1]` is `w_1` etc. and `w[0]` is unused.

Further, you should change `w[]` so that `w[]` can be used for the next input value (much like you did in lab). Care should be taken in rounding and being sure to keep all values in the correct representation. **You may assume that no overflow occurs.** [20]

(Additional room for your answer is given on the next page should you need it).

```c
short biquad(short x, short a, short b, short c, short d, short e, int w[])
{
    short v,y;
    v = (((w[1]+(1<<15))>>16)+x);
    y = (((int)v*c)<<1)+(1<<15))>>16;
    w[1]=w[2]+(((int)v*a)<<1);
    w[2]=((int)v*b)<<1;
    w[3]=w[4]+(((int)v*d)<<1);
    w[4]=((int)v*e)<<1;
    return(y);
}
```

There may be a few games that can be played with `v` to get a slightly better resolution.
This page left blank to give you room for the previous question.
6) Consider an FIR filter that has the following properties:
   - It is in Direct-Form (see figure above)
   - It has 15 $z^{-1}$ blocks.
   - The registers used to implement these $z^{-1}$ blocks are 8 bits in size as are the
     coefficients. Both are represented using Q7 notation.
   - The input and output of the filter are both represented in 8-bit Q7 notation.
   - The FIR filter samples data at a rate of 200 KHz.
   - The results of the multiplication are 16-bits and the result is exact (no overflow or
     loss of low-order bits occur).
   - The filter has linear phase in the passband.
   - The filter has a unity gain in the passband.
   - The filter has at least 30dB attenuation in the stopband.
   - The passband is from 0 to 30 KHz, the stopband starts at 35 KHz.

a) If the signal $0.3 \sin(2 \pi *300t)$ is put into the filter, what will be the phase and magnitude
   of the signal that comes out the other side? Your phase should be in radians and in the
   range of $-\pi$ to $\pi$. [5]

   **Magnitude=0.3, group delay=7.0\*0.005ms=.037ms.**
   **Phase=-0.037ms/3.333ms*2\pi = -.021\pi**
   **NOTE: 7.5 rather than 7.0 is actually the right value to use here…**

b) If the signal $\sin(2 \pi *40000t)$ is put into the filter, what is largest magnitude you would
   expect of the signal that comes out the other side (in steady state)? Show your work. [5]

   **0.0316**