MSE-Optimal Scalar Quantizers

densities and quantizers are symmetric about the origin, and densities have variance 1
data taken from Jayant and Noll, p. 134,135

<table>
<thead>
<tr>
<th>rate =</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>thresh</td>
<td>level</td>
<td>thresh</td>
<td>level</td>
</tr>
<tr>
<td>MSE =</td>
<td>0.25</td>
<td>0.0625</td>
<td>0.0156</td>
<td>0.00391</td>
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<tr>
<td>MSE =</td>
<td>0.363</td>
<td>0.117</td>
<td>0.0345</td>
<td>0.00951</td>
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<tr>
<td>MSE =</td>
<td>0.500</td>
<td>0.176</td>
<td>0.0544</td>
<td>0.0154</td>
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</tbody>
</table>
### Gaussian source density

<table>
<thead>
<tr>
<th>Rate</th>
<th>Unif Q</th>
<th>Opt Q</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.4</td>
<td>4.4</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>9.25</td>
<td>9.3</td>
<td>0.05</td>
</tr>
<tr>
<td>3</td>
<td>14.27</td>
<td>14.62</td>
<td>0.35</td>
</tr>
<tr>
<td>4</td>
<td>19.38</td>
<td>20.22</td>
<td>0.84</td>
</tr>
<tr>
<td>5</td>
<td>24.57</td>
<td>26.01</td>
<td>1.44</td>
</tr>
<tr>
<td>6</td>
<td>29.83</td>
<td>31.89</td>
<td>2.06</td>
</tr>
<tr>
<td>7</td>
<td>35.13</td>
<td>37.81</td>
<td>2.68</td>
</tr>
<tr>
<td>8</td>
<td>40.34</td>
<td></td>
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</tr>
</tbody>
</table>
Laplacian source density

SNR (dB)

Rate

Unif Q Opt Q Gain Panter Dite diff

1 3.01 3.01 0 -0.51 3.52

2 7.07 7.54 0.47 5.51 2.03

3 11.44 12.64 1.2 11.53 1.11

4 15.96 18.13 2.17 17.55 0.58

5 20.6 23.87 3.27 23.57 0.30

6 25.36 29.74 4.38 29.59 0.15

7 30.23 35.69 5.46 35.61 0.08

8 35.14
**uniform source density**

![Graph](image)

**Uniform density**

<table>
<thead>
<tr>
<th>Rate</th>
<th>Unif Q</th>
<th>Opt Q</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.02</td>
<td>6.02</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>12.04</td>
<td>12.04</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>18.06</td>
<td>18.06</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>24.08</td>
<td>24.08</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>30.1</td>
<td>30.1</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>36.12</td>
<td>36.12</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>42.14</td>
<td>42.14</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>48.17</td>
<td>48.17</td>
<td>0</td>
</tr>
</tbody>
</table>
nonuniform scalar quantization

SNR (dB) vs rate

- Uniform
- Gaussian
- Laplacian
uniform scalar quantization

SNR (dB) vs. rate for uniform, Gaussian, and Laplacian distributions.
Comparison: SQ-VL vs. SQ-FL

Why is SQ-VL so much better than SQ-FL for Laplacian?
Probably because with SQ-VL, we don't have to worry much about the overload region.
Comparison: SQ-VL vs. SQ-FL

<table>
<thead>
<tr>
<th></th>
<th>SQ-FL</th>
<th>SQ-VL</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_{sq,fl}(R) )</td>
<td>( \frac{1}{12} \sigma^2 \beta 2^{-2R} )</td>
<td>( \frac{1}{12} \sigma^2 \eta 2^{-2R} )</td>
</tr>
<tr>
<td>( S_{sq,fl}(R) )</td>
<td>( 6.02 R + 10 \log_{10} \frac{12}{\beta} )</td>
<td>( S_{sq,vl}(R) ) ( = 6.02 R + 10 \log_{10} \frac{12}{\eta} )</td>
</tr>
<tr>
<td>( \Lambda^*(x) = \text{constant} )</td>
<td></td>
<td>( \lambda^*(x) = c f_{X}^{1/3}(x) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>uniform</th>
<th>Gaussian</th>
<th>Laplacian</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>12</td>
<td>6\sqrt{3}\pi=32.65</td>
<td>54</td>
</tr>
<tr>
<td>( 10 \log_{10} \frac{12}{\beta} )</td>
<td>0 dB</td>
<td>-4.347 dB</td>
<td>-6.532 dB</td>
</tr>
<tr>
<td>( 10 \log_{10} \frac{12}{\eta} )</td>
<td>0 dB</td>
<td>-1.533 dB</td>
<td>-0.904 dB</td>
</tr>
<tr>
<td>VL gain</td>
<td>0 dB</td>
<td>2.81 dB</td>
<td>5.63 dB</td>
</tr>
<tr>
<td>( \eta = 2^{2h}/\sigma^2 )</td>
<td>12</td>
<td>2\pi e=17.08</td>
<td>2 e^2=14.78</td>
</tr>
<tr>
<td>( h )</td>
<td>( \frac{1}{2} \log_2 12 \sigma^2 )</td>
<td>( \frac{1}{2} \log_2 2\pi e \sigma^2 )</td>
<td>( \frac{1}{2} \log_2 2 e^2 \sigma^2 )</td>
</tr>
</tbody>
</table>

- Fact: \( \eta \leq \beta \). Equality holds iff \( f_X(x) \) has the same value wherever it is not zero, e.g. it is uniform. Proof comes later.
- As expected, optimal SQ-VL is at least as good as optimal SQ-FL. Indeed it is better, except for uniform densities.
- Also as expected, opt'l pt. dens. for SQ-VL makes cells have more disparate probabilities than opt'l pt dens. for SQ-FL. (Pr(cell cont'g x) is proportional to \( f_X(x) \) for SQ-VL, whereas it is proportional to \( f_X^{2/3}(x) \) for SQ-FL.)