EECS 455 Problem Set 2

Due: Friday September 24.

1. Problem 2.1 of Text

2. Problem 2.9 of Text

3. A communication system transmits one of three signals:

$$s_0(t) = A\cos\omega_c t\, p_T(t)$$
$$s_1(t) = 0$$
$$s_2(t) = -A\cos\omega_c t\, p_T(t)$$

over an additive white Gaussian noise channel with spectral density $N_0/2$. Let $r(t)$ denote the received signal ($r(t) = s_i(t) + n(t)$). The receiver computes the quantity

$$Z = \int_0^T r(t)\cos\omega_c t\,dt.$$

Assume $\omega_c T = 2\pi n$ for some integer n. Z is compared with a threshold γ and a threshold $-\gamma$. If $Z > \gamma$, the decision is made that $s_0(t)$ was sent. If $Z < -\gamma$, the decision is made that $s_2(t)$ was sent. If $-\gamma < Z < \gamma$ the the decision is made in favor of $s_1(t)$

   (a) Determine the three conditional probabilities of error: $P_{e,0} =$ probability of error given s_0 sent, $P_{e,1} =$probability of error given s_1 sent, and $P_{e,2}$

   (b) Determine the average error probability assuming that all three signals are equally probable of being transmitted.

4. A signal $s(t)$ of duration T consists of 15 consecutive pulses (of duration $T/15$) of amplitude ± 1. The sequence of amplitudes is (-1 -1 -1 -1 +1 -1 +1 -1 -1 +1 +1 -1 +1 +1 +1). Data is transmitted using this signal. The data sequence 011010 is transmitted in 6T seconds by transmitting the signal $s(t)$ for the first T seconds, $-s(t-T)$ during the interval $[T,2T]$, $-s(t-2T)$ during the interval $[2T,3T]$ and so on. The total signal transmitted is thus

$$s(t) - s(t-T) - s(t-2T) + s(t-3T) - s(t-4T) + s(t-5T)$$

Assume that this signal is input to a linear time-invariant system (filter) with impulse response $h(t) = s(T-t)$. Find (plot) the output of the filter.

5. A data signal consists of an infinite sequence of rectangular pulses of duration T. That is

$$s(t) = \sum_{l=-\infty}^{\infty} b_l p_T(t-lT)$$

where $p_T(t)$ is 1 for $0 \le t \le T$ and zero elsewhere. The data is represented by b_l and is either +1 or -1. The signal is filtered by a low pass RC filter with impulse response

$$h(t) = \alpha e^{-\alpha t} u(t)$$

where $u(t)$ is one for $t > 0$ and is 0 otherwise. The filter output is sampled every T seconds. Find the largest possible value (over all possible data sequences) of the sampled output and the smallest possible positive value for the sampled output.

6. White Gaussian noise with power spectral density $N_0/2$ is the input to an RC filter with impulse response

$$h(t) = e^{-\alpha t} u(t)$$

where $u(t)$ is one for $t > 0$ and is 0 otherwise. Find the variance of the noise at the output of the filter.