EECS 455 Problem Set 1

Due: Friday September 17.

1. Let $X(t)$ be a zero mean wide sense stationary Gaussian random process with power spectral density
$$S_X(\omega) = \begin{cases} 1\,, & |\omega| \le 5 \\ 0\,, & |\omega| > 5\,. \end{cases}$$
Let $H(\omega) = 5$ for all ω be the transfer function of a linear system with input $X(t)$ and output $Y(t)$. Find the mean and variance of the output $Y(t)$.

2. A zero mean white Gaussian random process $X(t)$ with power spectral density $N_0/2$ is the input to a linear time invariant system.

   (a) The inpulse response of the linear system is
   $$h(t) = A\cos(\omega_c t)\, p_T(t)$$
   where $p_T(t)$ is 1 for $0 \le t \le T$ and is zero elsewhere. Also $\omega_c T = 2\pi n$ (an integer number of cycles in T seconds). Find the mena and variance of the output of the filter.

   (b) If the transfer function of (a different) linear system is triangular,
   $$H(f) = \begin{cases} T(fT+1), & -\frac{1}{T} < f <\le 0 \\ T(1-fT), & 0 \le f < \frac{1}{T} \\ 0, & \text{elsewhere,} \end{cases}$$
   find the mean and variance of the output of the filter.

3. Problem 4.9 of Text

4. Problem 4.11 of Text

5. A signal $s(t)$ of duation T consists of 15 consectuive pusles (of duration $T/15$) of amplitude ± 1. The sequence of amplitudes is (-1 -1 -1 -1 +1 -1 +1 -1 -1 +1 +1 -1 +1 +1 +1). Assume that this signal is input to a linear time-invariant system (filter) with impulse response $h(t) = s(T-t)$. Find (plot) the output of the filter.

6. Consider the UWB channel which goes from 3.1GHz to 10.6 GHz. Suppose the noise power spectral density is $N_0 = kT = 1.38 \times 10^{-23}(290) = 4 \times 10^{-21}$ Watts/Hz. The allowed transmitted power density is -41.3dBm/MHz. Suppose the received power is related to the transmitted power by
   $$P_r = P_t/d^4$$
   where the d is the distance in meters (independent of frequency). Compute the largest possible data rate that can be communicated reliably at a distance of 100 m and 1000 m.