EECS 455 Problem Set 4

Due: Friday October 8.

1. A binary communications system operates over an AWGN channel with spectral density $N_0/2$. The transmitted signals are given by

$$s_0(t) = Ap_{T/2}(t) - Ap_{T/2}(t - T/2)$$

$$s_1(t) = 0$$

   (a) Give an expression (in terms of A, T, N_0, and $Q(x)$) for the minimum average error probability when the two signals are transmitted with equal probabilities (i.e. $\pi_0 = \pi_1$).

   (b) Assume that the optimum filter $h(t)$ is used for the signals above but the signal $s_0(t)$ is actually given by

$$s_0(t) = cAp_{T/2}(t) - cAp_{T/2}(t - T/2)$$

   instead of the one given above while $s_1(t)$ is the same. Give an expression (in terms of c, A, T, N_0 and Φ) for the average error probability if $\pi_0 = \pi_1$.

2. For each signal set below, find the minmax error probability for binary communication via an AWGN channel (spectral density $N_0/2$). Assume for each signal that the receiver consists of an ideal matched filter, a sampler which samples at an optimal time, and a threshold device with the optimum minimax threshold.

   (a)

$$s_0(t) = \begin{cases} A & 0 \leq t < T/3 \\ -A & 2T/3 \leq t < T \\ 0 & \text{elsewhere} \end{cases}$$

$$s_1(t) = \begin{cases} A & 2T/3 \leq t < T \\ -A & T/3 \leq t < 2T/3 \\ 0 & \text{elsewhere} \end{cases}$$

   (b)

$$s_0(t) = A|\cos\omega_0 t|p_T(t)$$

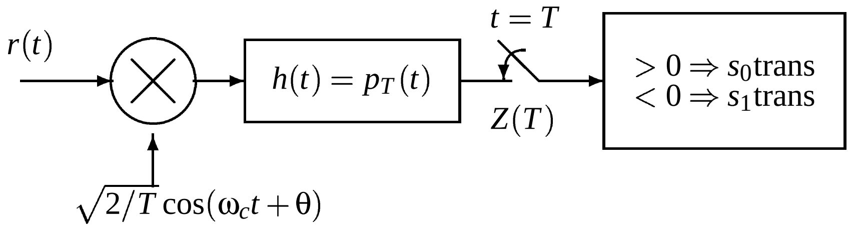
$$s_1(t) = A|\sin\omega_0 t|p_T(t)$$

   (c)

$$s_0(t) = A(1+\cos\omega_0 t)p_T(t)$$

$$s_1(t) = A(1+\sin\omega_0 t)p_T(t)$$

   In parts (b) and (c), assume $\omega_0 T = 2\pi n$ for some integer n.



3. Consider a binary communication system that transmits one of two signal $s_0(t)$ and $s_1(t)$ over an additive white Gaussian noise channel (spectral density $N_0/2$) where

$$s_0(t) = A_0 p_{T/2}(t)$$

and

$$s_1(t) = A_1 p_{T/2}(t - T/2)\ ;$$

that is s_0 is a pulse of amplitude A_0 from 0 to $T/2$ and s_1 is a pulse of amplitude A_1 from $T/2$ to T. The receiver shown below consist of a filter $h(t)$ which is sampled at time T and a threshold device.

   (a) If $h(t) = p_{T/2}(t) - p_{T/2}(t - T/2)$ find the threshold that will minimize the *maximum* of the error probabilities $P_{e,0}$ and $P_{e,1}$. Find the error probability $P_{e,m} = \max(P_{e,0}, P_{e,1})$.

   (b) Find the matched filter for the same system and find the corresponding threshold that minimizes $P_{e,m}$. Also find $P_{e,m}$.

   (c) If the s_0 is transmitted with probability $\pi_0 = 1/4$ and s_1 is transmitted with probability $\pi_1 = 3/4$ and the filter of part (a) is used find the threshold that minimizes the *average* error probability. What is the average error probability with this threshold?

   (d) Repeat part (c) if the matched filter is used.

4. A binary communication system transmits one of two equally likely signals $s_0(t)$ and $s_1(t)$ of duration T given by

$$s_i(t) = A(-1)^i \cos(\omega_0 t) p_T(t)$$

The noise in the system is white Gaussian noise with power spectral density $N_0/2$. The receiver shown below is used to demodulate the signal. However, as shown, the phase of the received signal is not know completely accurately. In fact there is a discrepancy of θ radians between the received signal (in the absence of noise) and the local reference signal. Determine the error probability at the output of the demodulator as a function of θ. (Assume $\omega_c T = 2\pi n$ for some integer n).

5. A transmitter uses one of four equally likely signals to convey two bits of information. The signals are $s_0(t), s_1(t), s_2(t)$, and $s_3(t)$. The following table indicates the mapping between information bits and signals.

Information bits	Signals
00	$s_0(t)$
01	$s_1(t)$
11	$s_2(t)$
10	$s_3(t)$

The signals are received in the presence of white Gaussian noise with power spectral density $N_0/2$. The receiver consist of a filter, a sampler and a threshold device. The sampled output is denoted by $Z(T)$. The threshold device uses the following table to make a decision.

$Z(T) > 2$	decide $s_0(t)$ transmitted
$0 < Z(T) < 2$	decide $s_1(t)$ transmitted
$-2 < Z(T) < 0$	decide $s_2(t)$ transmitted
$Z(T) < -2$	decide $s_2(t)$ transmitted

It is known that the output of the filter due to the signals alone at the sampling time is

$$\begin{aligned}\hat{s}_0(T) &= +3\\ \hat{s}_1(T) &= +1\\ \hat{s}_2(T) &= -1\\ \hat{s}_3(T) &= -3\end{aligned}$$

It is also know that the variance of the output due to noise alone is $\sigma^2 = 4$.

(a) Determine the probability of error given signal $s_i(t)$ is transmitted for $i = 0, 1, 2, 3$. (Express your answers in terms of the Q function).

(b) Determine the probability that the receiver makes an error in the first bit given the first bit is 0. (The first bit being 0 means either signal $s_0(t)$ was transmitted or $s_1(t)$ was transmitted).

Hint: Let b_0 represent the first bit. Let $\hat{b}_0$ represent the decision on the first bit. The probability of error for the first bit is then $P\{\hat{b}_0 = 1 | b_0 = 0\}$.